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Financial and Insurance Mathematics

Master thesis

ECONOMETRIC MODELLING OF MONETARY POLICY DESIGN FOCUSED ON INFLATION EXPECTATION, MULTILATERAL PRODUCTIVITY AND PRICE COMPETITIVENESS

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Contents

Introduction

Inflation is a quantitative measure of the rate at which the average price level of a basket of selected goods and services in an economy increases over a period of time. The demand for real balances is described as $L(i, Y)$, where *i* is the nominal interest rate and *Y* is real income. $L_i < 0$ and $L_Y > 0$, which means if nominal interest rate increases then demand for real balances decreases and if real income increases then demand for real balances also increases. The following equality is a condition of equilibrium in the money market

$$\frac{M}{P} = L(i, Y) \tag{1.1}$$

where P - price level, M - money stock. From the last condition it follows that the price level is defined as

$$P = \frac{M}{L(i,Y)} \,. \tag{1.2}$$

The price level can increase due to rises in interest rate and money supply or decreases in money demand. Economists consider the most important factor: growth of the money supply. An explanation for this is that none of the other factors can lead to such large rises in price level. Components that determine the real – interest - rate has limited variation. In addition, there is no reason to expect a significant falls in money demand when *i* and *Y* are given. On the other hand, we observe huge negative (positive) variation in money supply during deflation (hyperinflation), that is money supply can grow at almost any rate.

As money growth varies more than other factors that determine inflation, this leads to the fact that money growth plays a big role in determining inflation.

As money growth is the main factor affecting inflation, we examine its effects in detail. From previous findings, we know that the money supply does not affect real output or the real interest rate, so we assume that these $\bar{Y}$ and $\bar{r}$ are constant. So using definition $r = i - \pi^e$ or $i = r + \pi^e$ (1.3) and assumption that r and Y are constant, we have

$$P = \frac{M}{L(\bar{r}+\pi^e,\bar{Y})}. \quad (1.3)$$

We assume that M and P are increasing together at some steady state and that $\pi^e = \pi$. Next let's consider that at time t_0 , there is a permanent rise in money growth. As consequences of this change, as money demand is increasing at a new steady rate, *r* and *Y* are fixed, $\frac{M}{P}$ is also constant. That is contented with price level increasing at the equal rate as M and with expected inflation equal to the new rate of money growth.

As the price level increases at the higher rate after the change, so π^e increase in the time of change. Thus, the nominal interest rate jumps, and therefore the amount of real balances required periodically falls. It results in sharply increase in the price level at the time t_0.

The change in inflation due to changes in the growth of money is reflected one by one in the nominal interest rate. The claim that a 1% increase in inflation leads to the same increase in the nominal rate is known as the Fisher effect.

The real money stock decreases due to higher growth rate of the nominal money stock. The rise in money growth result in rising in π^e, accordingly affecting nominal interest rate. As cost of holding money increases this lead to reduces in the quantity of money that people want to hold. Thus to hold system in equilibrium we need that price level rises more than money. Thus, in a certain period, the inflation rate must exceed the growth rate of money. In our case, this occurs at the moment when money growth rises.

Thus, monetary policy, which is consistent with a steady decline in inflation, is a sudden jump in money supply, accompanied by low growth.

The main purpose of monetary authority is stabilization policy, by stabilization policy we mean the ability of authority to influence inflation and the output gap. We assume that stabilization policy is aimed to keep low and stable inflation and minimize deviation of output gap from its potential level.

Constant inflation simply adds the same amount to the growth rates of all prices, services, salaries and nominal interest rate.

As prices are not adjusted continuously this is serves as cost of inflation. Thus, even stable inflation causes fluctuations in relative prices, as different firms adjust their prices at different times, as results this volatility in levels of prices is not in line with social cost. In this way, inflation increase lead to unequal distribution of wealth among public.

In addition, inflation distorts tax system. Since income from capital and interest income mostly calculated in nominal terms, inflation determines future decisions about saving and investment.

In worth to note, that inflation is costly not only due to above reasons, but mainly due to fact, that people simply don't like it. Since, there are a lot of dollarized countries, people mostly pay attention to their revenues in dollar term. In this way, they very sensitive to change in dollars prices and salaries, even if there is no effect on real wealth and income.

In addition, high inflation also serves as cost of inflation, as inflation become less predictable and policy authority need to put more effort to bring it to low and stable level.

Chapter 1

Backward- Looking Model

1.1 Model overview

The goals of stabilization policy should be models that give accurate statements about how the policy should be provided .Next we consider a base model where private behavior is backward- looking.

We suppose that the economy is described by two equations, one characterizing aggregate demand and the other characterizing aggregate supply. The equation of aggregate demand indicates that output responses to lagged real interest rate negatively. The equation aggregate supply suggests that the change in the price level depend on its previous level and on the output deviation from it flexible level. The change in the real interest rate does not affect the domestic output until the next period and does not affect inflation until the period after that. This means that the policy is lagging and that it affects output faster than inflation. The model is described by such system of equation:

$$y_t = -\beta r_{t-1} + u_t^{IS}\ , \beta > 0 \tag{1.4}$$

$$\pi_t = \pi_{t-1} + \alpha(y_{t-1} - y_{t-1}^n), \alpha > 0 \tag{1.5}$$

$$u_t^{IS} = \rho_{IS} u_{t-1}^{IS} + \varepsilon_t^{IS}, -1 < \rho_{IS} < 1 \tag{1.6}$$

$$y_t^n = \rho_Y y_{t-1}^n - \varepsilon_t^Y, 0 < \rho_Y < 1 \tag{1.7}$$

$$y_t^* - y_t^n = \Delta,\ \Delta \geq 0 \tag{1.8}$$

where y_t^n is the flexible price level and y_t^* is Walrasian demand.

Domestic output is presented by the first equation. The real interest rate (r_{t-1}) is defined as difference between nominal interest rate and expected inflation $(i_{t-1} - E_{t-1}[\pi_t])$. The inflation behavior is described by Phillips curve. Inflation can change only gradually, this fact is captured by lagged term, also inflation is sensitive to the gap between the actual and flexible level of output. The following two equation describe the behavior of the autoregressive shocks to the IS curve and to the flexible- price level of output. The equation (1.8) suggests that difference between flexible output and Walrasian output is fixed.

Monetary policy set real interest rate after observing shocks. The Central bank does not like deviation of inflation from its target value and deviation of output from Walrasian level. So Central bank minimizes $E[(y-y^{*})^{2}]+\lambda E[\pi^{2}]$, where λ is a weight that monetary authority put on inflation stabilization, here we assume that target level of inflation equal zero.

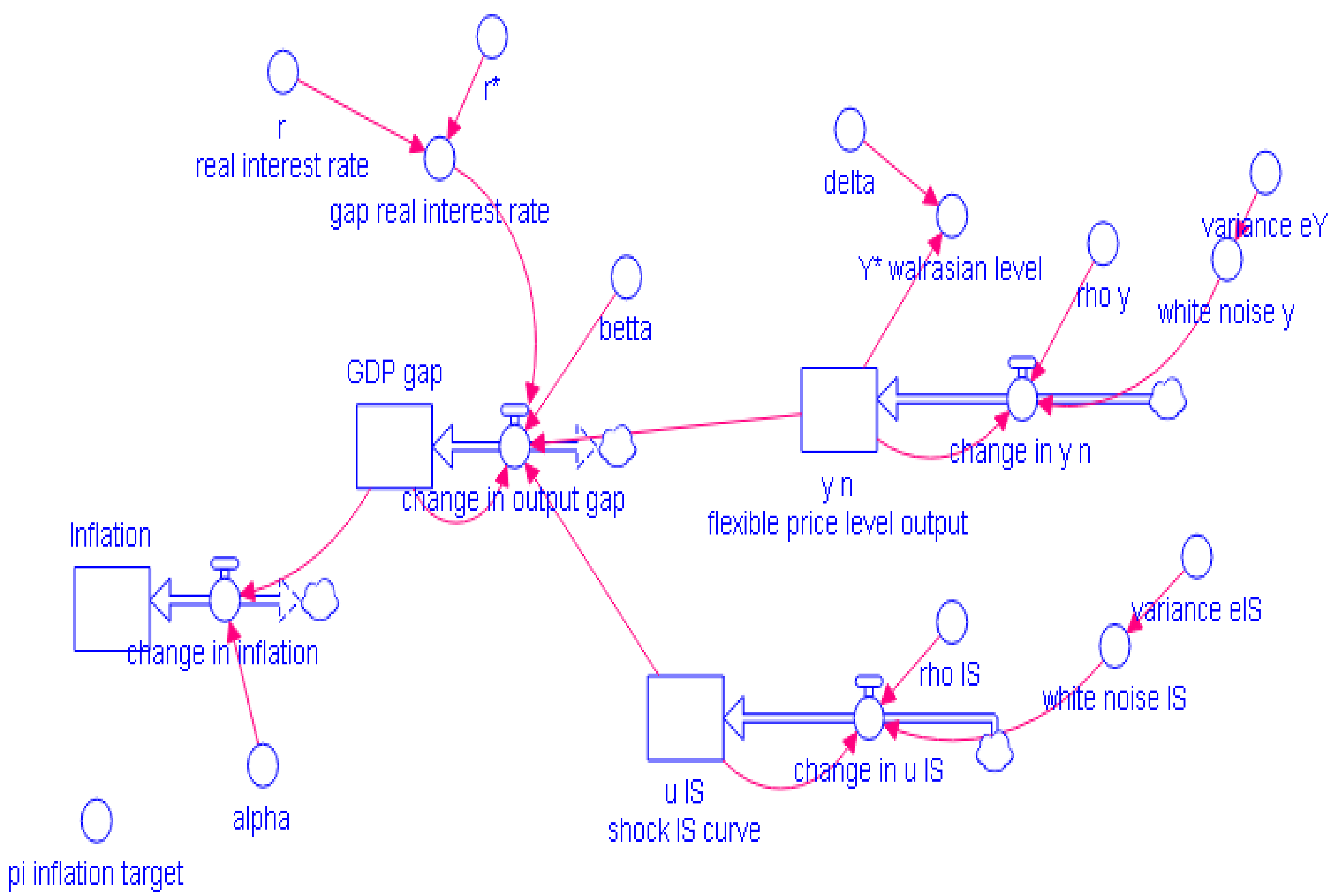


Figure 1.1: Simple Backward- looking Model in System Dynamics

If in this model put permanent shock in ε_t^{IS}, we have such dynamics:

