Заняття 7
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Exercise 2 (3.1). (IM : Integrated Monopolist)

$c>0;\quad X(p)=1-p \quad 0<p<1;$

$\pi^{IM}(p) = p\,X(p) - c\cdot X(p)$

$\pi^{IM}(p) = p(1-p) - c(1-p);$

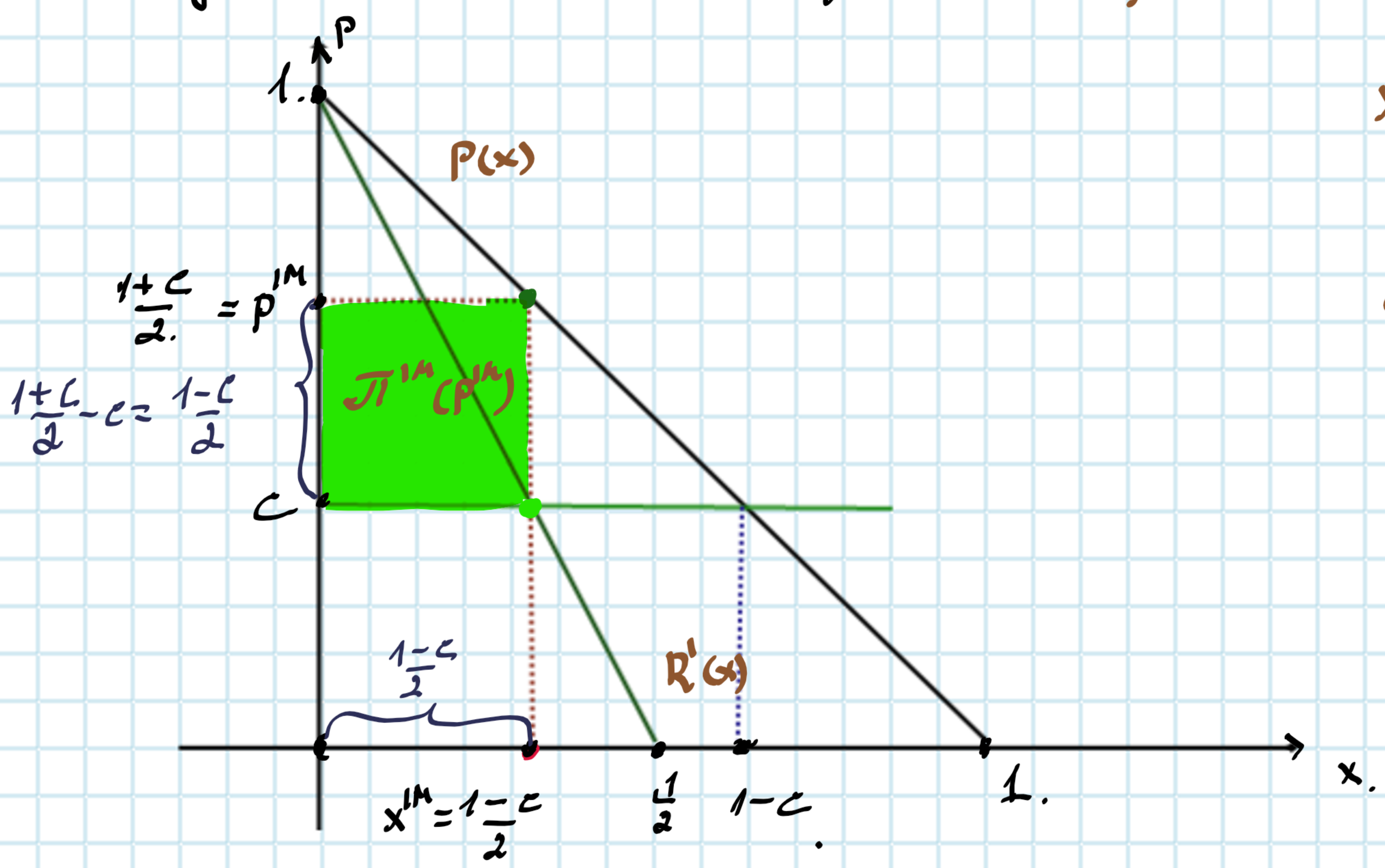
$$\pi^{IM}(p) = (p-c)(1-p) \qquad (1)$$

$\pi^{IM}(p) \to \max_p \; \{\Leftrightarrow\} \; \pi^{IM}_p(p) = 0 \; \{\Leftrightarrow\} \; \pi^{IM}_p = 1-p-p+c = 1-2p+c=0 \; \{\Rightarrow\}$

$$p^{IM} = \frac{1+c}{2} \qquad (2)$$

$$\pi^{IM}(p^{IM}) = \left(\frac{1+c}{2}-c\right)\left(1-\frac{1+c}{2}\right) = \frac{(1-c)^2}{4} \qquad (3)$$

