Заняття 7
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Exercise 2 (3.1). (IM : Integrated Monopolist)

$c \geq 0$; $\quad X(p) = 1 - p \quad 0 < p < 1$;

$\Pi^{IM}(p) = p\,X(p) - c \cdot X(p)$

$\Pi^{IM}(p) = p(1-p) - c(1-p)$;

$$\Pi^{IM}(p) = (p-c)(1-p) \qquad (1)$$

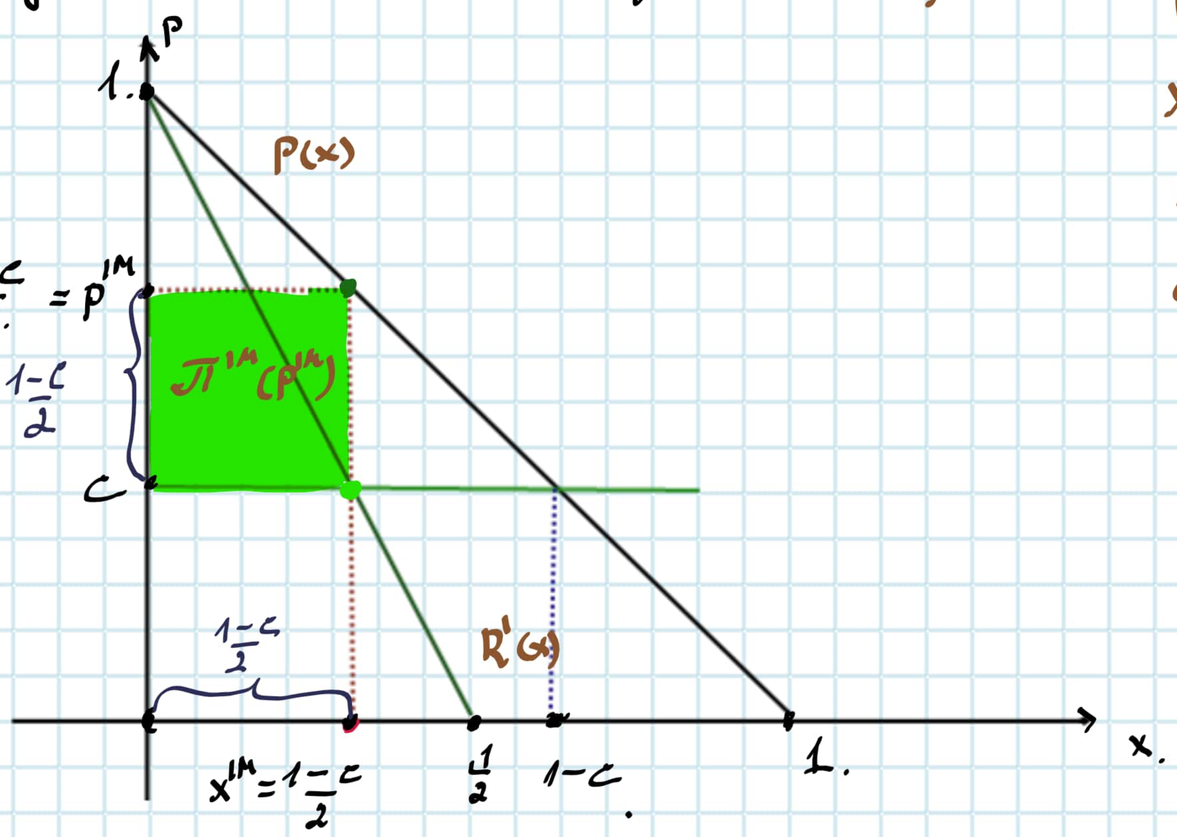
$\Pi^{IM}(M) \to \max\limits_{p}$: $\{ \Leftrightarrow \}$ $\Pi^{IM}_{p}(p) = 0$ $\{ \Leftrightarrow \}$ $\Pi^{IM}_{p} = 1 - p - p + c = 1 - 2p + c = 0$ $\{ \Rightarrow \}$

$$p^{IM} = \frac{1+c}{2} \qquad (2)$$

$$\Pi^{IM}(p^{IM}) = \left(\frac{1+c}{2} - c\right)\left(1 - \frac{1+c}{2}\right) = \frac{(1-c)^2}{4} \qquad (3)$$

SOC: $\Pi^{IM}_{pp} \overset{(1)}{=} (1 - 2p + c)'_p = -2 \Rightarrow \Pi^{IM}_{pp}(p^{IM}) = -2 < 0$ $\{ \Rightarrow \}$ строго увігнута

(за умови існування вн. розв.): $p^{IM} = \arg\max\limits_{p} \Pi^{IM}(p)$



$X(p) = 1 - p$ $\{ \Rightarrow \}$ $P(x) = 1 - x$.