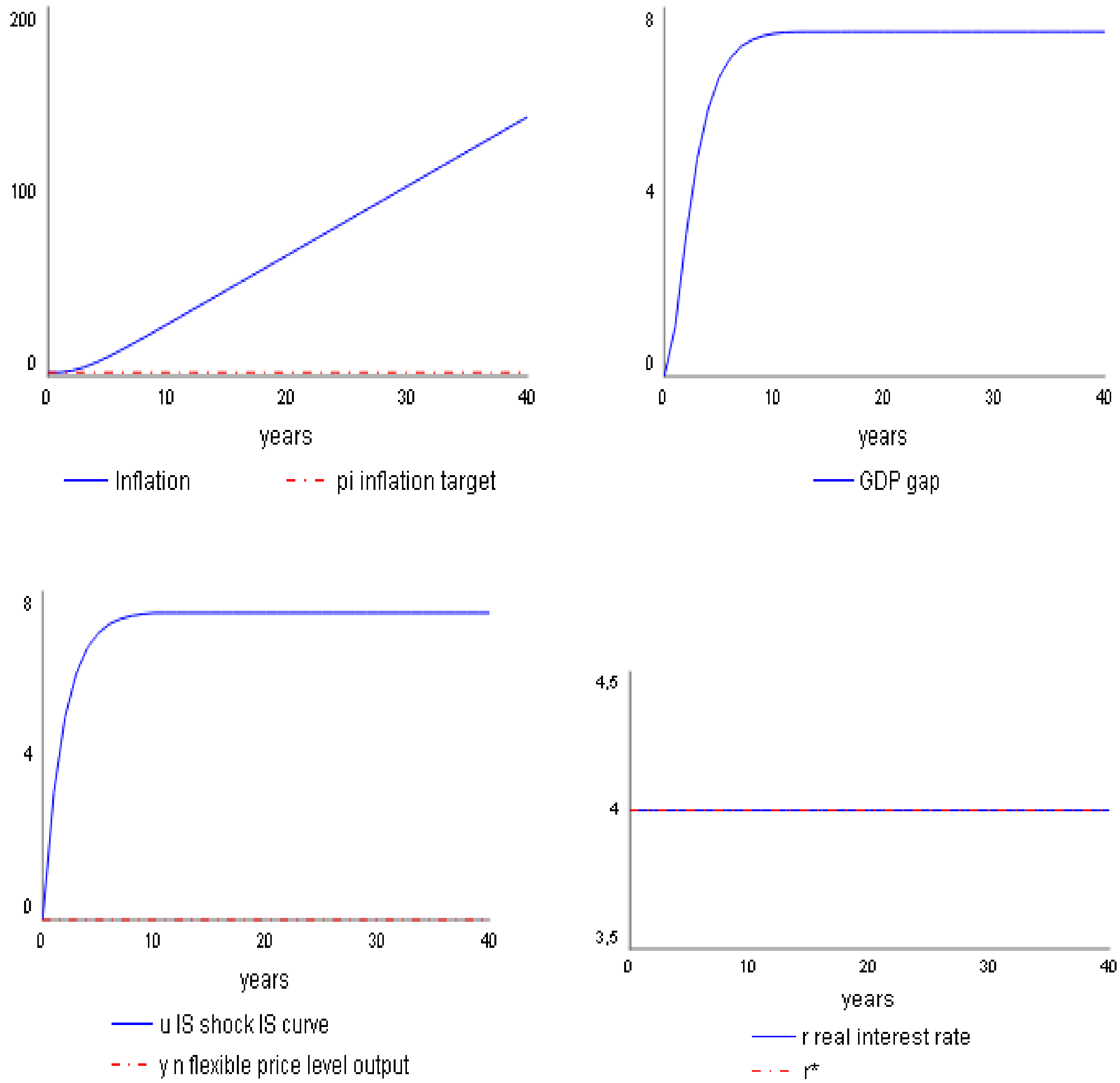


Figure 1.2: Permanent shock in ε_t^{IS}

1.2 The model analysis

Next we assume that $\tilde{y} = y - y^n$ and rewrite (1. 5) and (1. 6) as

$$\tilde{y}_t = -\beta r_{t-1} + u_t^{IS} - y_t^n \tag{1.9}$$

$$\pi_t = \pi_{t-1} + \alpha \tilde{y}_{t-1} \tag{1.10}$$

In this way, the real interest rate does not affect $\tilde{y}_t, \pi_t, \pi_{t+1}$. It’s choice first affecton $\widetilde{y_{t+1}}$, and it is only via $\widetilde{y_{t+1}}$ that it affects inflation and output in next periods. Thus, a policy can be considered for the expectation, but not for interest rate. In this way, we are now thinking about monetary choice as: $-\beta r_t + \rho_{IS} u_t^{IS} - \rho_Y y_t^n = E_t[\widetilde{y_{t+1}}]$.

The paths of inflation and output beginning in period *t+1* and determined by $E_t[\widetilde{y_{t+1}}]$ (determined in period *t*), π_{t+1} and future shocks. The desirable policy will make $E_t[\widetilde{y_{t+1}}]$ a function of π_{t+1}. The inflation equation, (1.10), means it is required that the average value of $\tilde{y}$ be equal to zero to result in bounded inflation. When lead inflation (π_{t+1}) is zero, the monetary policy sets $E_t[\widetilde{y_{t+1}}]$ to zero. In this way, the optimal policy is:

$$E_t \tilde{y}_{t+1} = -q\pi_{t+1}. \tag{1.11}$$

To determine q, we need to present $E[(y - y^*)^2] + \lambda E[\pi^2]$ as a function of q. From equation (1. 9) follows:

$$\tilde{y}_t = E_{t-1}\tilde{y}_t + \varepsilon_t^{IS} - \varepsilon_t^Y = \tag{1.12}$$

$$= -q\pi_t + \varepsilon_t^{IS} - \varepsilon_t^Y,$$

here we use lagged one period expected deviation of output. From equation (1.10) we get:

$$\pi_{t+1} = \pi_t + \alpha\tilde{y}_t = \tag{1.13}$$

$$= (1 - \alpha q)\pi_t + \alpha\varepsilon_t^{IS} - \alpha\varepsilon_t^Y.$$

In addition, the long run behavior of inflation is determined as $E[{\pi_t}^2] = [{\pi_{t+1}}^2]$. As consequence, we can solve (1.13) for expected inflation deviation from target level. It gives:

$$E[\pi^2] = \frac{\alpha^2}{1 - (1 - \alpha q)^2}(\sigma_Y^2 + \sigma_{IS}^2)$$

$$= \frac{\alpha^2}{\alpha q(2 - \alpha q)}(\sigma_Y^2 + \sigma_{IS}^2), \tag{1.14}$$

Note that σ_Y^2 and σ_{IS}^2 are the variances of the residuals ε^Y and ε^{IS}.

In order to find$E[(y-y^*)^2]$, we can rewrite $y-y^*$ as $\tilde{y}-\Delta= (y-y^n)-(y^*-y^n)$. We can use (1.12) to get:

$$E[(y-y^*)^2] = \Delta^2 + q^2E[\pi^2] + \sigma_Y^2 + \sigma_{IS}^2 \qquad (1.15)$$

Equations (1.14) and (1.15) give to us the central bank's loss function value, $E[(y-y^*)^2] + \lambda E[\pi^2]$, as a function of q. The first order condition for q:

$$E[(y-y^*)^2] + \lambda E[\pi^2] = \Delta^2 + q^2E[\pi^2] + \sigma_Y^2 + \sigma_{IS}^2 + \frac{\lambda\alpha^2}{\alpha q(2-\alpha q)}(\sigma_Y^2 + \sigma_{IS}^2) =$$

$$= \Delta^2 + q^2\frac{\alpha^2}{\alpha q(2-\alpha q)}(\sigma_Y^2 + \sigma_{IS}^2) + \sigma_Y^2 + \sigma_{IS}^2 + \frac{\lambda\alpha^2}{\alpha q(2-\alpha q)}(\sigma_Y^2 + \sigma_{IS}^2) =$$

$$= \Delta^2 + \frac{q\alpha}{2-\alpha q}(\sigma_Y^2 + \sigma_{IS}^2) + \sigma_Y^2 + \sigma_{IS}^2 + \frac{\lambda\alpha}{q(2-\alpha q)}(\sigma_Y^2 + \sigma_{IS}^2)$$

$$E[(y-y^*)^2] + \lambda E[\pi^2] \to min$$

$$(\sigma_Y^2 + \sigma_{IS}^2)\left(\frac{\alpha(2-\alpha q) - q\alpha(-\alpha)}{(2-\alpha q)^2}\right) - (\sigma_Y^2 + \sigma_{IS}^2)\left(\frac{\alpha\lambda(2-2q\alpha)}{q^2(2-\alpha q)^2}\right) =$$

$$= (\sigma_Y^2 + \sigma_{IS}^2)\left(\frac{q^2\alpha(2-\alpha q) - q^3\alpha(-\alpha) - \alpha\lambda(2-2q\alpha)}{q^2(2-\alpha q)^2}\right) = (\sigma_Y^2 + \sigma_{IS}^2)\left(\frac{q^2+\alpha\lambda q-\lambda}{q^2(2-\alpha q)^2}\right)$$

$$(\sigma_Y^2 + \sigma_{IS}^2)\left(\frac{q^2+\alpha\lambda q-\lambda}{q^2(2-\alpha q)^2}\right) = 0$$

$$q^2 + \alpha\lambda q - \lambda = 0$$

As quadratic equation has two solutions, positive and negative. We use only positive value, as the negative one leads to the infinity variances of y and π.

The solution is

$$q^* = \frac{-\lambda\alpha + \sqrt{\alpha^2\lambda^2 + 4\lambda}}{2}. \tag{1.16}$$

The policy of the central bank is presented by $E_t[\tilde{y}_{t+1}] = -q^*\pi_{t+1}$. To interpret (1.16) for q^*, it is helpful to consider how q^* varies with the weight the monetary policy places on inflation deviation from target. From expression (1.16) follows that if λ approaches zero, q^* approaches zero: the monetary authority conduct policy in way: $E_t[\tilde{y}_{t+1}] = 0$. In this way, domestic output is white noise with zero mean and inflation is random walk.

If central bank weight on inflation stabilization is rises, q^* rises: since the central bank pays more attention to inflation deviation, it lead to of output deviation from its natural rate to return inflation back to its target level.

If λ reaches infinity, q^* reaches$\frac{1}{\alpha}$. Monetary policy want to stabilize inflation as soon as possible after shock. With $q^*= \frac{1}{\alpha}$, $E_t[\tilde{y}_{t+1}] = -\frac{1}{\alpha}\pi_{t+1}$. Then it follows that $E_t[\pi_{t+2}]$ =0. Note that if λ approaches infinity, monetary policy still care about output deviation (the variance of output does not approach infinity.

This optimal policy is type of inflation targeting regime. Equation (1.13) implies that $E_t[\pi_{t+2}] = (1 - \alpha q)\pi_{t+1}$. As $0 \leq q \leq \frac{1}{\alpha}$, $0 \leq 1 - \alpha q \leq 1$. So, the optimal can be presented for expected inflation in the form:

$$E_t[\pi_{t+2}] = \phi\pi_{t+1} \tag{1.17}$$

where $0 \leq \phi \leq 1$. If $\lambda = \infty$, q = $\frac{1}{\alpha}$, and so ϕ = 0. In this way, $E_t[\pi_{t+2}]$ = 0: the monetary policy always tries to bring inflation to target as soon as possible. The central bank with such behavior is follow strict inflation targeting regime.

For $0 < \lambda < \infty$ and $0 < \phi < 1$, Central bank follow flexible inflation targeting. The action of policy take the form of trying to bring inflation back target level after shock.

The greater is λ the more effort monetary policy put to stabilize inflation, the lower is ϕ the faster it act.

Next we define neutral interest rate, r_t^n. Neutral interest rate makes output to be equal its flexible-price level. Since r_t affects y_{t+1}, r_t^n is the value of r_t that yields $y_{t+1} = y_{t+1}^n$. From (1.4) or (1.9) ,this interest rate is

$$r_t^n = -\frac{1}{\beta}(y_{t+1}^n - u_{t+1}^{IS}). \tag{1.18}$$

Using this definition, we can rewrite (1.10) as

$$\tilde{y}_t = -\beta(r_{t-1} - r_{t-1}^n). \tag{1.19}$$

It means that

$$E_t[\tilde{y}_{t+1}] = -\beta(r_t - E_t[r_t^n]). \tag{1.20}$$

($E_t[r_t^n]$ appears in this expression rather than r_t^n because r_t^n depends on u_{t+1}^{IS} and y_{t+1}^n, which are known at moment t). Since monetary policy set interest rate so that $E_t[\tilde{y}_{t+1}] = -q\pi_{t+1}$, and that $\pi_{t+1} = \pi_t + \alpha\tilde{y}_t$. Using these facts and (1.21) we have

$$-q[\pi_t + \alpha\tilde{y}_t] = -\beta(r_t - E_t[r_t^n]), \tag{1.21}$$

or

$$r_t = E_t[r_t^n] + \frac{q}{\beta}\pi_t + \frac{\alpha q}{\beta}\tilde{y}_t. \tag{1.22}$$