SOC: $\pi^{IM}_{pp} \overset{(1)}{=} (1-2p+c)'_p = -2 \;\Rightarrow\; \pi^{IM}_{pp}(p^{IM}) = -2<0 \; \{\Rightarrow\}$ строго увігнута
(за умови існування вн. розв.): $p^{IM} = \arg\max_p \pi^{IM}(p)$



$X(p) = 1-p \;\{\Leftrightarrow\}\; P(x) = 1-x.$

$R(p) = (1-p)\,p \;\Rightarrow\; R'(p) = 1-2p.$

$C(x) = c\cdot x \;\Rightarrow\; c'(x) = c.$

$R'(x) = c'(x)$

$x^{IM} = 1-p^{IM} = 1-\frac{1+c}{2} = \frac{1-c}{2}$

$\pi^{IM}(p^{IM}) = (1-p^{IM})(p^{IM}-c) = \left(1-\frac{1+c}{2}\right)\left(\frac{1+c}{2}-c\right) = \frac{1-c}{2}\cdot\frac{1-c}{2}.$

Exercise 2 (3.2). (R : Retailer)

I крок.

$$\pi^R(p) = p\,X(p) - s\cdot X(p)$$

$\pi^R(p) = p(1-p) - s(1-p)$

$$\pi^R(p) = (p-s)(1-p) \qquad (4)$$

$\pi^R(p) \to \max_p \; \{\Leftrightarrow\} \; \pi^R_p(p) = 0 \; \{\Leftrightarrow\} \; 1-p-p+s=0 \; \{\Rightarrow\} \; 1-2p+s=0$

$$p^R(s) = \frac{1+s}{2} \qquad (5)$$

$$\pi^R(p^R(s),s) \overset{(5)\to(4)}{=} \left(\frac{1+s}{2}-s\right)\left(1-\frac{1+s}{2}\right) = \frac{(1-s)^2}{4} \qquad (6)$$

SOC: $\pi^R_{pp} = (1-2p+s)'_p = -2<0;\quad \pi^R_{pp}(p^R(s)) = -2<0 \; \{\Rightarrow\}$

$\pi^R(p^R(s),s)$ - строго увігнута $\{\Rightarrow\}$ $p^R(s) = \frac{1+s}{2} = \arg\max_p \pi^R(p^R(s),s)$

II крок. M : Manufacture.

$$\pi^M(s) = s\cdot X(p^R(s)) - c\cdot X(p^R(s)) \qquad (7)$$

$$\pi^M(s) = s\cdot(1-p^R(s)) - c\,(1-p^R(s)) = (s-c)(1-p^R(s)) \overset{(5)}{=} (s-c)\left(1-\tfrac{1+s}{2}\right):$$

$$\pi^M(s) = (s-c)\,\frac{1-s}{2}. \qquad (8)$$

$$\pi^M(s) \to \max_s \Rightarrow \left\{ FOC: \ \pi^M(s)=0 \iff \tfrac{1}{2}(1-s-s+c)=0 \right\} \iff$$

$$s^M = \frac{1+c}{2} \qquad (9)$$

$$\pi^M(s^M) = \max_s \pi^M(s) \overset{(8)\to(9)}{=} \tfrac{1}{2}\left(\tfrac{1+c}{2}-c\right)\left(1-\tfrac{1+c}{2}\right) = \tfrac{1}{8}(1-c)^2$$

