Macro Topics: Introduction to Matlab

Fall semester 2016

Lecture notes (December 6)

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Topics Covered Today

Search Models

- Worker's Problem
- Changes in Unemployment Compensation
- Extension: What If People Can Be Fired?
- Equilibrium Wage Dispersion
- Matching and Bargaining

This lecture is based on Ljungquist & Sargent book, Chapter 6.

Search and Matching

1. Labor search literature models frictions in the labor market.
2. Addresses questions like
   - Why do workers sometimes turn down offers?
   - Why are otherwise identical workers often paid differently?
   - Why do we have unemployment and unfulfilled vacancies at the same time?
3. Some of those questions are not possible to answer in a frictionless (supply/demand) framework.

Labor Search

- An unemployed worker faces a distribution of potential wage offers.
- When an offer arrives, the worker must choose whether to accept it or reject it:

$$\textit{value} = \max\{\textit{value of accepting},\ \textit{value of rejecting}\}.$$

- We will study how the worker makes decisions over time.
- We will also look at how the worker responds to changes in unemployment benefits, probability of being fired, changes in the wage distribution.

Setup of the Model

- Consider an unemployment worker who is searching for a job.
- Each period, the worker draws an offer $w \in [0, B]$ from some wage distribution $F(w) = prob(\tilde{w} \leq w)$, $F(0) = 0$, and $F(B) = 1$.
- If the worker rejects the offer, he receives unemployment compensation c per period.
- If the worker accepts the offer, he gets w forever.
- The worker is risk-neutral and maximizes the present discounted value of income:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t y_t, \quad 0 < \beta < 1.$$

Worker's Problem

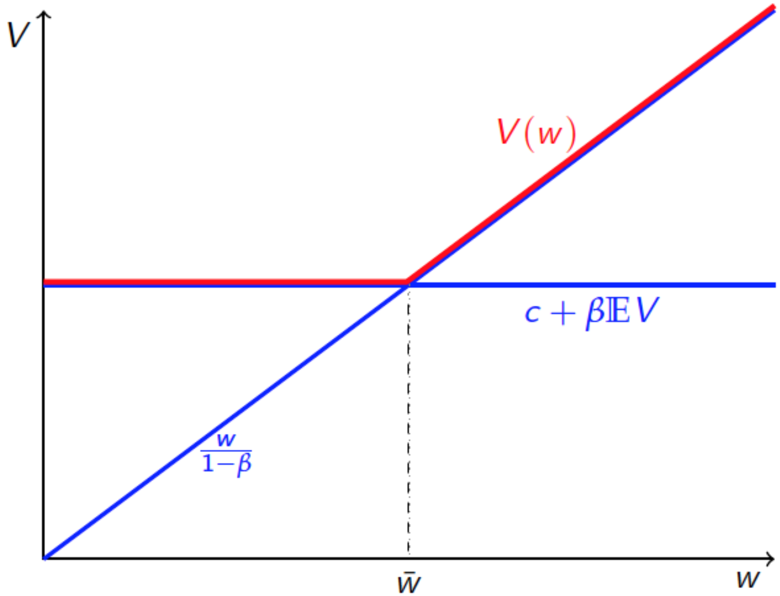
- $V(w)$ is the value of having a job offer w:

$$V(w) = \max \left\{ \frac{w}{1-\beta}, c + \beta \int_0^B V(w')dF(w') \right\}.$$

- The solution is a reservation wage strategy.
- There is a wage offer $\bar{w} \in [0, B]$ such that
  - accept the wage offer if $w \geq \bar{w}$;
  - reject the wage offer if $w \leq \bar{w}$.
- The reservation wage satisfies

$$\frac{\bar{w}}{1-\beta} = c + \beta \int_0^B V(w')dF(w').$$

Worker's Problem



Solving for the Reservation Wage

- The value function $V(w)$ is given by

$$V(w) = \begin{cases} \frac{\bar{w}}{1-\beta} & \text{if } w \leq \bar{w}, \\ \frac{w}{1-\beta} & \text{if } w \geq \bar{w}. \end{cases}$$

- Write the reservation wage equation as follows:

$$\frac{\bar{w}}{1-\beta} = c + \beta \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF(w') + \beta \int_{\bar{w}}^{B} \frac{w'}{1-\beta} dF(w').$$

- Rearranging,

$$\bar{w} - c = \frac{\beta}{1-\beta} \int_{\bar{w}}^{B} (w' - \bar{w}) dF(w'). \qquad (1)$$

- Cost of searching one more time equals expected discounted benefit of searching one more time.