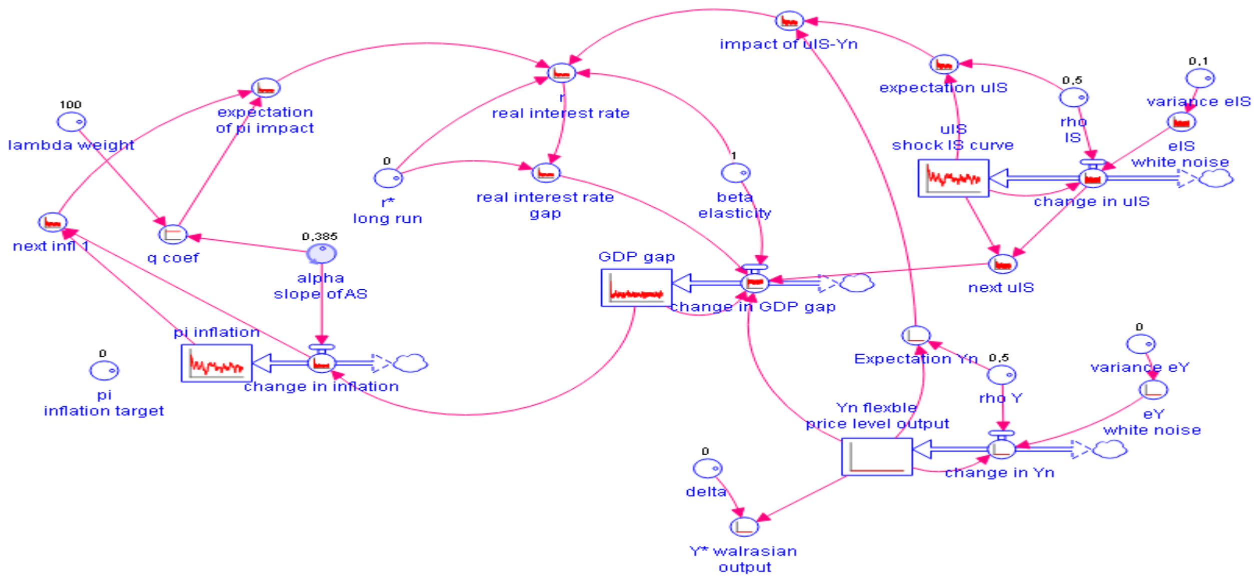
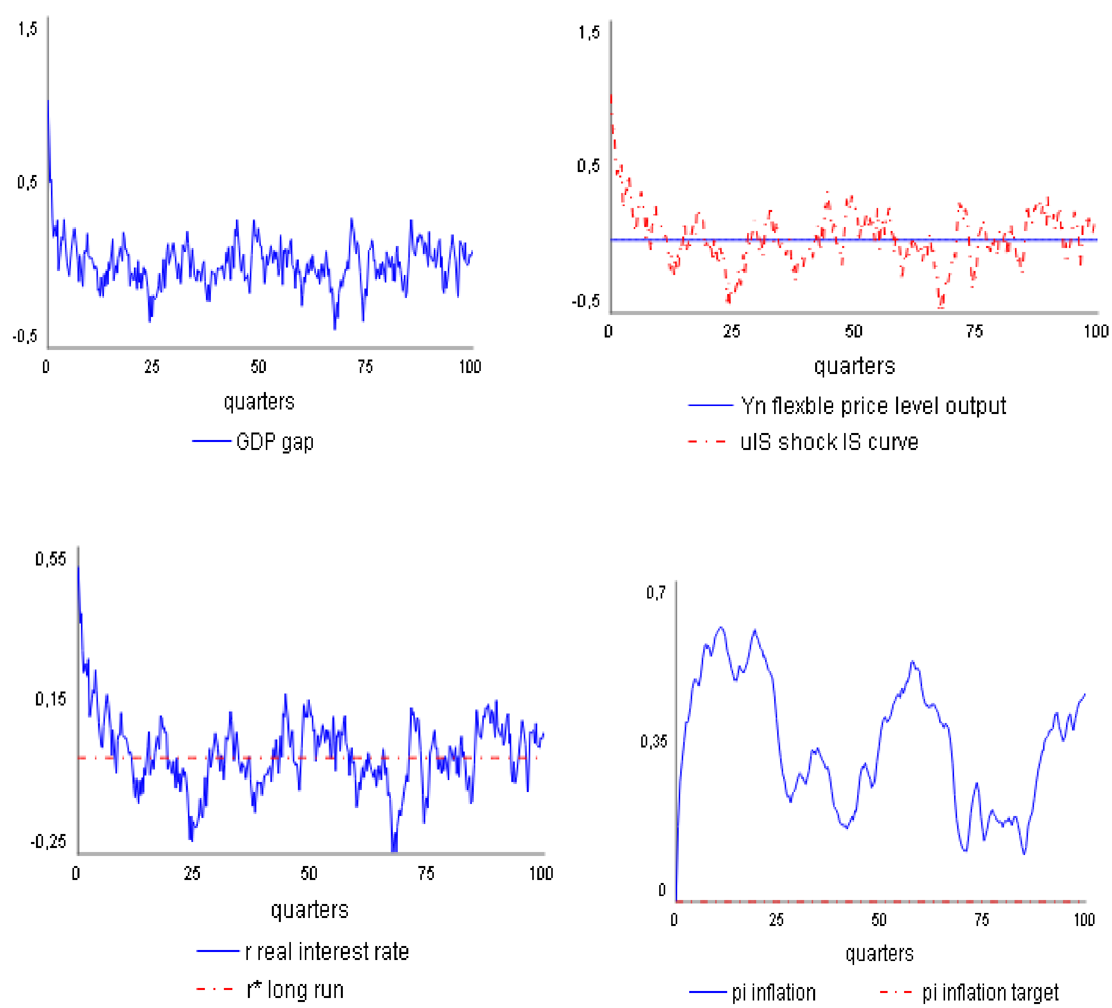


Figure 1.3: Simple Backward-looking Model in System Dynamics



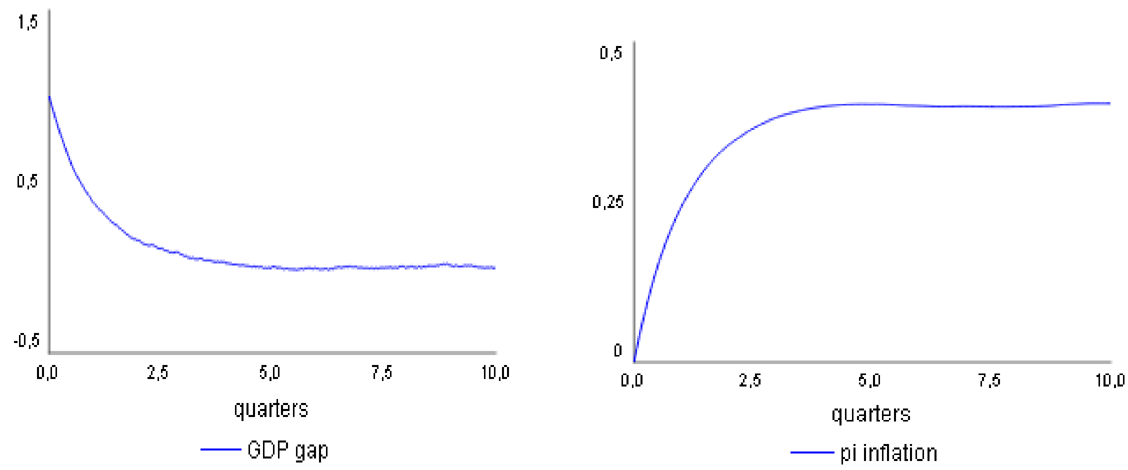


Figure 1.4: The choice of Central Bank $\lambda = 0$

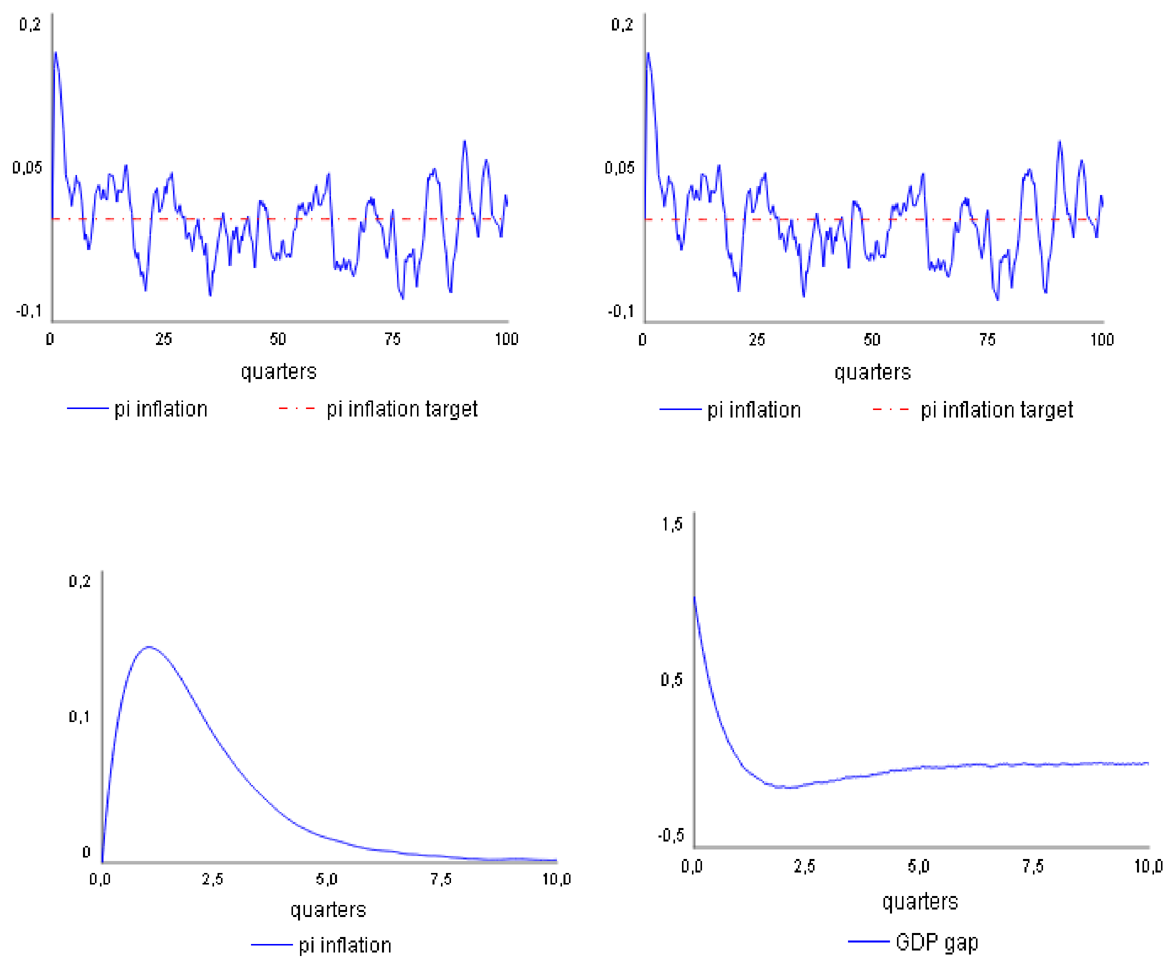


Figure 1.5: The choice of Central Bank $\lambda = 200$

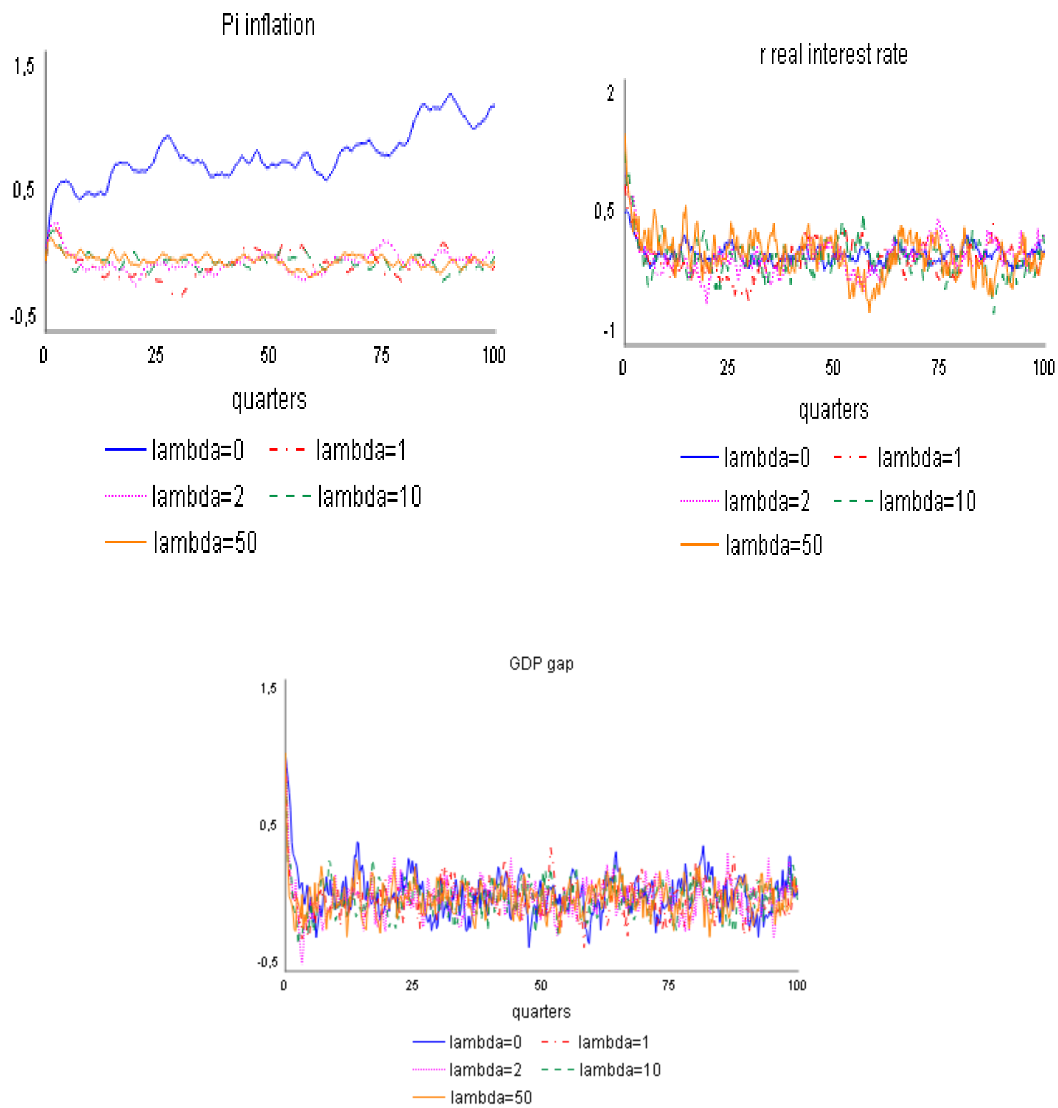


Figure 1.6: The choice of Central Bank $0 \leq \lambda \leq 50$

Finally, optimal policy is interest-rate rule based. Monetary policy sets key interest rate as combination of natural real, output and inflation. As result of this analysis, we can conclude that not all policies are optimal.