$$\pi^M(s^M) = \tfrac{1}{8}(1-c)^2 \qquad (10)$$

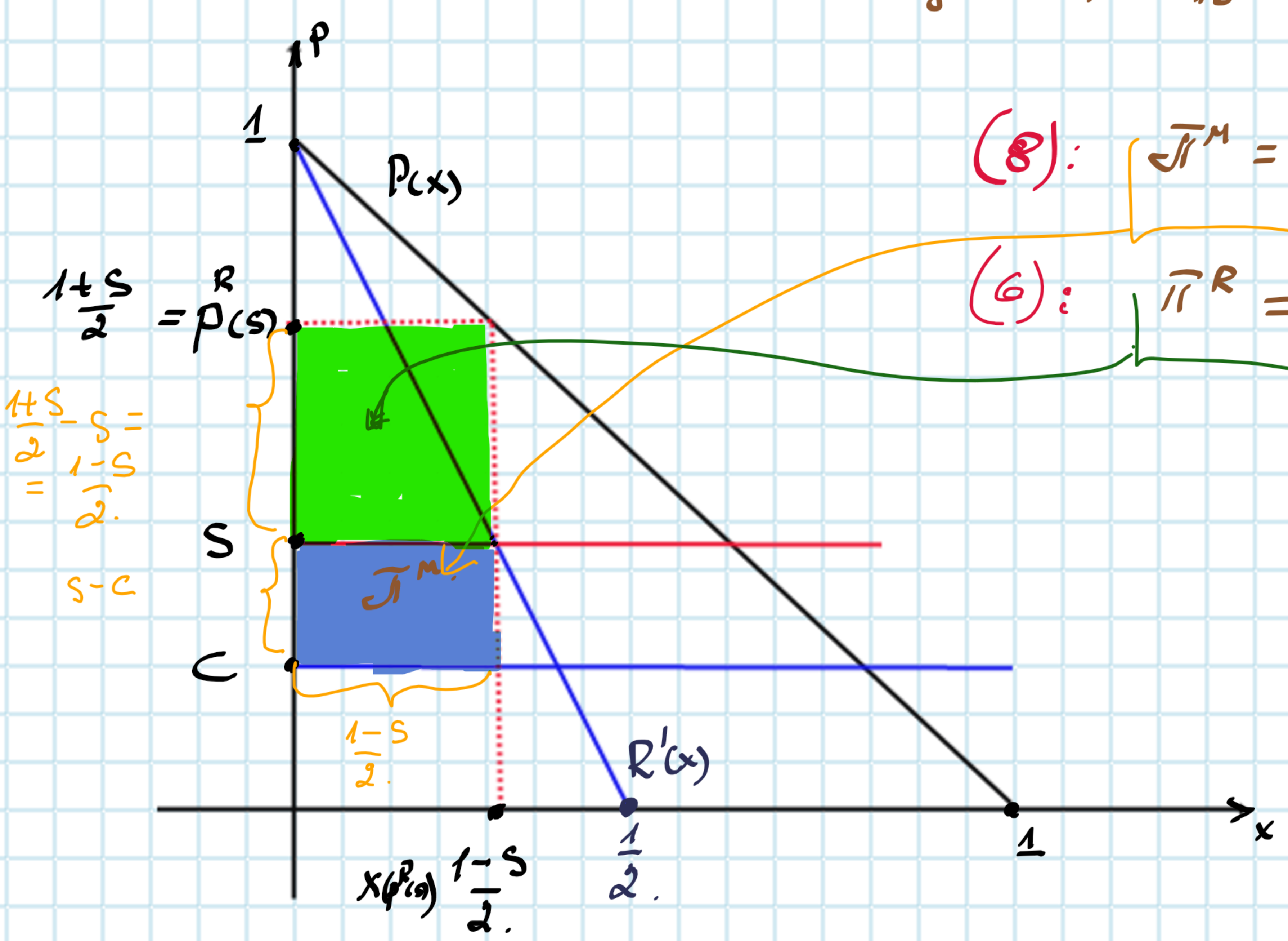
SOC: $\pi^M_{ss}(s) = \left(\tfrac{1}{2}(1-2s+c)\right)'_{ss} = -1 < 0 \Rightarrow \pi^M(s)$ – строго увігнута:

$$s^M = \arg\max_s \pi^M(s)$$

Обчислимо прибуток неінтегрованого монополіста і порівняємо його із прибутком інтегрованого.