$R(p) = (1-p)\,p \Rightarrow R'(p) = 1 - 2p$.

$C(x) = c \cdot x \Rightarrow C'(x) = c$.

$R'(x) = C'(x)$

$x^{IM} = 1 - p^{IM} = 1 - \frac{1+c}{2} = \frac{1-c}{2}$

$\Pi^{IM}(p^{IM}) = (1 - p^{IM})(p^{IM} - c) = \left(1 - \frac{1+c}{2}\right)\left(\frac{1+c}{2} - c\right) = \frac{1-c}{2} \cdot \frac{1-c}{2}$

Exercise 2 (3.2). (R : Retailer)

I крок.

$$\Pi^{R}(p) = p\,X(p) - s \cdot X(p)$$

$\Pi^{R}(p) = p(1-p) - s(1-p)$

$$\Pi^{R}(p) = (p-s)(1-p) \qquad (4)$$

$\Pi^{R}(p) \to \max\limits_{p}$ $\{ \Leftrightarrow \}$ $\Pi^{R}_{p}(p) = 0$ $\{ \Leftrightarrow \}$ $1 - p - p + s = 0$ $\{ \Rightarrow \}$ $1 - 2p + s = 0$

$$p^{R}(s) = \frac{1+s}{2} \qquad (5)$$

$$\Pi^{R}(p^{R}(s), s) \overset{(5)\to(4)}{=} \left(\frac{1+s}{2} - s\right)\left(1 - \frac{1+s}{2}\right) = \frac{(1-s)^2}{4} \qquad (6)$$

SOC: $\Pi^{R}_{pp} = (1 - 2p + s)'_p = -2 < 0$; $\Pi^{R}_{pp}(p^{R}(s)) = -2 < 0$ $\{ \Rightarrow \}$

$\Pi^{R}(p^{R}(s), s)$ - строго увігнута $\{ \Rightarrow \}$ $p^{R}(s) = \frac{1+s}{2} = \arg\max\limits_{p} \Pi^{R}(p^{R}(s), s)$

II крок. M : Manufacture.

$$\Pi^M(s) = s \cdot X(p^R(s)) - c \cdot X(p^R(s)) \tag{7}$$

$$\Pi^M(s) = s\cdot(1-p^R(s)) - c\,(1-p^R(s)) = (s-c)(1-p^R(s)) \overset{(5)}{=} (s-c)\left(1-\frac{1+s}{2}\right):$$

$$\Pi^M(s) = (s-c)\,\frac{1-s}{2}. \tag{8}$$

$$\Pi^M(s) \to \max_s \Rightarrow \left\{ \text{FOC: } \Pi^M(s) = 0 \Leftrightarrow \frac{1}{2}(1-s-s+c) = 0 \right\} \Leftrightarrow$$

$$s^M = \frac{1+c}{2} \tag{9}$$

$$\Pi^M(s^M) = \max_s \Pi^M(s) \overset{(8)\to(9)}{=} \frac{1}{2}\left(\frac{1+c}{2} - c\right)\left(1-\frac{1+c}{2}\right) = \frac{1}{8}(1-c)^2$$