Chapter 2

Price Competitiveness

In economics, real exchange rate is widely used macroeconomic indicator; with it help economists can judge about competitiveness of their country. In literature, indicator of price competitiveness is defined as deviation of real exchange rate from some benchmark. This indicator serves as important policy tool for policy makers. To compute price competitiveness indicator I use productivity-based approach that was proposed by Fisher and Hossfeld (2014). This approach has a list of desirable properties, among them: benchmark level is model based and calculated indicator of price competitiveness based on large number of economies.

2.1 The data and sample

Analysis cover up to 20 years (2000-2020) and based on large number of economies (54), among them developed and emerging countries. Price and productivity data in levels were used in this approach. All data are semi-annual. As price level, I used Big Mac burger price, as McDonald's is present in almost every country. In this way, ingredients for burger the same in different economies. The Economist publishes the Big Mac price data two times per year. Productivity data are taken from International Monetary Fund. GDP per capita is used as productivity measure approach.

The data panel is unbalanced. To compute indicators of price competitiveness for all 54 countries the base country is needed. USA is chosen as base country**.**

In analysis, I use log price of Big Mac of specific country relative to the base country and log productivity level of county relative to the base country.

2.2 Preliminary data analysis and estimation

To assess the time series properties and check the existence of cointegration among variables, I present here results of panel unit root and cointegration tests.

The results of Augmented Dickey-Fuller and Phillips-Perron test (Table 3.1) imply that both time series are nonstationary on level and stationary in first differences, series are I(1).

Table 3.1: Unit root tests

Variable	ADF	PP	Status
Relative price level	0.89	0.44	Non- stationary
Relative productivity	0.68	0.79	Non- stationary
D(Relative price level)	0.00	0.00	Stationary
D(Relative productivity)	0.00	0.00	Stationary

The results of panel cointegration test in Table 3.2 indicates that there is long-run relationship among series at 1% (or 5%) significance level. As result, to determine the relative price level in the long-run, it is sufficiently to take relative productivity.

Table 3.2: Cointegration test

	Statistic	Probability
Pedroni test	Panel v – statistic	0.02
	Panel rho - statistic	0.00
	Panel PP - statistic	0.00
	Panel ADF - statistic	0.00
	Group rho- statistic	0.03
	Group PP - statistic	0.00
	Group ADF - statistic	0.00

The fixed effects panel regression is used to compute benchmark for real exchange rate:

$$q_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it} \tag{3.1}$$

where α_i is fixes effect of country i, q_{it} and x_{it} are productivity and price level of country i relative to USA (in log), ε_{it}- error term. Based on panel fixed effects OLS regression, the estimated elasticity is *0.27.* The coefficient is significant at the *1%* level. The estimated elasticity in line with previous findings for similar group of country.

Table 3.3: Estimated long-term elasticity

Own estimation	**Previous findings**	
	C. Fischer & O. Hossfeld (2014)	Cheung (2007)
0.27* **	**0.35**	**from 0.25 to 0.39**

2.3 Computation of benchmarks for real exchange rate

In the estimation of equation (3.1), all observation receive the same weights, in this way, we assume that all foreign partners play important role for country *i*. However, we want that indicator of price competitiveness be multilateral one, so all observations should receive weight according to their importance. To construct multilateral indicator of competitiveness, we define relative price level ($\widetilde{q_{it}}$) and productivity ($\widetilde{x_{it}}$) of country i compared to other countries as:

$$\widetilde{q_{it}} = q_{it} - \sum_{j=1}^{54} w_{ij} q_{jt}$$

$$\widetilde{x_{it}} = x_{it} - \sum_{j=1}^{54} w_{ij} x_{jt}$$

where w_{ij} -constant trade weight (average during 2004-2020)of country j for country i.

The trade weight is combination of import and trading weight:

$$\text{Import weight: } w_{ij}^{m} = \frac{m_j^i}{\sum_{j=1}^{N} m_j^i}$$

$$\text{Export weight: } w_{ij}^{x} = \frac{x_j^i}{\sum_{j=1}^{N} x_j^i}$$

$$\text{Trade weight: } w_{ij} = \frac{m_i}{x_i+m_i} * w_{ij}^{m} + \frac{x_i}{x_i+i} * w_{ij}^{x}$$

where m_j^i is import of country j to country i and x_j^i is export of country j to country i.

The benchmark for the relative price level of country i is constructed according to approaches (a) and (b) as follows:

$$\widetilde{q_{(a)\iota t}^{*}} = \hat{\beta}\widetilde{x_{\iota t}}$$

$$\widetilde{q_{(b)\iota t}^{*}} = \breve{\alpha} + \hat{\beta}\widetilde{x_{\iota t}}$$

where $\breve{\alpha} = \widehat{\alpha_\iota} - \sum_{j=1}^{54} w_{ij}\,\widehat{\alpha_j}$.

The indicator of price competitiveness (in log) ,m_i , for country i is defined as deviation of relative price level from the benchmark:

$$\widetilde{m_{\iota t}} = \widetilde{q_{\iota t}} - \widetilde{q_{\iota t}^{*}}, \qquad \widetilde{M_{\iota t}} = e^{\widetilde{m_{\iota t}}}.$$

2.4 Results

Results for the multilateral indicator of price competitiveness for several economies are shown in Figure 3.1-Figure 3.8.

Figure 3.1 suggests that Ukrainian relative price and productivity levels exceeded benchmarks level, stayed in less favorable territory, over 2004s2-2008s2. In addition, relative productivity level increased faster than those of trading partners over 2004s2 -

2008s2. Crisis of 2008 - 2009 pushed price competitiveness below the benchmark levels and forced price and productivity level decrease compared to trading partners. Today Ukraine remains in a less favorable zone of competitiveness.

Figure 3.4. shows the result for China. Relative productivity increased and price level decreased over 2000s1 -2005s2. However, relative productivity and price level of China compared to trading partners have been increased over the last fifteen years. China's price competitiveness stayed in favorable territory from 2005s1-2013s2, in 2014s1 price competitiveness moved into slightly unfavorable territory. The results are in line with those Fisher and Hossfeld (2014).

Figure 3.5 shows the case of Japan. Relative price level persistently below benchmark level (b). In addition, relative price and productivity increased slower than those of trading partners.

Figure 3.6. displays low price competitiveness of Brazil.

