

Macro Topics: Introduction to Matlab

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Lecture notes (December 13)

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Topics Covered Today

Neal's Model of Carer Choice

- ▶ Idea of the Model
- ▶ Setup of the Model
- ▶ Value Function
- ▶ Illustration of the Solution
- ▶ Solution of the Model
- ▶ Intuition behind the Figures
- ▶ Special Case
- ▶ Conclusions and Extensions

This lecture is based on Ljungquist & Sargent book, Chapter 6.5.

Idea of the Model

Neal's model of career choice aims to explain

- ▶ why young men switch jobs and careers often early in their work histories,
- ▶ then later focus their search on jobs within a single career, and
- ▶ finally settle down in a particular job.

Setup of the Model

A worker chooses career-job (θ, ϵ) pairs subject to the following conditions:

- ▶ There is no unemployment.
- ▶ The worker's earnings at time t are $\theta_t + \epsilon_t$.
- ▶ The worker maximizes $\mathbb{E} \sum_{t=0}^{\infty} \beta^t (\theta_t + \epsilon_t)$.
- ▶ A **career** is a draw of θ from c.d.f. F ; a **job** is a draw of ϵ from c.d.f. G .
- ▶ Successive draws are independent, and $G(0) = F(0) = 0$, $G(B_\epsilon) = F(B_\theta) = 1$.
- ▶ The worker can draw a new career only if he also draws a new job.
- ▶ However, the worker is free to retain his existing career (θ) , and to draw a new job (ϵ') .
- ▶ The worker decides at the beginning of a period whether to stay in the current career-job pair, stay in his current career but draw a new job, or to draw a new career-job pair.
- ▶ There is no recalling past jobs or careers.

Value Function

Let $v(\theta, \epsilon)$ be the optimal value of the problem at the beginning of a period for a worker with career-job pair (θ, ϵ) who is about to decide whether to draw a new career and/or job.

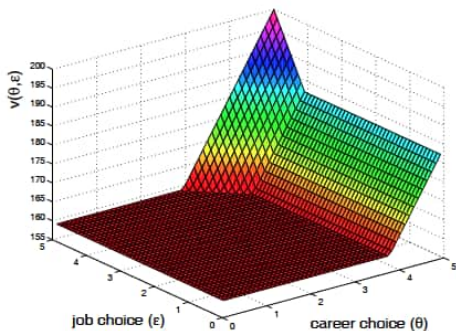
The Bellman equation is

$$v(\theta, \epsilon) = \max \left\{ \underbrace{\theta + \epsilon + \beta v(\theta, \epsilon)}_{\text{retain the present job-career pair}}, \right. \\ \left. \underbrace{\theta + \int_0^{B_\epsilon} (\epsilon' + \beta v(\theta, \epsilon')) dG(\epsilon')}_{\text{retain the present career but draw a new job}}, \right. \\ \left. \underbrace{\int_0^{B_\epsilon} \int_0^{B_\theta} (\theta' + \epsilon' + \beta v(\theta', \epsilon')) dF(\theta') dG(\epsilon')}_{\text{draw both a new job and a new career}} \right\}. \quad (1)$$

The value function is increasing in both θ and ϵ .

Illustration of the Solution: Figure 1

Optimal value function for Neal's model is computed by iterating to convergence on the Bellman equation.

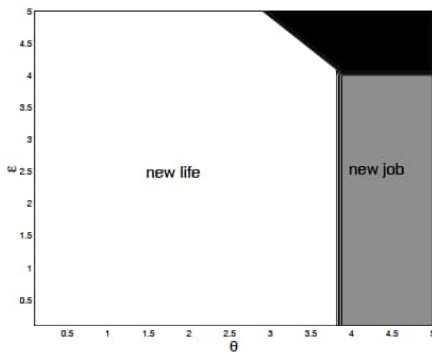


The value function is

- ▶ flat in the reject (θ, ϵ) region,
- ▶ increasing in θ only in the keep-career-but-draw-new-job region, and
- ▶ increasing in both θ and ϵ in the stay-put region.

Illustration of the Solution: Figure 2

Optimal decision rule for Neal's model.



- ▶ For high enough values of $\epsilon + \theta$, the worker stays put (black area).
- ▶ For high θ but low ϵ , the worker retains his career but searches for a better job (grey area).
- ▶ For low values of $\theta + \epsilon$, the worker finds a new career and a new job (white area).

Solution of the Model

When the career-job pair (θ, ϵ) is such that the worker chooses to stay put, the value function in (1) attains the value $(\theta + \epsilon)/(1 - \beta)$.

