Macro Topics: Introduction to Matlab

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Lecture notes (December 13)

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Topics Covered Today

Neal's Model of Carer Choice

- Idea of the Model
- Setup of the Model
- Value Function
- Illustration of the Solution
- Solution of the Model
- Intuition behind the Figures
- Special Case
- Conclusions and Extensions

This lecture is based on Ljungquist & Sargent book, Chapter 6.5.

Idea of the Model

Neal's model of career choice aims to explain

- why young men switch jobs and careers often early in their work histories,
- then later focus their search on jobs within a single career, and
- finally settle down in a particular job.

Setup of the Model

A worker chooses career-job (θ, ϵ) pairs subject to the following conditions:

- There is no unemployment.
- The worker's earnings at time t are $\theta_t + \epsilon_t$.
- The worker maximizes $\mathbb{E} \sum_{t=0}^{\infty} \beta^t (\theta_t + \epsilon_t)$.
- A **career** is a draw of θ from c.d.f. F; a **job** is a draw of ϵ from c.d.f. G.
- Successive draws are independent, and $G(0) = F(0) = 0$, $G(B_\epsilon) = F(B_\theta) = 1$.
- The worker can draw a new career only if he also draws a new job.
- However, the worker is free to retain his existing career (θ), and to draw a new job (ϵ').
- The worker decides at the beginning of a period whether to stay in the current career-job pair, stay in his current career but draw a new job, or to draw a new career-job pair.
- There is no recalling past jobs or careers.

Value Function

Let $v(\theta, \epsilon)$ be the optimal value of the problem at the beginning of a period for a worker with career-job pair (θ, ϵ) who is about to decide whether to draw a new career and/or job.

The Bellman equation is

$$v(\theta, \epsilon) = \max \Big\{ \underbrace{\theta + \epsilon + \beta v(\theta, \epsilon),}_{\text{retain the present job-career pair}}$$

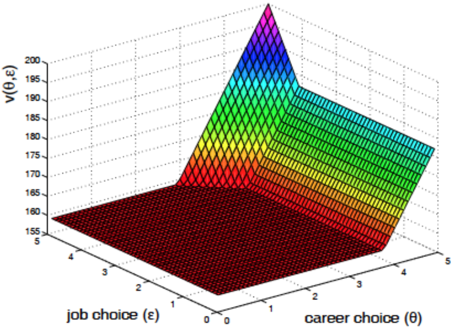
$$\underbrace{\theta + \int_0^{B_\epsilon} (\epsilon' + \beta v(\theta, \epsilon')) dG(\epsilon'),}_{\text{retain the present career but draw a new job}} \tag{1}$$

$$\underbrace{\int_0^{B_\epsilon} \int_0^{B_\theta} (\theta' + \epsilon' + \beta v(\theta', \epsilon')) dF(\theta') dG(\epsilon') \Big\}}_{\text{draw both a new job and a new career}}.$$

The value function is increasing in both θ and ϵ.

Illustration of the Solution: Figure 1

Optimal value function for Neal's model is computed by iterating to convergence on the Bellman equation.

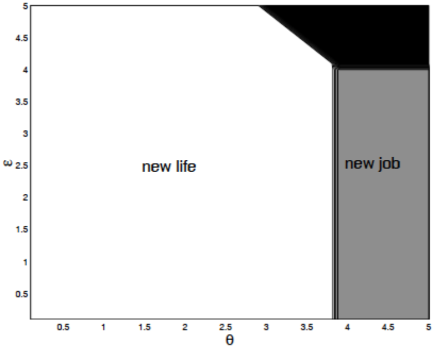


The value function is

- flat in the reject (θ, ϵ) region,
- increasing in θ only in the keep-career-but-draw-new-job region, and
- increasing in both θ and ϵ in the stay-put region.

Illustration of the Solution: Figure 2

Optimal decision rule for Neal's model.



- For high enough values of $\epsilon + \theta$, the worker stays put (black area).
- For high θ but low ϵ, the worker retains his career but searches for a better job (grey area).
- For low values of $\theta + \epsilon$, the worker finds a new career and a new job (white area).

Solution of the Model

When the career-job pair (θ, ϵ) is such that the worker chooses to stay put, the value function in (1) attains the value ${(\theta+\epsilon)}/{(1-\beta)}$.