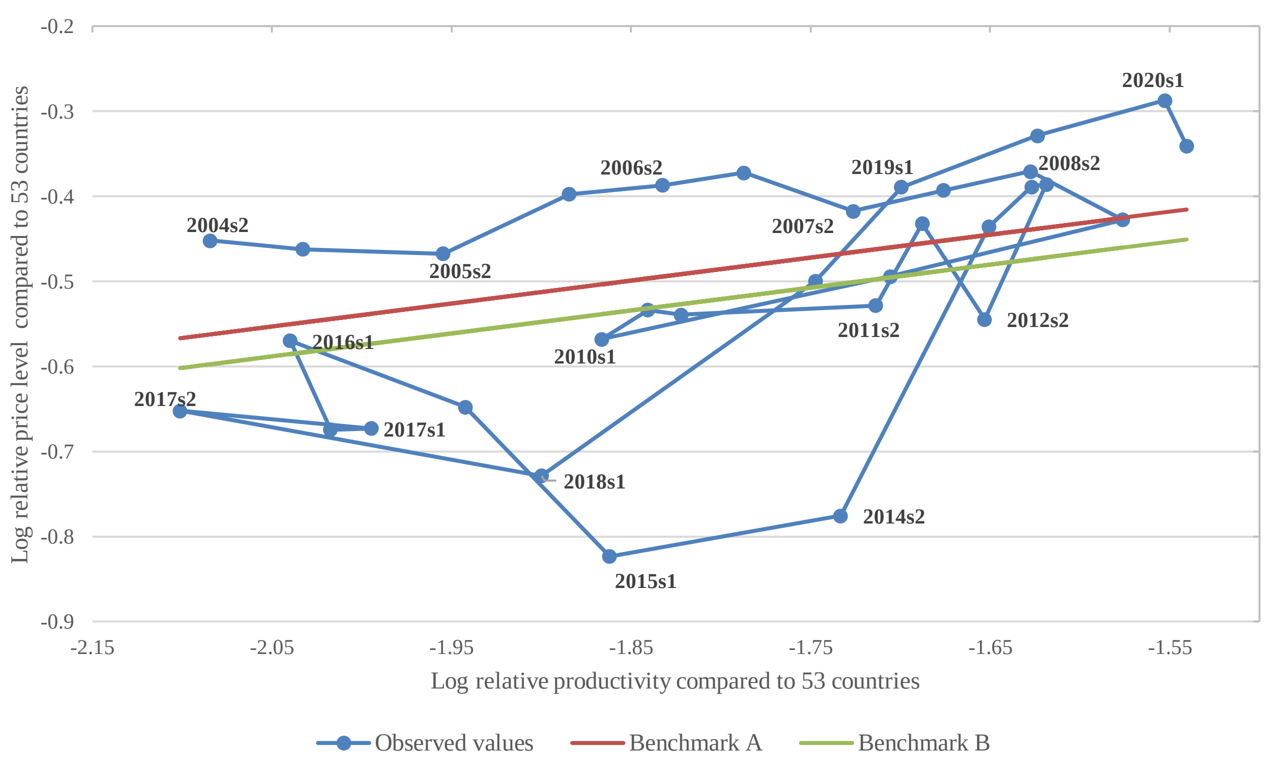


Figure 3.1: Indicator of price competitiveness for Ukraine

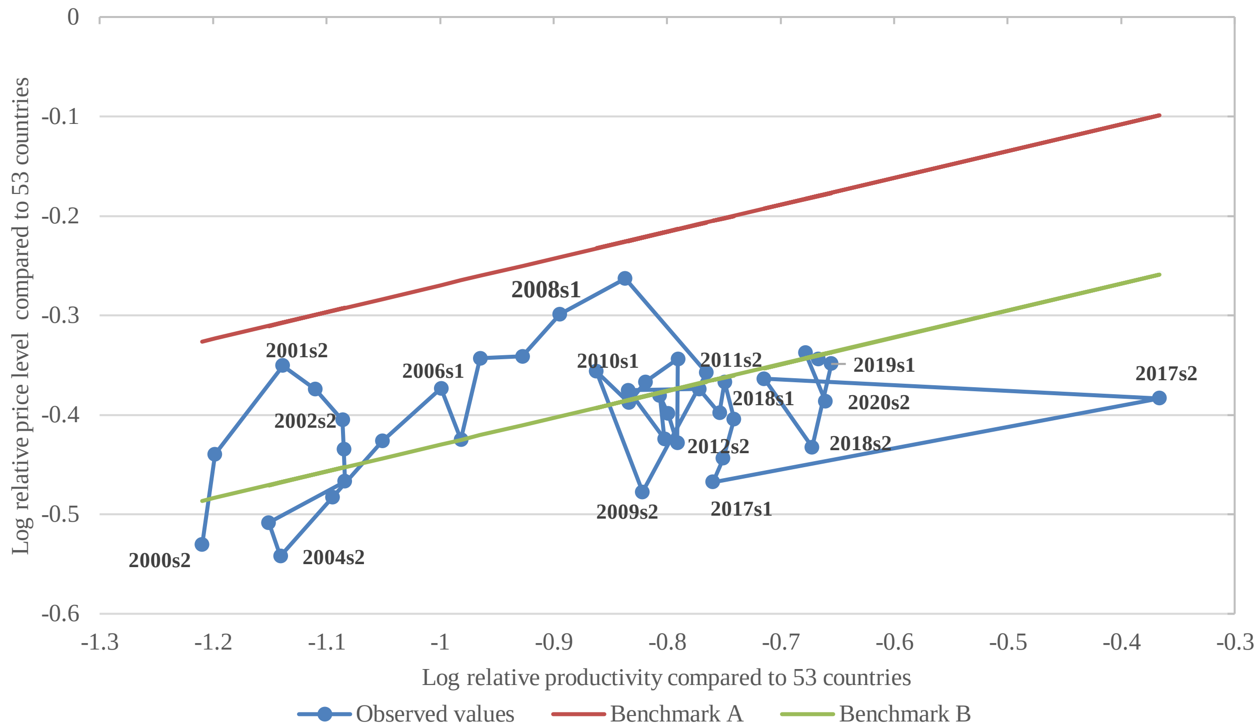


Figure 3.2: Indicator of price competitiveness for Poland

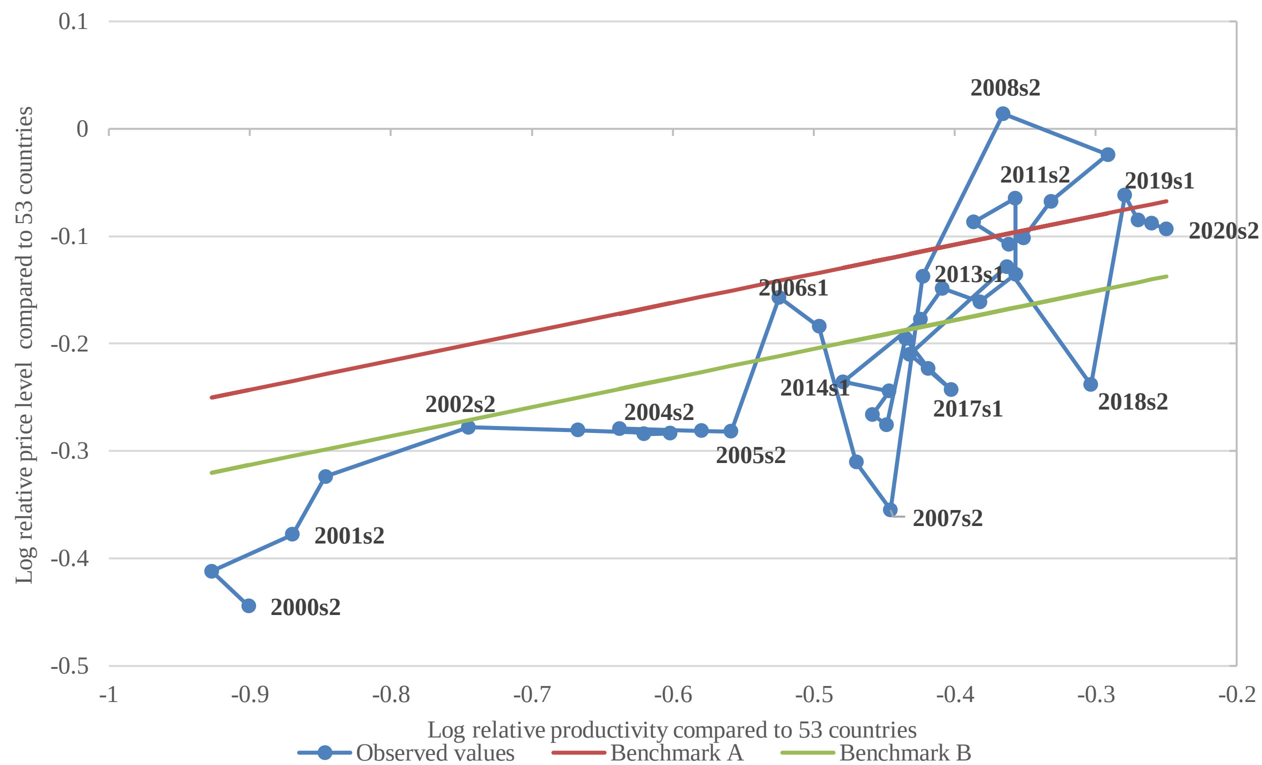


Figure 3.3: Indicator of price competitiveness for Czech Republic

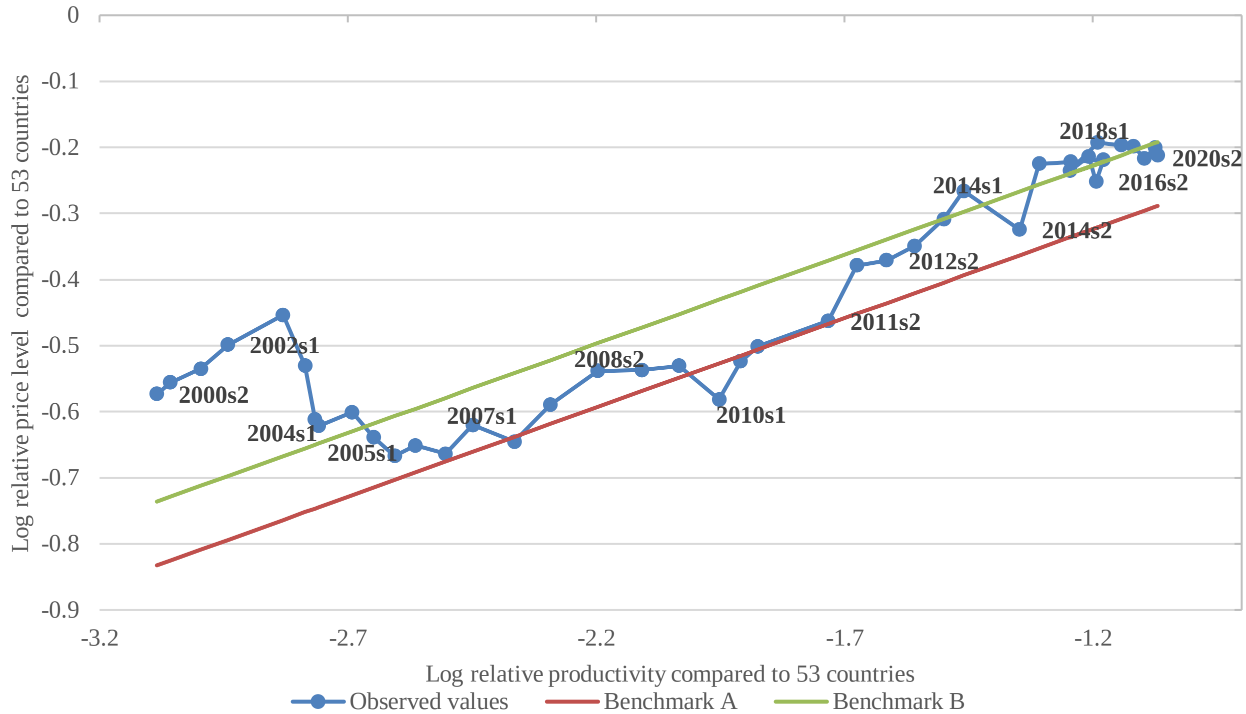


Figure 3.4: Indicator of price competitiveness for China

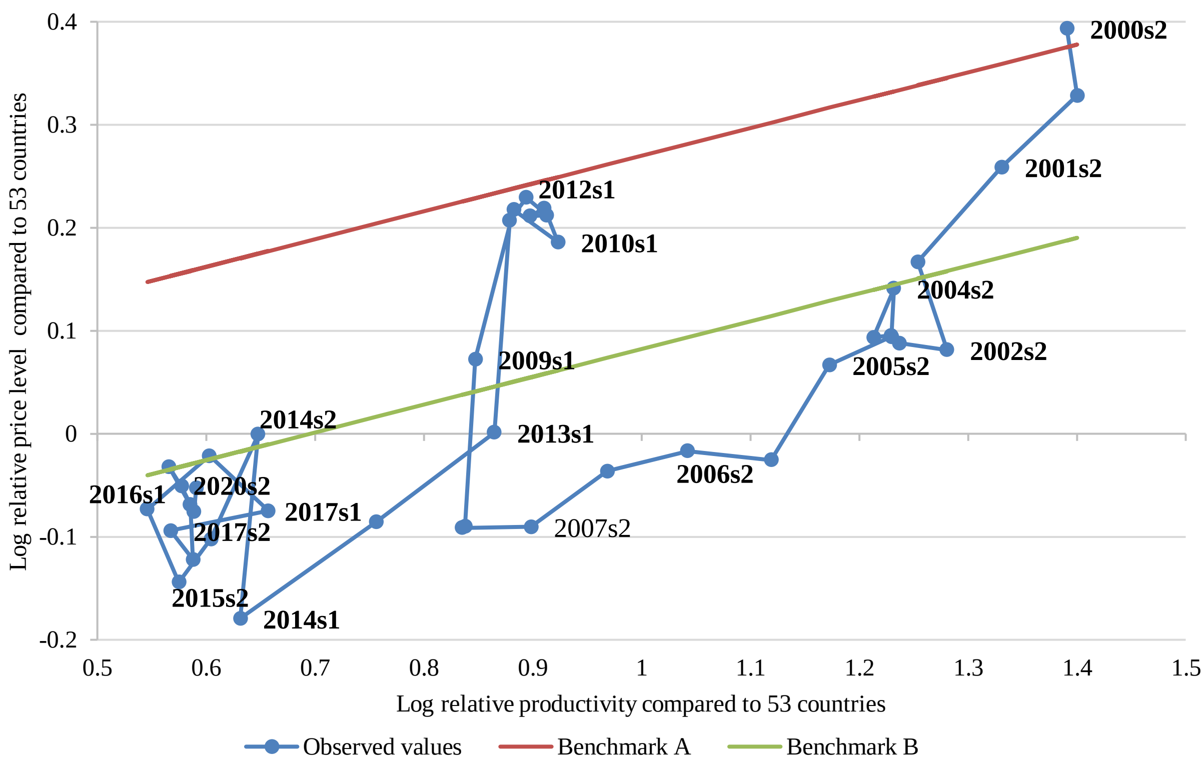


Figure 3.5: Indicator of price competitiveness for Japan

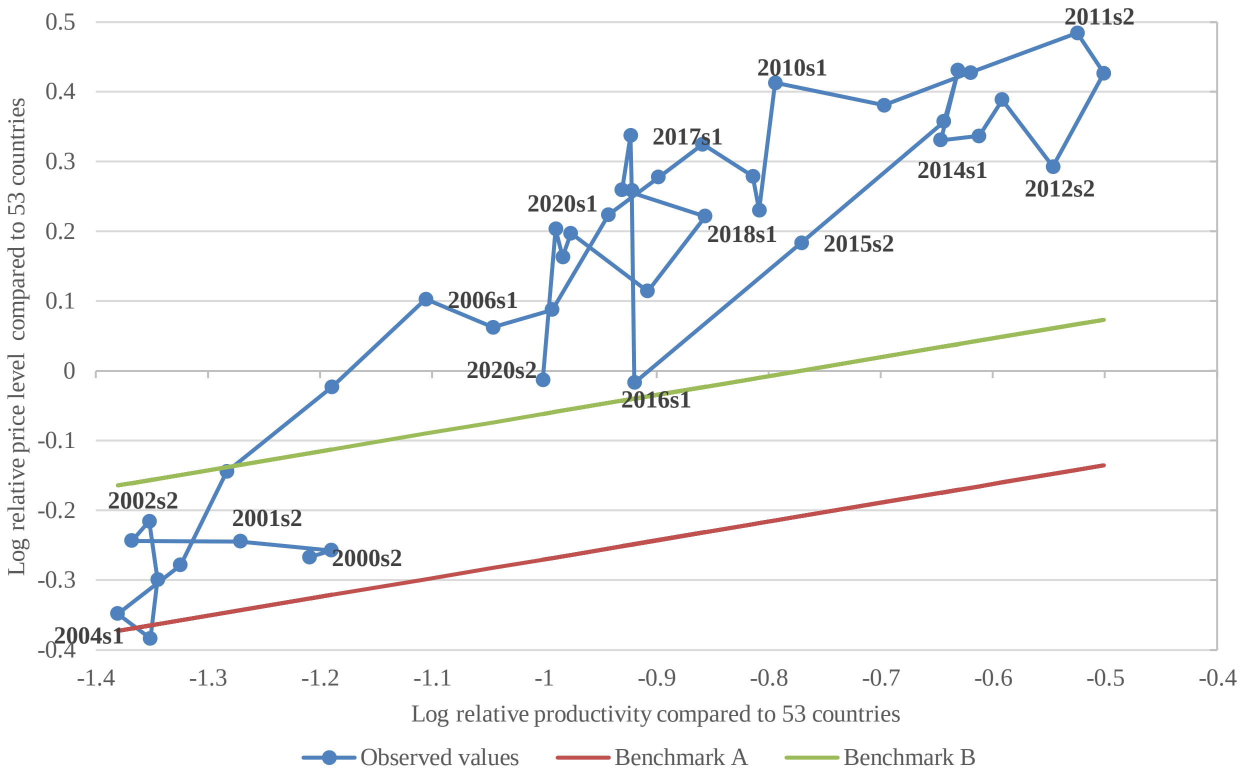


Figure 3.6: Indicator of price competitiveness for Brazil

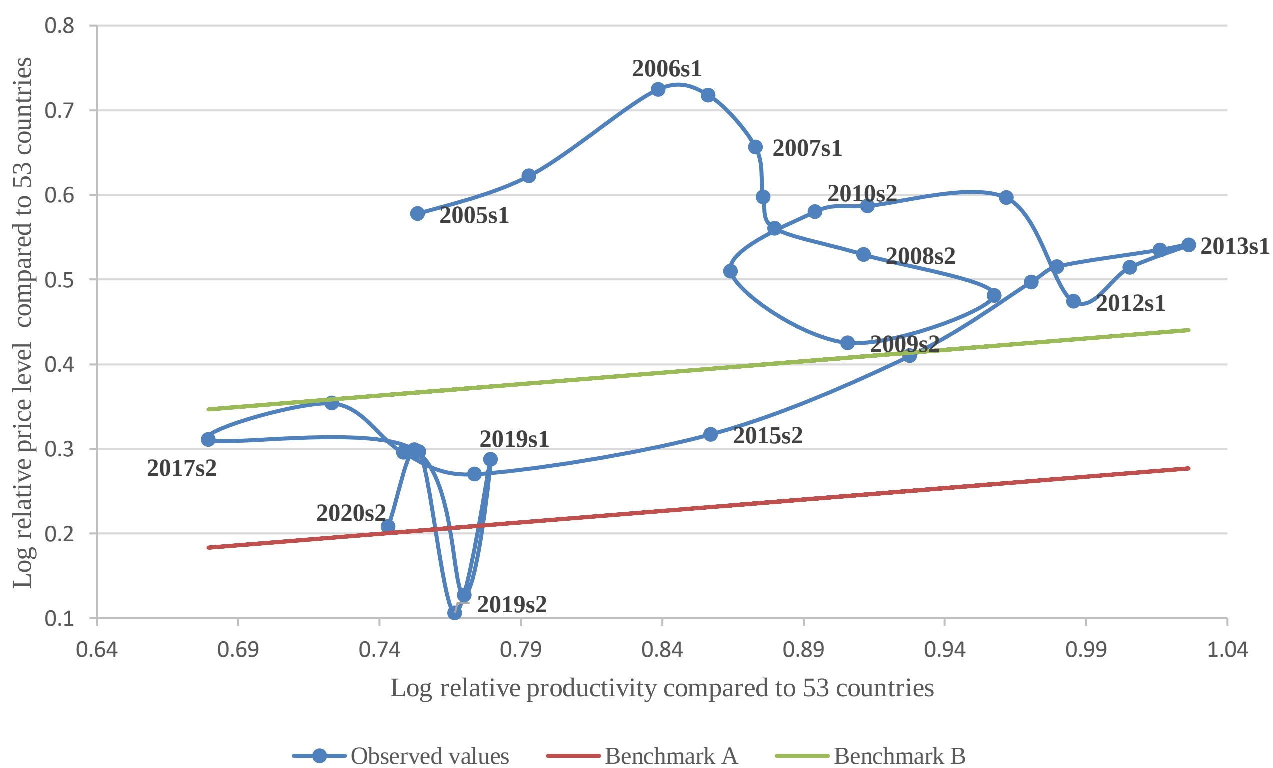


Figure 3.7: Indicator of price competitiveness for the Norway

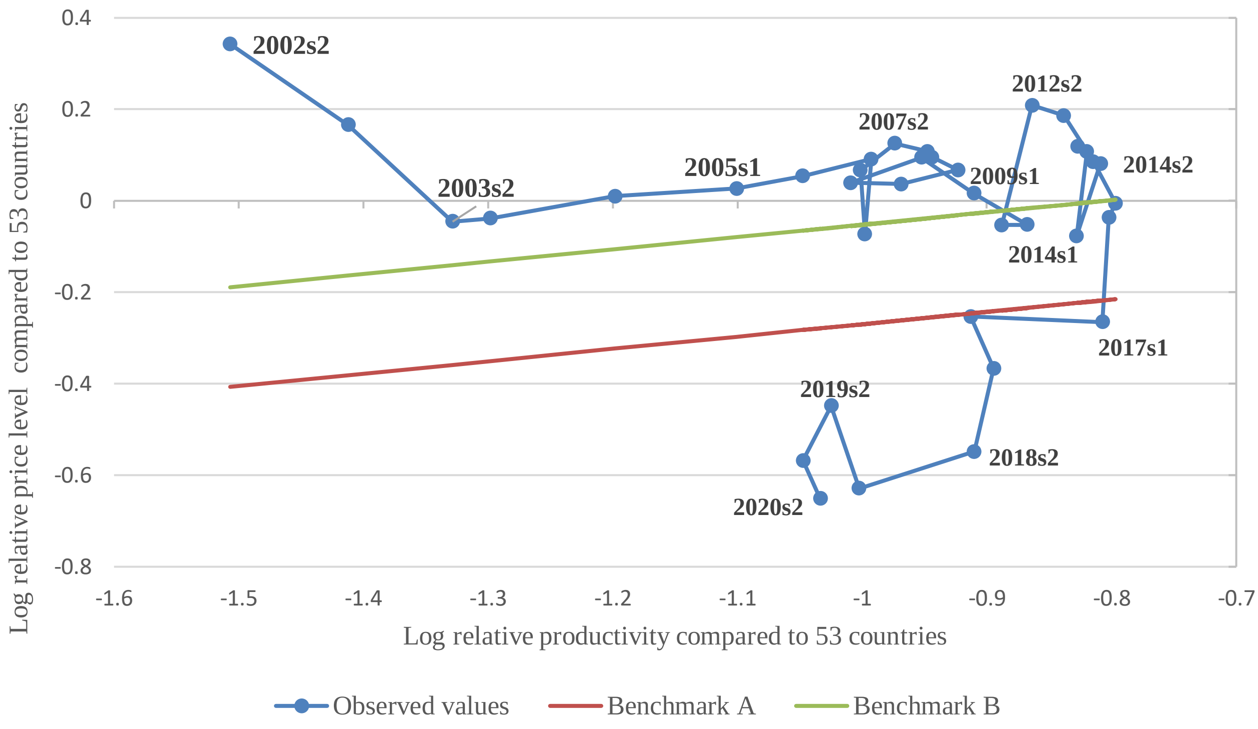


Figure 3.8: Indicator of price competitiveness for the Turkey

Chapter 3

Endogenous Monetary Credibility

Monetary policy action can affect economy via various transmission channels, for example, by changing cost of borrowing money. However, expectations channel is one of the most important for Central bank under inflation targeting regime. As, when economic agent make decisions about their investment and saving behavior, they take into account not only current economic situation but also expectations about future economic stance. In this way, success of monetary policy strongly depend on expectations in economy. Moreover, outcome of the same monetary strategy under different expectation can be opposite. In this regard, monetary authority around the world pay a lot of attention to communication with public and transparency to build credibility.

3.1 Literature review on policy credibility

Credibility is a question of high importance for countries under Inflation Targeting (IT) regime, especially for Emerging Market Economies (EMEs). Central banks make a lot of effort to build credibility.

However, why credibility is so important to Central banks? How Central bank can build credibility? Finally, what we mean by credibility?

Blinder (2000) state that "A Central bank is credible if people believe it will do what it say." A similar definition of credibility was proposed by Cukierman and Meltzer (1986). They state that policy is more credible if there is low discrepancy between planned action of policy and economic agent's beliefs about those plans.

Blinder (2000) state that high interest degree to policy credibility to some extent related to fact that "under certain assumptions, including rational expectations, a completely credible central bank can engineer a disinflation without suffering any adverse effects on employment". He found that high credibility is important to Central banks mainly due to next reasons: it makes disinflation process less costly, it helps curb inflation when it is low and gain public support for central bank independence.

According to Blinder research, both economists and Central banks think that it is necessary to Central bank "have a history of doing what it says it will do" to be credible.

Various methods of credibility modeling are described in the literature, but there is still no definitive way to measure credibility.

Bomfim and Rudebusch (2000) suggest that Central Bank credibility can be established according to outcome or behavior mechanisms. According to first, inflation expectations are become more anchored to target if Central bank managed to reach target in past; and according to latest, if agents believe that future inflation will be close to target.

Lalonde (2005) suggest similar approach to Bomfim and Rudebusch (2000), but with two distinguishes. Lalonde (2005) assumes that credibility is nonlinear function of distance between inflation and target, while latest propose opposite approach. Also Landone (2005) assume that credibility is based on both: action and behavior mechanisms.

Credibility developed by Lalonde (2005) have symmetric form, credibility is affected by upward and downward deviation of inflation from target.