This happens when the decision to stay put weakly dominates the other two actions, which occurs when

$$\frac{(\theta + \epsilon)}{(1 - \beta)} \geq \max\{C(\theta), Q\}, \quad (2)$$

where Q is the value of drawing both a new job and a new career,

$$Q \equiv \int_0^{B_\epsilon} \int_0^{B_\theta} (\theta' + \epsilon' + \beta v(\theta', \epsilon')) dF(\theta') dG(\epsilon'),$$

and $C(\theta)$ is the value of drawing a new job but keeping θ :

$$C(\theta) = \theta + \int_0^{B_\epsilon} (\epsilon' + \beta v(\theta, \epsilon')) dG(\epsilon').$$

Solution of the Model

- ▶ For a given career θ , a job $\bar{\epsilon}(\theta)$ makes equation (2) hold with equality. $\bar{\epsilon}(\theta)$ solves

$$\bar{\epsilon}(\theta) = \max\{(1 - \beta)C(\theta) - \theta, (1 - \beta)Q - \theta\}.$$

- ▶ The decision to stay put is optimal for any career-job pair (θ, ϵ) that satisfies $\epsilon \geq \bar{\epsilon}(\theta)$.
- ▶ When this condition is not satisfied, the worker will either draw a new career-job pair (θ', ϵ') or only a new job ϵ' .
- ▶ Retaining the current career θ is optimal when

$$C(\theta) \geq Q. \quad (3)$$

- ▶ From (3), the critical career value $\bar{\theta}$ satisfies

$$C(\bar{\theta}) = Q. \quad (4)$$

- ▶ Independently of ϵ , the worker will never abandon any career $\theta \geq \bar{\theta}$.

Solution of the Model

- ▶ The decision rule for accepting the current career can be expressed as follows: accept the current career θ
 - ▶ if $\theta \geq \bar{\theta}$ or
 - ▶ if the current career-job pair (θ, ϵ) satisfies $\epsilon \geq \bar{\epsilon}(\theta)$.
- ▶ When $\theta \geq \bar{\theta}$, because we know that the worker will keep θ forever, it follows that

$$C(\theta) = \frac{\theta}{1-\beta} + \int_0^{B_\epsilon} J(\epsilon') dG(\epsilon'),$$

where $J(\epsilon)$ is the optimal value of $\sum_{t=0}^{\infty} \beta^t \epsilon_t$ for a worker who has just drawn ϵ , who has already decided to keep his career θ , and who is deciding whether to try a new job next period.

- ▶ The Bellman equation for J is

$$J(\epsilon) = \max \left\{ \frac{\epsilon}{1-\beta}, \epsilon + \beta \int_0^{B_\epsilon} J(\epsilon') dG(\epsilon') \right\}. \quad (5)$$

Solution of the Model

The optimal policy is of the reservation-job form:

- ▶ keep the job ϵ for $\epsilon \geq \bar{\epsilon}$,
- ▶ otherwise try a new job next period.

The absence of θ from (5) implies that in the range $\theta \geq \bar{\theta}$, $\bar{\epsilon}$ is independent of θ .

Intuition behind the Figures

- ▶ At the boundary separating the “new life” and “new job” regions of the (θ, ϵ) plane, equation (4) is satisfied.
- ▶ At the boundary separating the “new job” and “stay put” regions,

$$\frac{\theta + \epsilon}{1 - \beta} = C(\theta) = \frac{\theta}{1 - \beta} + \int_0^{B_\epsilon} J(\epsilon') dG(\epsilon').$$

- ▶ Finally, between the “new life” and “stay put” regions,

$$\frac{\theta + \epsilon}{1 - \beta} = Q,$$

which defines a diagonal line in the (θ, ϵ) plane (see Figure 2).

- ▶ The value function is the constant value Q in the “get a new life” region (i.e., draw a new (θ, ϵ) pair).

Special Case

- ▶ Probably the most interesting feature of the model is that it is possible to draw a (θ, ϵ) pair such that the value of keeping the career (θ) and drawing a new job match (ϵ') exceeds both the value of stopping search, and the value of starting again to search from the beginning by drawing a new (θ', ϵ') pair.
- ▶ This outcome occurs when a large θ is drawn with a small ϵ .
- ▶ In this case, it can occur that $\theta \geq \bar{\theta}$ and $\epsilon < \bar{\epsilon}(\theta)$.

Conclusions and Extensions

- ▶ A normative model for young workers: don't shop for a firm until you have found a career you like.
- ▶ The model predicts that workers will not switch careers after they have settled on one.
- ▶ Extending the model to include learning could help explain the later career switches that the model misses.