Заняття 5
05.10.2021

Exercise 2

1. $\tilde{\Pi}(x,q) = P(x,q)\cdot x - C(x,q)$

$\tilde{\Pi}(x,q) \to \max_{x,q}$

1) $\frac{\partial \tilde{\Pi}}{\partial x} = 0 \Rightarrow \left((1-x+q)\cdot x - q^2 - \frac{1}{2}x\right)'_x =$

$= 1 - 2x + q - \frac{1}{2} = 0$

$\frac{\partial \tilde{\Pi}}{\partial q} = 0 \Rightarrow \left(x - x^2 + qx - q^2 - \frac{1}{2}x\right)'_q =$

$= x - 2q = 0$

$x = 2q$

$1 - 4q + q - \frac{1}{2} = 0$

$q^M = \frac{1}{6}; \; x^M = \frac{1}{3}$

2) $H(x,q) = \begin{pmatrix} \frac{\partial^2 \tilde{\Pi}}{\partial x^2} & \frac{\partial^2 \tilde{\Pi}}{\partial x \partial q} \\ \frac{\partial^2 \tilde{\Pi}}{\partial q \partial x} & \frac{\partial^2 \tilde{\Pi}}{\partial q^2} \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$

$x = (x_1, \dots, x_n)$

$x^0 = (x_1^0, \dots, x_n^0)$

$f \in C^2(Q_\delta(x^0))$:

$f(x) = f(x^0) + \frac{1}{1!} df(x^0) + \frac{1}{2!} d^2 f(x^0) + o(\rho^2(dx))$

$\forall x \in Q_\delta(x^0)$

$f(x) - f(x^0) \simeq \frac{1}{2} d^2 f(x^0)$

$\wedge \quad <=> \quad \wedge$

$0 \qquad\qquad 0$

$d^2 f(x^0) = \frac{\partial^2 f(x^0)}{\partial x_1^2} dx_1^2 + \dots + \frac{\partial^2 f(x^0)}{\partial x_n^2} dx_n^2 + 2\frac{\partial^2 f(x^0)}{\partial x_1 \partial x_2} dx_1 dx_2 + \dots + 2\frac{\partial^2 f(x^0)}{\partial x_n \partial x_{n-1}} dx_n dx_{n-1}$

! max

Знаковизначеність $H(x^M, q^M) = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$

$\Delta_1 = -2 < 0 \; ; \; \Delta_2 = \det H(x^M, q^M) = 3 > 0$

кут. $\Leftrightarrow$ Silv.

1, 2 $\Rightarrow$ $x^M = \frac{1}{3}; \; q^M = \frac{1}{6}$ $\quad \Pi(x,q)$.

2. $W(x,q) = \int_0^x P(s)ds - C(x) = \int_0^x (1 - s + q)\,ds - q^2 - \frac{1}{2}x =$

$= \left(s - \frac{s^2}{2} + qs\right)\Big|_0^x - q^2 - \frac{1}{2}x = \frac{1}{2}x - \frac{x^2}{2} + xq - q^2$

$W_q = x - 2q \; ; \; x^M = \frac{1}{3} \to$

$q^* = \frac{x^M}{2} \; ; \; q^* = q^M = \frac{1}{6}$.

$(x^*, q^*) - ?$ $\quad W(x,q) \to \max$.

$W^M = \frac{5}{36}$

4) $W_q = x - 2q = 0 \quad \Rightarrow x = 2q$

$W_x = \frac{1}{2} - x + q = 0$

$\frac{1}{2} - 2q + q = 0$

$q = \frac{1}{2}; \; x = 1$

$(x^*; q^*) = (1; \frac{1}{2})$

$W^* = \frac{1}{4}$