Isard, Laxton, and Eliasson (2001) consider two-stage regime economy: low- and high-inflation states. They assume that policy is credible if economy stand in low-inflation state, and noncredible if inflation converge to high inflation level, that is, the economy is on the second state. They assume that the relationship between credibility and distance of actual inflation to high and low inflation rates is nonlinear.

Argov, Epstein and Karam (2007) extend semi-structural model for policy analysis and forecasting with endogenous policy credibility process, for the case of Israel. They found that responses of extended model to shocks more closely reproduce country's properties.

3.2 Key equations of the model

This section presents explanation of the main equations of the Quarterly Projection Model, which is a semi-structural gap model. By gap we mean deviations of actual values of variable from its trend values, in percentage.

The key special feature included in our model is the presence of the endogenous process of monetary policy credibility.

IS curve

The equation for domestic aggregate demand is written for the output gap ($\hat{y}_t$).

$$\hat{y}_t = \alpha_1 \hat{y}_{t-1} - \beta_1 * rmc + \delta_1 \hat{y}_t^* + \theta_1 \widehat{tot}_t - \mu_1 \widehat{prem}_t + \varepsilon_{1,t}$$

$$rmc = \gamma_1 \left(-\widehat{lr}_t\right) + (1 - \gamma_1)\hat{z}_t \quad (4.1)$$

To present the persistence of business cycle, the output gap is related to its own lagged value($\hat{y}_{t-1}$).

Next term of equation, real monetary conditions(rmc), is reflect the impact of monetary policy on domestic economy through interest rate and exchange rate transmission channels. Real monetary conditions variable is the weighted average of the real credit rate gap ($\widehat{lr}_t$) and real effective exchange rate gap ($\hat{z}_t$). The decisions of the economic participants about consumption and saving are captured by real credit rate. Process of substitution the domestic goods by foreign and vice versa is reflected by real effective exchange.

In addition, domestic demand depend on foreign output gap ($\hat{y}_t^*$) and terms-of-trade gap ($\widehat{tot}_t$).

Incorporation of risk premium gap ($\widehat{prem}_t$) into output equation reflect the fact that the riskier the investment, the more demand decreases.

The term $\varepsilon_{1,t}$ represents a demand shock.

Philips curve

Annualized quarterly inflation (π_t) depend on expected inflation (π_t^e) and marginal cost, which is presented by second term of equation.

Marginal cost is a weighted average of output gap ($\hat{y}_t$) and real effective exchange rate ($\hat{z}_t$), this gaps are approximation of domestic and importers marginal cost respectively.

$$\pi_t = \pi_t^e + \beta_2(\gamma_2\hat{y}_t + (1-\gamma_2)\hat{z}_t) + \varepsilon_{2,t} \tag{4.2}$$

The last one term ($\varepsilon_{2,t}$) is the supply shock.

Inflation expectations

The inflation expectation is determined by equation (4.3). We assume that in economy is constant share of backward-looking ($\pi_{b_t}^e$) and forward-looking agents ($\pi_{f_t}^e$).

$$\pi_t^e = \alpha_2\pi_{b_t}^e + (1-\alpha_2)\pi_{f_t}^e \tag{4.3}$$

Backward-looking agents form their expectation according to equation (4.4). Their expectations are the weighed sum of inflation target (π_t^T) and recently realized inflation(π_{t-1}). Weight depend on outcome credibility (ψ_t^b) and parameter δ_2 .

Credibility can vary from 0 (no credibility) to 1 (full credibility).

Parameter δ_2 can be interpreted as weight that agents assign to inflation target in fully credible economy.

$$\pi_{b_t}^e = \psi_t^b\delta_2\pi_t^T + \left(1-\psi_t^b\delta_2\right)\pi_{t-1} \tag{4.4}$$

Expectations of forward-looking agents are modeled as follow:

$$\pi^e_{f_t} = \psi^f_t \delta_2 \pi^T_t + (1 - \psi^f_t \delta_2)\pi_{t+1} \tag{4.5}$$

Forward-looking agents in forming their expectation put some weight on inflation target (π^T_t) and model-consistent inflation expectations(π_{t+1}). Unlike backward-looking agents, forward-looking agents consider action credibility (ψ^f_t) to shape their expectations.

Credibility stock

Outcome and action credibility is presented by equation (4.6) and (4.7) respectively. Both credibility is modeled as stocks that can fluctuate from 0 (no credibility) to 1 (full credibility).