This happens when the decision to stay put weakly dominates the other two actions, which occurs when

$$\frac{(\theta+\epsilon)}{(1-\beta)} \geq \max\{C(\theta), Q\}, \tag{2}$$

where Q is the value of drawing both a new job and a new career,

$$Q \equiv \int_0^{B_\epsilon} \int_0^{B_\theta} (\theta' + \epsilon' + \beta v(\theta', \epsilon')) dF(\theta') dG(\epsilon'),$$

and $C(\theta)$ is the value of drawing a new job but keeping θ:

$$C(\theta) = \theta + \int_0^{B_\epsilon} (\epsilon' + \beta v(\theta, \epsilon')) dG(\epsilon').$$

Solution of the Model

- For a given career θ, a job $\bar{\epsilon}(\theta)$ makes equation (2) hold with equality. $\bar{\epsilon}(\theta)$ solves

$$\bar{\epsilon}(\theta) = \max\{(1-\beta)C(\theta) - \theta, (1-\beta)Q - \theta\}.$$

- The decision to stay put is optimal for any career-job pair (θ, ϵ) that satisfies $\epsilon \geq \bar{\epsilon}(\theta)$.
- When this condition is not satisfied, the worker will either draw a new career-job pair (θ', ϵ') or only a new job ϵ'.
- Retaining the current career θ is optimal when

$$C(\theta) \geq Q. \tag{3}$$

- From (3), the critical career value $\bar{\theta}$ satisfies

$$C(\bar{\theta}) = Q. \tag{4}$$

- Independently of ϵ, the worker will never abandon any career $\theta \geq \bar{\theta}$.

Solution of the Model

- The decision rule for accepting the current career can be expressed as follows: accept the current career θ
  - if $\theta \geq \bar{\theta}$ or
  - if the current career-job pair (θ, ϵ) satisfies $\epsilon \geq \bar{\epsilon}(\theta)$.
- When $\theta \geq \bar{\theta}$, because we know that the worker will keep θ forever, it follows that

$$C(\theta) = \frac{\theta}{1-\beta} + \int_0^{B_\epsilon} J(\epsilon')dG(\epsilon'),$$

  where $J(\epsilon)$ is the optimal value of $\sum_{t=0}^{\infty} \beta^t \epsilon_t$ for a worker who has just drawn ϵ, who has already decided to keep his career θ, and who is deciding whether to try a new job next period.
- The Bellman equation for J is

$$J(\epsilon) = \max\left\{\frac{\epsilon}{1-\beta}, \epsilon + \beta \int_0^{B_\epsilon} J(\epsilon')dG(\epsilon')\right\}. \tag{5}$$

Solution of the Model

The optimal policy is of the reservation-job form:

- keep the job ϵ for $\epsilon \geq \bar{\epsilon}$,
- otherwise try a new job next period.

The absence of θ from (5) implies that in the range $\theta \geq \bar{\theta}$, $\bar{\epsilon}$ is independent of θ.

Intuition behind the Figures

- At the boundary separating the "new life" and "new job" regions of the (θ, ϵ) plane, equation (4) is satisfied.
- At the boundary separating the "new job" and "stay put" regions,

$$\frac{\theta+\epsilon}{1-\beta} = C(\theta) = \frac{\theta}{1-\beta} + \int_0^{B_\epsilon} J(\epsilon')dG(\epsilon').$$

- Finally, between the "new life" and "stay put" regions,

$$\frac{\theta+\epsilon}{1-\beta} = Q,$$

  which defines a diagonal line in the (θ, ϵ) plane (see Figure 2).
- The value function is the constant value Q in the "get a new life" region (i.e., draw a new (θ, ϵ) pair).

Special Case

- Probably the most interesting feature of the model is that it is possible to draw a (θ, ϵ) pair such that the value of keeping the career (θ) and drawing a new job match (ϵ') exceeds both the value of stopping search, and the value of starting again to search from the beginning by drawing a new (θ', ϵ') pair.
- This outcome occurs when a large θ is drawn with a small ϵ.
- In this case, it can occur that $\theta \geq \bar{\theta}$ and $\epsilon < \bar{\epsilon}(\theta)$.

Conclusions and Extensions

- A normative model for young workers: don't shop for a firm until you have found a career you like.
- The model predicts that workers will not switch careers after they have settled on one.
- Extending the model to include learning could help explain the later career switches that the model misses.