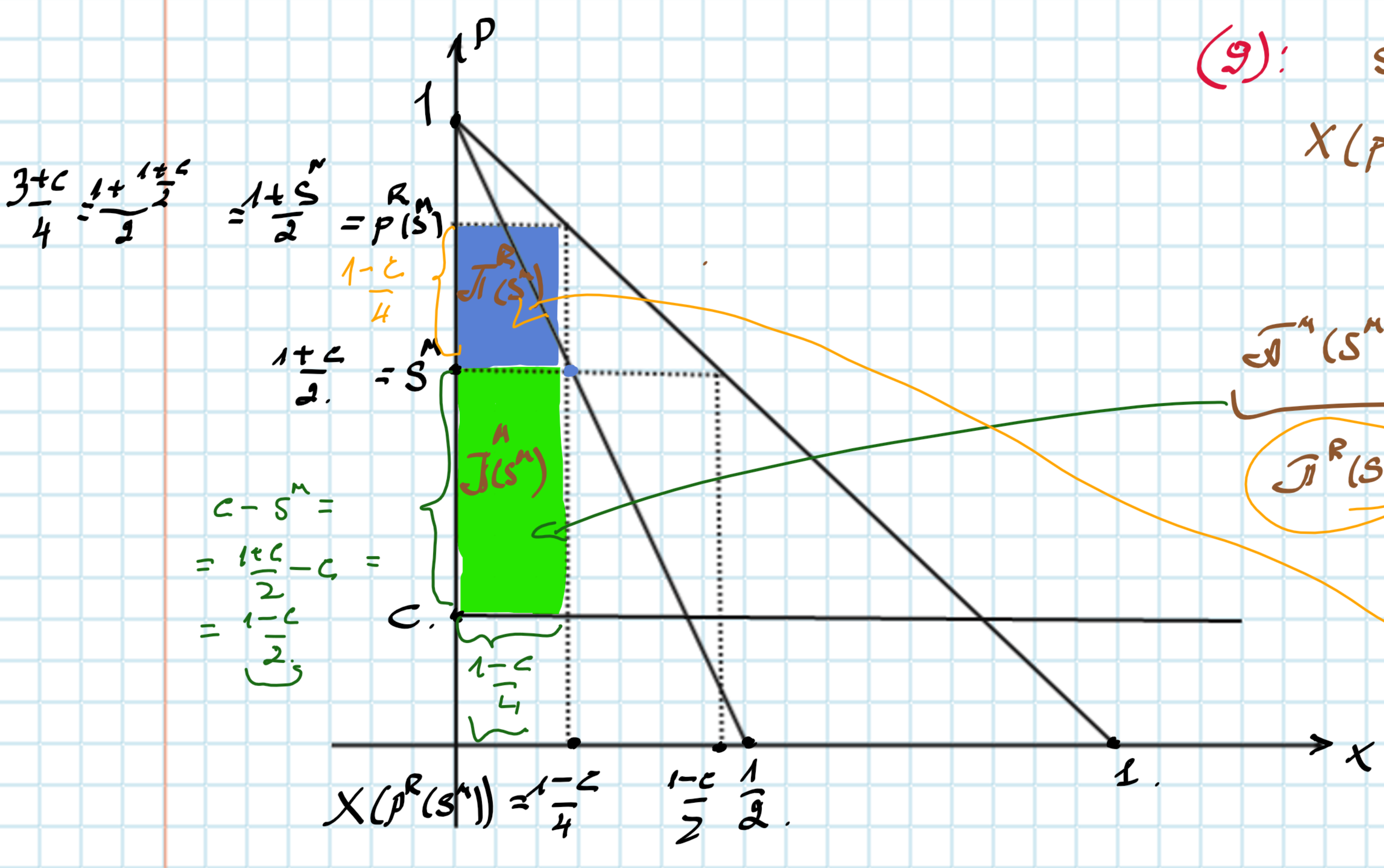
$$\pi^M(s^M) + \pi^R(s^M) \overset{(10),(6)}{=} \tfrac{1}{8}(1-c)^2 + \tfrac{1}{4}(1-s^M)^2 \overset{(9)}{=} \tfrac{1}{8}(1-c)^2 + \tfrac{1}{4}\left(1-\tfrac{1+c}{2}\right)^2 =$$

$$= \tfrac{1}{8}(1-c)^2 + \tfrac{1}{16}(1-c)^2 = \tfrac{3}{16}(1-c)^2 \overset{(3)}{<} \tfrac{1}{4}(1-c)^2 = \pi^{IM}(p^{IM}).$$



(8): $\pi^M = \frac{1-s}{2}\cdot(s-c)$

(6): $\pi^R = \frac{(1-s)^2}{4}$



(9): $s^M = \frac{1+c}{2}$.

$$X(p^R(s^M)) = 1 - p^R(s^M) = 1 - \frac{3+c}{4} = \frac{1-c}{4}$$

$$\pi^M(s^M) \overset{(10)}{=} \tfrac{1}{8}(1-c)^2 = \frac{1-c}{2}\cdot\frac{1-c}{4}$$

$$\pi^R(s^M) \overset{(6)}{=} \frac{(1-s^M)^2}{4} \overset{(9)}{=} \frac{\left(1-\frac{1+c}{2}\right)^2}{4} = \frac{(1-c)^2}{16} =$$

$$= \frac{1-c}{4}\cdot\left(p^R(s^M) - s^M\right) =$$

$$= \frac{1-c}{4}\left(\frac{3+c}{4} - \frac{1+c}{2}\right) =$$

$$= \frac{1-c}{4}\cdot\frac{3+c-2-2c}{4} = \frac{1-c}{4}\cdot\frac{1-c}{4} =$$

$$= \left(\frac{1-c}{4}\right)^2$$