To capture the fact that credibility can change only gradually, outcome and action credibility are related to their own lagged values.

$$\psi^b_t = \mu_b \psi^b_{t-1} + (1 - \mu_b) e^{-\frac{(\pi 4_{t-1} - \pi 4^T_{t-1})^2}{2\theta^2_2}} \tag{4.6}$$

Outcome credibility depend on distance between past year-on-year inflation ($\pi 4_{t-1}$) and past inflation target($\pi 4^T_{t-1}$). Credibility is high if monetary policy has managed to reach the target in past.

$$\psi^f_t = \mu_f \psi^f_{t-1} + (1 - \mu_f) e^{-\frac{(\pi 4_{t+4} - \pi 4^T_{t+4})^2}{2\theta^2_2}} \tag{4.7}$$

Action credibility depend on distance between expected year-on-year inflation for four quarter ahead ($\pi 4_{t+4}$) and future inflation target($\pi 4^T_{t+4}$). Due to action credibility monetary policy is assumed to be credible if economic participants expect that monetary policy will be able to meet inflation target in future.

Credibility parameter θ presents the sensitivity of agents to the deviation of inflation from the target. If θ is close to zero, then any slight deviation of inflation from the target lead to almost zero signal to credibility.

Overall monetary policy credibility c_t is the weighed average of outcome and action credibility.

$$c_t = \alpha_2 \psi_t^b + (1 - \alpha_2)\psi_t^f \tag{4.8}$$

Monetary policy rule

The monetary policy reaction function is represented by Taylor rule in the following way:

$$i_t^P = \alpha_3 i_{t-1}^P + (1 - \alpha_3)(\bar{r}_t^P + \pi_{t+1} + \beta_3(\pi_{t+1} - \pi_{t+1}^T) + \gamma_3 \hat{y}_t) + \varepsilon_{3,t} \tag{4.9}$$

Nominal short-term policy interest rate (i_t^P) is the key monetary policy instrument. To capture the fact that Central bank adjust policy interest rate only gradually, it is related to its lagged value (i_{t-1}^P).

Policy makers change interest rate in response to following reasons: to deviation of expected inflation from target $(\pi_{t+1} - \pi_{t+1}^T)$ and to deviation of actual output from it potential level($\hat{y}_t$).

In long run, when there is absence of mentioned above gaps, policy interest rate is converge to its neutral level ($\bar{r}_t^P + \pi_{t+1}$).

The monetary policy shock is presented by $\varepsilon_{3,t}$ term.

Uncovered interest parity (UIP)

Nominal exchange rate (s_t) is presented by modified condition of uncovered interest parity (UIP), which reflects assumption about no-arbitrage opportunity.

$$s_t = s_{t+1} + interv_t + \frac{i_t^* - i_t + prem_t}{4} - \gamma_4 \widehat{tot}_t + \varepsilon_{4,t} \tag{4.10}$$

No-arbitrage state that market in equilibrium, agents nothing gain from investing in foreign assets rather than in domestic one.

Nominal exchange rate is a function of expected nominal exchange rate(s_{t+1}), foreign (i_t^*) and domestic nominal interest rate(i_t), and additional sovereign risk premium$(prem_t)$.

Equation (10) include foreign exchange market interventions$(interv_t)$, which serve as Central bank's tool in smoothing undesirable volatility in exchange rate.

Nominal exchange rate also adjusted by term of trade gap$(\widehat{tot}_t)$.

The term $\varepsilon_{4,t}$ is exchange rate shock.

3.3 Estimation coefficients of main behavioral equation

The model has 14 measurement variables. However, all the gap and trend variables are not directly observable. Nevertheless, those variable were obtained using Hodrick-Prescott filter. Data was mainly taken from National Bank of Ukraine.

There are varies methods to parametrize model. One can choose calibration, using some benchmark from literature, empirical knowledge or expert judgments about economy. However, instead of following calibration procedure that a commonly used by researchers, we choose Bayesian approach for estimation of model's parameters.

Application of Bayesian technique allow us to incorporate priors about parameters and about distribution. In fact, prior distribution allow researchers to include some additional information into estimation of parameters. In this way, estimated parameters consistent with available historical data and priors about them.

The presented here model has 48 parameters, 42 of them are estimated with Bayesian techniques. Economic knowledge allow us to set lower and upper bounds on parameters, which in turn help us to judge about prior distributions. In cases where parameters value range from zero to one, we use Beta distribution. As prior distribution of the standard deviations of shocks we use Inverse Gamma.

We provide estimated coefficients that were proposed by Grui (2020) as a mean priors values of parameters.

The model is estimated over inflation targeting regime in Ukraine (2015:1 to 2020:1).

The estimation results are reported in Tables 4.1 – 4.4. The results of external parameters are not reported.

In the IS curve (Table 4.1), estimated weight on lagged output gap (α_1) is rather high (0.86), indicating strong persistence of domestic aggregate demand. The obtained posterior coefficients on monetary conditions (β_1) and interest rate transmission channel (γ_1) imply the stronger impact of the monetary policy action on economy through exchange rate channel. In addition, estimated coefficients assigned to the gap in term-of-trade (θ_1), risk premium gap (μ_1) and foreign output gap (δ_1) are lower than the priors.

The results of Phillips curve and credibility stock (Table 4.2) suggest that economic agents put lower weigh on inflation target (δ_2) in forming their expectation. Also, estimated value of agents sensitivity to inflation deviation from target (θ_2) is lower than its prior, in this way, deviation from inflation target more than 2 p.p. lead to almost zero signal to credibility. Finally, the posterior sensitivity of inflation to domestic and foreign marginal cost (β_2) is lower than prior.

In monetary policy rule (Table 4.3), posterior persistence parameter (α_3) is higher than was originally estimated by Grui (2020). In addition, monetary policy sensitivity to deviation of inflation from its target (β_3) is lower (0.7).

Table 4.4 show that nominal exchange rate is less sensitive to term-of-trade gap (γ_4).

Table 4.1: Prior and posterior distribution. IS curve

	Prior			Posterior
	Distribution	**Mean**	**Std. deviation**	**Mode**
α_1	Beta	0.79	0.15	0.86
β_1	Inverse Gamma	0.05	0.15	0.02
δ_1	Inverse Gamma	0.45	0.15	0.34
θ_1	Inverse Gamma	0.09	0.15	0.04
μ_1	Inverse Gamma	0.06	0.15	0.03
γ_1	Beta	0.4	0.15	0.37
$\varepsilon_{1,t}$	Inverse Gamma	1	1	0.21

Table 4.2: Prior and posterior distribution. Phillips curve and credibility stock

	Prior			Posterior
	Distribution	**Mean**	**Std. deviation**	**Mode**
β_2	Inverse Gamma	0.4	0.15	0.35
γ_2	Beta	0.4	0.15	0.4
α_2	Beta	0.6	0.15	0.61
δ_2	Beta	0.5	0.15	0.44
μ_b	Beta	0.6	0.15	0.63
μ_f	Beta	0.6	0.15	0.63
θ_2	Inverse Gamma	0.7	0.15	0.64
$\varepsilon_{2,t}$	Inverse Gamma	1	1	1.75

Table 4.3: Prior and posterior distribution. Taylor rule

	Prior			Posterior
	Distribution	**Mean**	**Std. deviation**	**Mode**
α_3	Beta	0.7	0.15	0.78

β_3	Inverse Gamma	1	0.15	0.7
γ_3	Inverse Gamma	0.4	0.15	0.39
$\varepsilon_{3,t}$	Inverse Gamma	1	1	0.56

Table 4.4: Prior and posterior distribution. Uncovered interest parity

	Prior			**Posterior**
	Distribution	**Mean**	**Std. deviation**	**Mode**
γ_4	Inverse Gamma	0.07	0.15	0.04
$\varepsilon_{4,t}$	Inverse Gamma	1	1	0.87

3.4 Impulse response functions

In this section, we provide impulse response functions of the key macroeconomic variables to different shock under two scenario.

By first scenario we mean model with endogenous credibility, by second- linear model without incorporated credibility to monetary policy. The only distinguish between models is equation of Phillips curve.

The Phillips curve for linear model we can get from equations (2)-(6) putting ψ_t^b and ψ_t^f equal to one. In can be presenting in following way:

$$\pi_t = \delta_2 \pi_t^T + \alpha_2(1-\delta_2)\pi_{t-1} + (1-\alpha_2)(1-\delta_2)\pi_{t+1} + \beta_2(\gamma_2 \hat{y}_t + (1-\gamma_2)\hat{z}_t) + \varepsilon_{2,t} \quad (4.11)$$

Here we assume, that agents in forming their expectation put constant weight to inflation target, last realized inflation and future expected inflation regardless to Central bank ability to reach target.

All shocks are temporary and hit economy in first period. Responses of variables to shocks are presented as deviation from control.

3.4.1 Supply shock

In Figure 4.1, the supply shock is modeled as an exogenous positive three percentage point increase in domestic Philips curve.

Outcome credibility fall by 55 percent after 5 quarter due to recent inability of monetary policy to reach target. As result of substantially drop in credibility, economic agents put much less weigh to inflation target in forming their expectation.

In response to the deviation of inflation from the target level, the monetary authority raises the key policy rate by 0.35 p.p. and 0.29 p.p. respectively in endogenous credibility and baseline scenarios. In first case, Central bank react more aggressively to rebuild stock of credibility.

The tightening monetary policy action leads to currency appreciation. Under the base model scenario, the nominal exchange rate appreciates by 0.18 p.p. after 3 quarter, compared to 0.27 p.p. under endogenous credibility scenario. Furthermore, negative output gap opens up in economy. Time-varying monetary credibility lead to tighter monetary condition than under base case, which in turn results in higher drop in domestic demand.

Nominal exchange rate appreciation and reduction in demand stimulate inflation to return to the target.

Finally, it is costlier for monetary policy to bring inflation back to target, in case of endogenous credibility, as Central bank should put additional effort to offset loss in credibility.

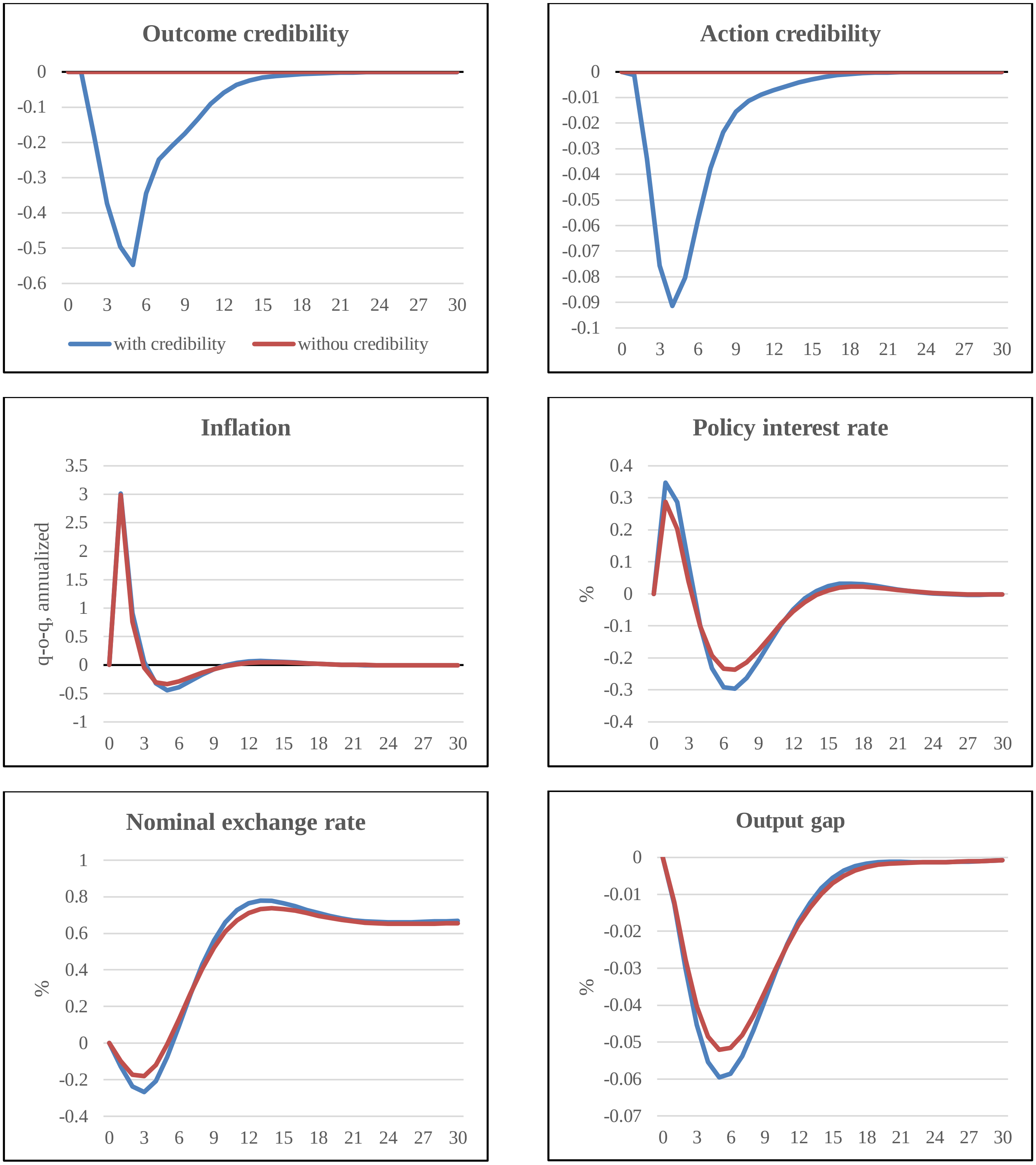


Figure 4.1: Impulse response functions to supply shock ($\varepsilon_{2,t}$)

