Заняття 5
05.10.2021

Exercise 2

1. $\Pi(x,q) = P(x,q)\cdot x - C(x,q)$

$\Pi(x,q) \to \max_{x,q}$

1. $\frac{\partial \Pi}{\partial x} = 0 \Rightarrow \left((1-x+q)\cdot x - q^2 - \frac{1}{2}x\right)'_x = 1 - 2x + q - \frac{1}{2} = 0$

$\frac{\partial \Pi}{\partial q} = 0 \Rightarrow \left(x - x^2 + qx - q^2 - \frac{1}{2}x\right)'_q = x - 2q = 0$

$x = 2q$

$1 - 4q + q - \frac{1}{2} = 0$ $\boxed{q^M = \frac{1}{6};\ x^M = \frac{1}{3}}$

2. $H(x,q) = \begin{pmatrix} \frac{\partial^2 \Pi}{\partial x^2} & \frac{\partial^2 \Pi}{\partial x \partial q} \\ \frac{\partial^2 \Pi}{\partial q \partial x} & \frac{\partial^2 \Pi}{\partial q^2} \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$

$x = (x_1, \ldots, x_n)$

$x^0 = (x_1^0, \ldots, x_n^0)$

$f \in C^2(Q_\delta(x^0))$:

$f(x) = f(x^0) + \frac{1}{1!} d f(x^0) + \frac{1}{2!} d^2 f(x^0) + o(\rho^2(dx))$

$\forall x \in Q_\delta(x^0)$

$f(x) - f(x^0) \approx \frac{1}{2} d^2 f(x^0)$

$\wedge\ 0 \iff \wedge\ 0$

! max

$d^2 f(x^0) = \frac{\partial^2 f(x^0)}{\partial x_1^2} dx_1^2 + \ldots + \frac{\partial^2 f(x^0)}{\partial x_n^2} dx_n^2 + 2\frac{\partial^2 f(x^0)}{\partial x_1 \partial x_2} dx_1 dx_2 + \ldots + 2\frac{\partial^2 f(x^0)}{\partial x_n \partial x_{n-1}} dx_n dx_{n-1}$

Знаковизначеність $H(x^M, q^M) = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$

$\Delta_1 = -2 < 0$; $\Delta_2 = \det H(x^M, q^M) = 3 > 0$ $\overset{\text{крит}}{\iff}$ Сильв.

1, 2. $\Rightarrow \boxed{x^M = \frac{1}{3};\ q^M = \frac{1}{6}}$ $\Pi(x,q)$.

2. $W(x,q) = \int_0^x P(s)ds - C(q) = \int_0^x (1 - s + q)\, ds - q^2 - \frac{1}{2}x =$

$= \left(s - \frac{s^2}{2} + qs\right)\Big|_0^x - q^2 - \frac{1}{2}x = \frac{1}{2}x - \frac{x^2}{2} + xq - q^2$

$W_q^{(x^M,q)} = x^M - 2q$; $x^M = \frac{1}{3} \rightarrow$

$q^* = \frac{x^M}{2}$; $q^* = q^M = \frac{1}{6}$

$(x^*, q^*) - ?$ $W(x,q) \to \max$

$W^M = \frac{5}{36}$

$W_q = x - 2q = 0 \Rightarrow x = 2q$

$W_x = \frac{1}{2} - x + q = 0$

$\frac{1}{2} - 2q + q = 0$

$q = \frac{1}{2}$; $x = 1$

$(x^*; q^*) = (1; \frac{1}{2})$

$W^* = \frac{1}{4}$