$$\Pi^M(s^M) = \frac{1}{8}(1-c)^2 \tag{10}$$

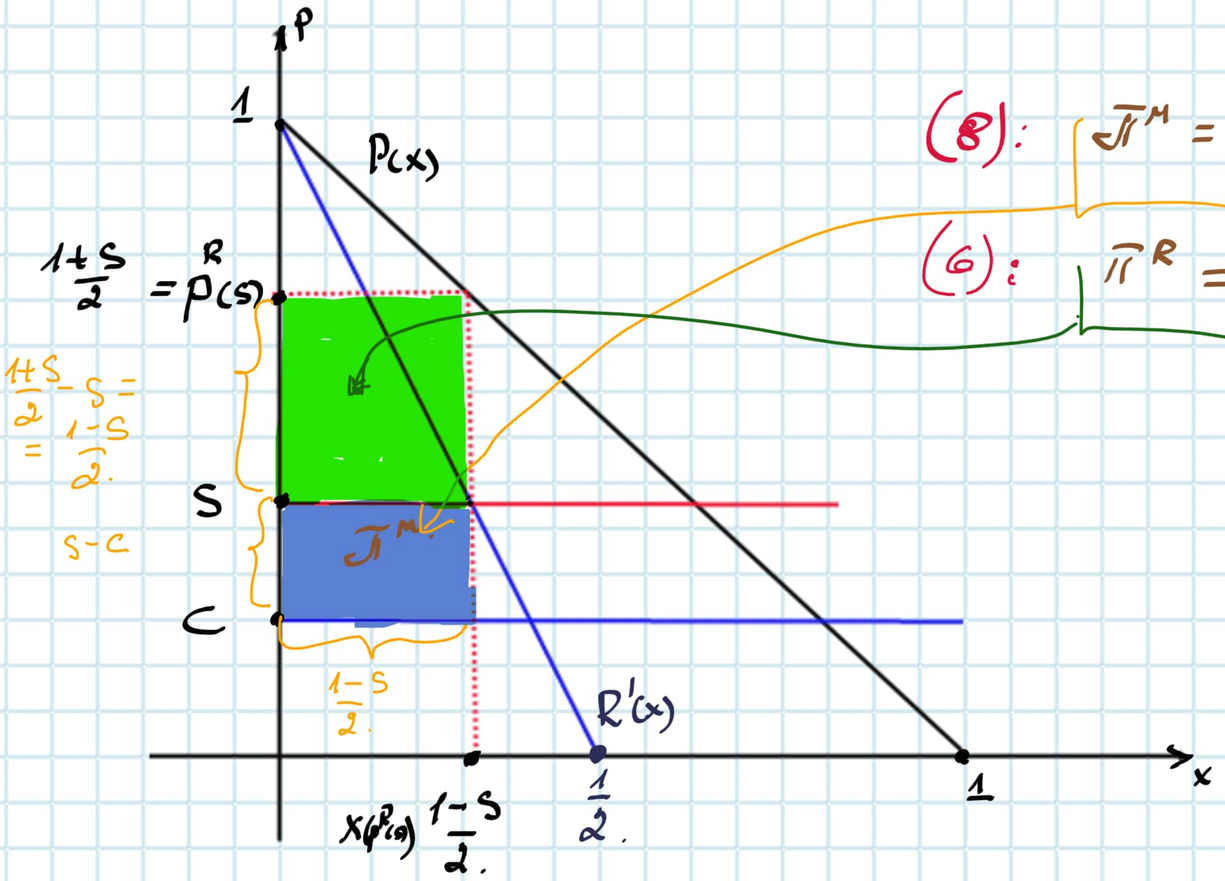
SOC: $\Pi^M_{ss}(s) = \left(\frac{1}{2}(1-2s+c)\right)'_{ss} = -1 < 0 \Rightarrow$ $\Pi^M(s)$ – строго увігнута:

$$s^M = \arg\max_s \Pi^M(s)$$

Обчислимо прибуток неінтегрованого монополіста і порівняємо його із прибутком інтегрованого:

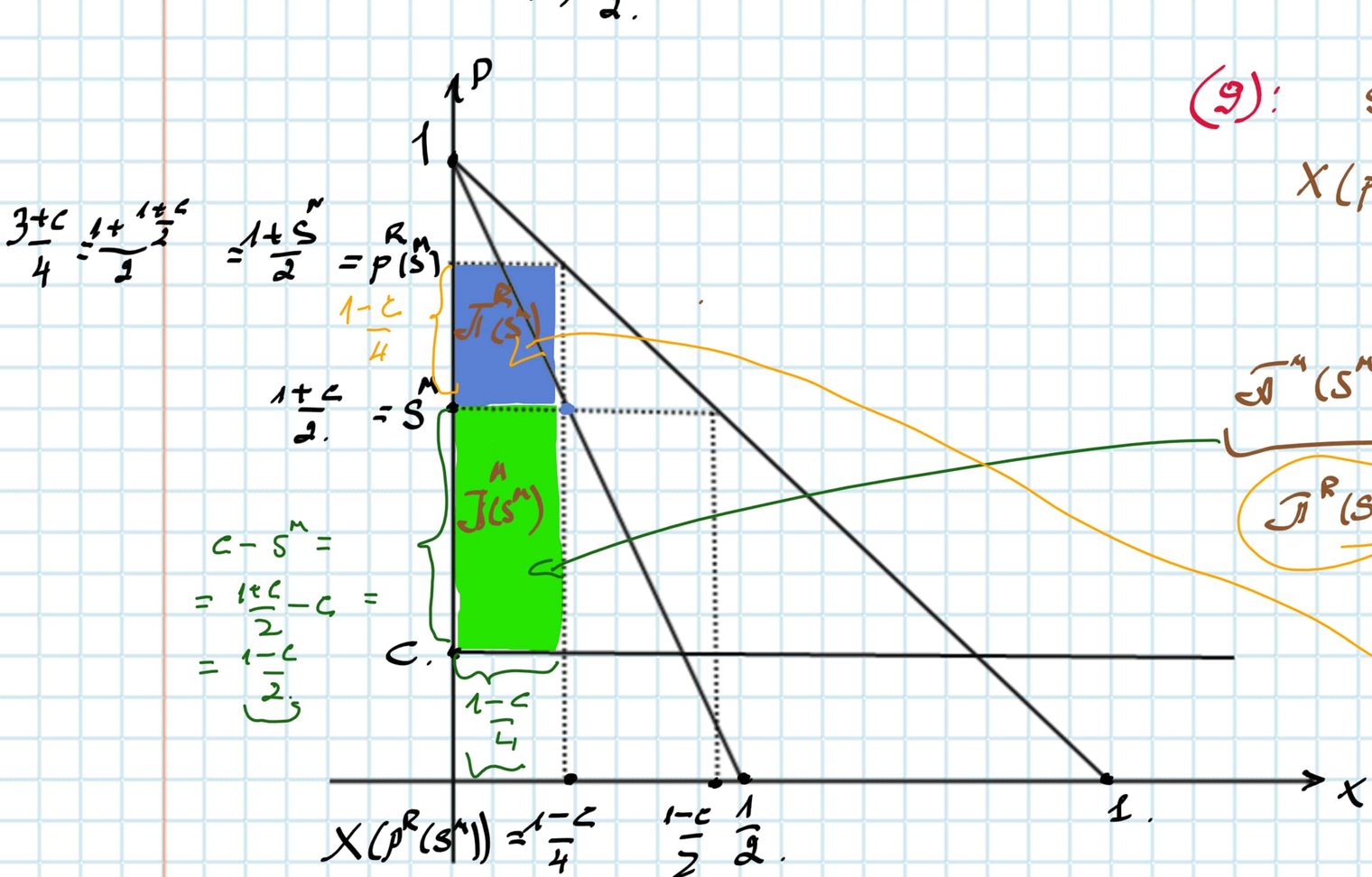
$$\Pi^M(s^M) + \Pi^R(s^M) \overset{(10),(6)}{=} \frac{1}{8}(1-c)^2 + \frac{1}{4}(1-s^M)^2 \overset{(9)}{=} \frac{1}{8}(1-c)^2 + \frac{1}{4}\left(1-\frac{1+c}{2}\right)^2 =$$

$$= \frac{1}{8}(1-c)^2 + \frac{1}{16}(1-c)^2 = \frac{3}{16}(1-c)^2 \overset{(3)}{<} \frac{1}{4}(1-c)^2 = \Pi'^M(p'^M)$$



(8): $\Pi^M = \frac{1-s}{2}\cdot(s-c)$

(6): $\Pi^R = \frac{(1-s)^2}{4}$



(9): $s^M = \frac{1+c}{2}$.

$$X(p^R(s^M)) = 1 - p^R(s^M) = 1 - \frac{3+c}{4} = \frac{1-c}{4}$$

$$\Pi^M(s^M) \overset{(10)}{=} \frac{1}{8}(1-c)^2 = \frac{1-c}{2}\cdot\frac{1-c}{4}$$

$$\Pi^R(s^M) \overset{(6)}{=} \frac{(1-s^M)^2}{4} \overset{(9)}{=} \frac{\left(1-\frac{1+c}{2}\right)^2}{4} = \frac{(1-c)^2}{16} =$$

$$= \frac{1-c}{4}\cdot\left(p^R(s^M) - s^M\right) =$$

$$= \frac{1-c}{4}\left(\frac{3+c}{4} - \frac{1+c}{2}\right) =$$

$$= \frac{1-c}{4}\,\frac{3+c-2-2c}{4} = \frac{1-c}{4}\,\frac{1-c}{4} =$$

$$= \frac{(1-c)^2}{4}$$