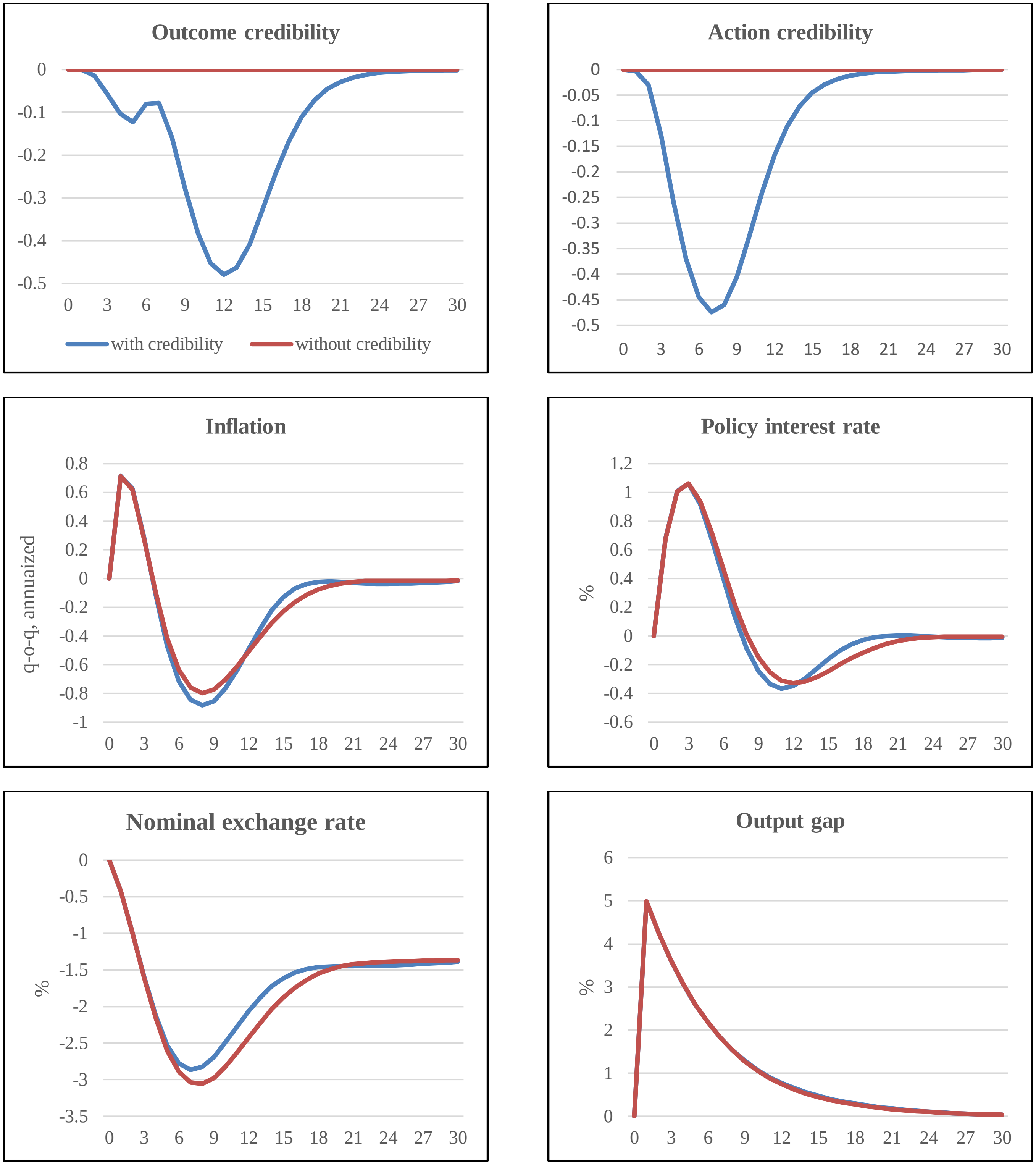
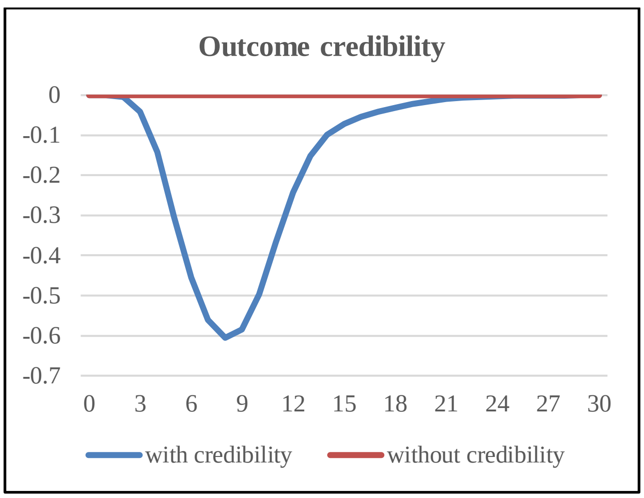


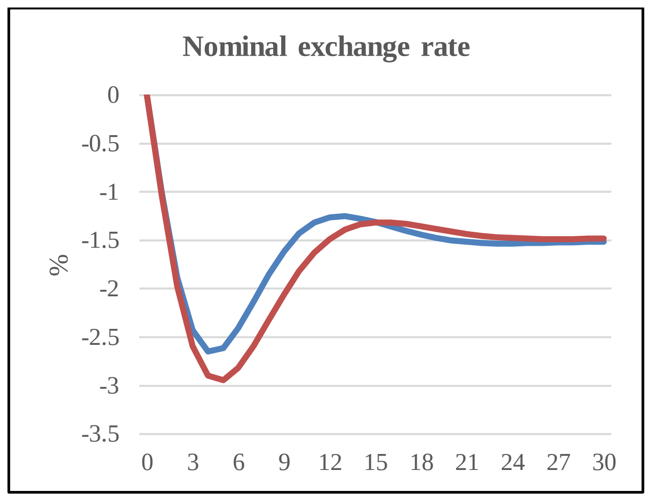
Figure 4.2: Impulse response functions to demand shock ($\varepsilon_{1,t}$)

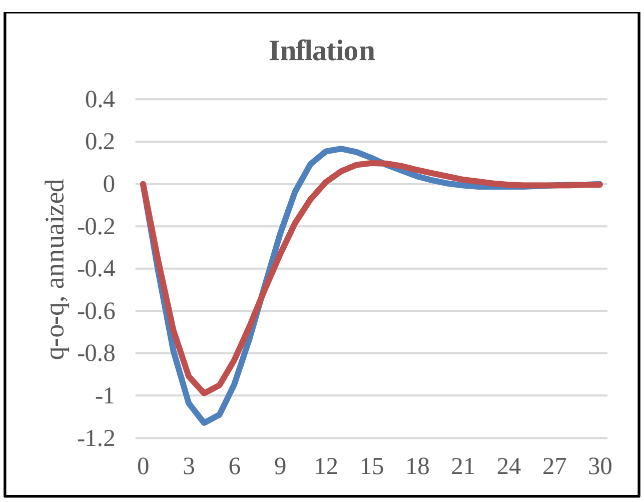
3.4.2 Monetary policy shock

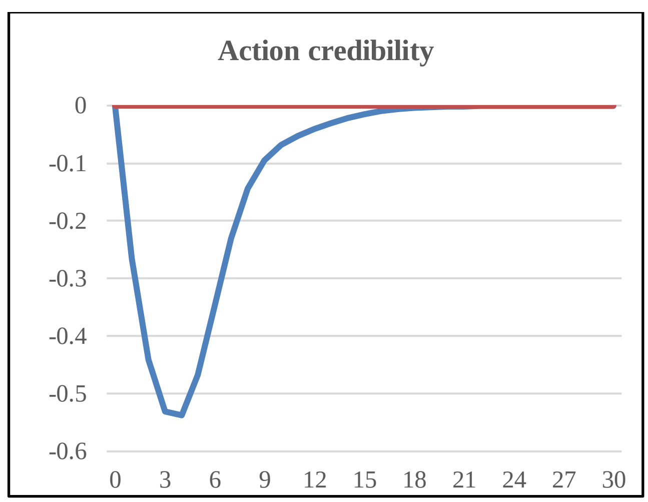
The shock of monetary policy is simulated as an increase in the key policy rate by three percentage points.

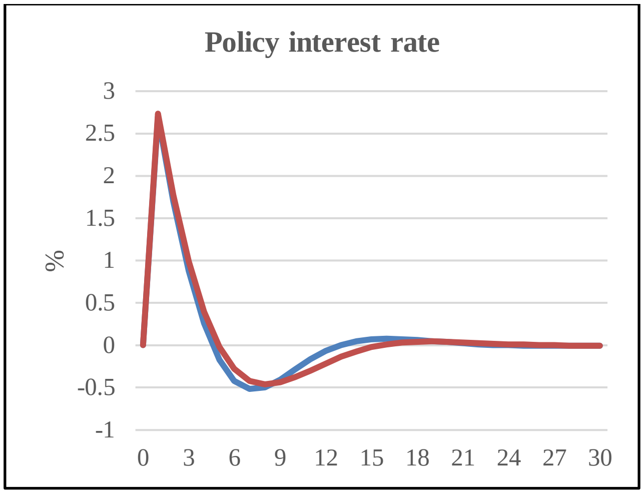
In the linear model, the increase in the policy rate leads to more appreciated currency, which peaks at 2.9 p.p. below control in 4 quarter, compared to 2.6 p.p. in 5 quarter under nonlinear mode. In addition, inflation fall below target level. Higher inflation drop under credibility scenario (by 1.1 p.p.) is caused by loss in both credibility stocks. Action and outcome credibility fall by 54 per cent after 4 quarter and 61 per cent after 8 quarter respectively due to change in agent's future expectation and past observed failures of Central bank in reaching target.











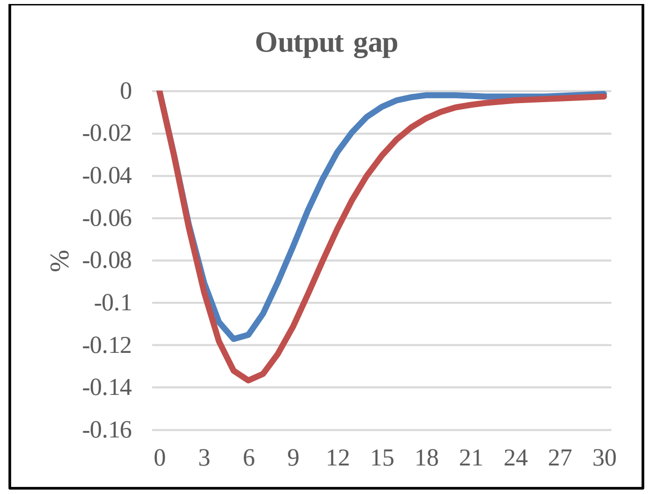


Figure 4.3: Impulse response functions to monetary policy shock ($\varepsilon_{3,t}$)

Summary

In this work, I consider backward-looking model for policy analysis. I show that Central banks conduct their monetary policy to minimize inflation deviation from target and output gap from potential level. To stabilize inflation, Central banks provide optimal rule-based policy. Monetary authority set key policy rate as function of neutral interest rate, inflation and output gap. Central banks can follow strict or flexible inflation targeting regime, which depend on weight that they put to inflation stabilization. In addition, it is shown behavior of monetary policy and economic development under different inflation targeting scenario.

Also, simple productivity-based approach for computation a multilateral indicators of price competitiveness was considered. The results for emerging and developed economies are in line with expectations. A relative price level of Brazil currently exceed benchmark level, which is approved by economic situation in country. As for Poland, Turkey and Czech Republic their price competitiveness is higher than of their trading partners. Economists and policy makers can use this framework to analyze changes in price competitiveness.

Finally, I provide empirical evidence that the incorporation of endogenous credibility in the model for policy analysis is an important aspect. Model with endogenous credibility better present process of inflation expectation formation in economy. Building of monetary policy credibility is essential for Central banks. Credible monetary policy can effect economy in way that is more effective.

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