Solving for the Reservation Wage

- Define a function h by the right-hand side of equation (1):

$$h(w) = \frac{\beta}{1-\beta} \int_{w}^{B} (w' - w) dF(w').$$

- The function h has the following properties:

$$h(0) = \frac{\beta}{1-\beta} \mathbb{E} w,$$

$$h(B) = 0,$$

$$h'(w) = -\frac{\beta}{1-\beta}(1 - F(w)) < 0,$$

$$h''(w) = \frac{\beta}{1-\beta} F'(w) > 0.$$

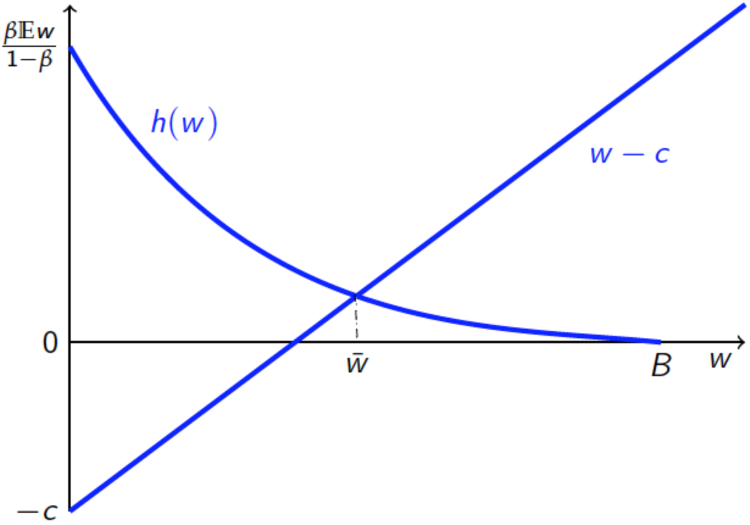
Changes in Unemployment Compensation

- The reservation wage is determined by

$$\bar{w} - c = h(\bar{w}).$$

- Increase in c leads to an increase in the reservation wage.
- Increase in β increases the reservation wage as well.

Solving for the Reservation Wage



Alternative Characterization of the Reservation Wage

- Rewrite the reservation wage equation as follows:

$$\bar{w} - c = \frac{\beta}{1-\beta}(\mathbb{E}w - \bar{w}) - \frac{\beta}{1-\beta}\int_0^{\bar{w}}(w' - \bar{w})dF(w').$$

- Rearranging,

$$\bar{w} - (1-\beta)c = \beta\mathbb{E}w - \beta\int_0^{\bar{w}}(w' - \bar{w})dF(w').$$

- Integrating by parts and rearranging,

$$\bar{w} - c = \beta(\mathbb{E}w - c) + \beta\int_0^{\bar{w}} F(w')dw'. \tag{2}$$

Alternative Characterization of the Reservation Wage

- Define a function g by the last term of equation (2):

$$g(w) = \int_0^w F(w')dw'.$$

- The function g has the following properties:

$$g(0) = 0.$$

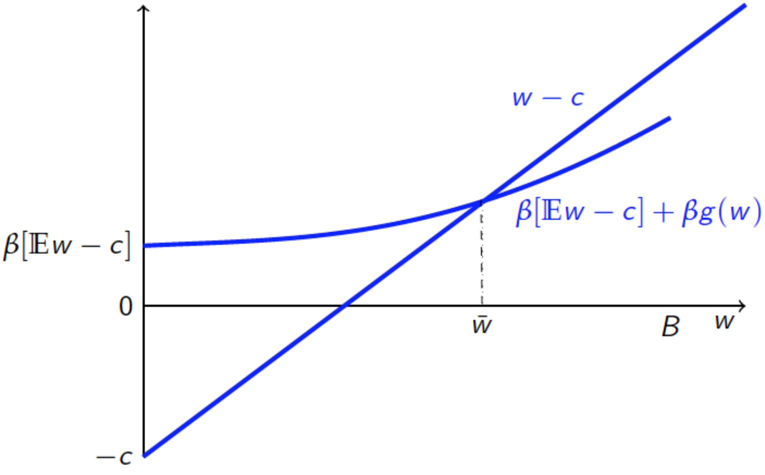
$$g'(w) = F(w) > 0,$$

$$g''(w) = F'(w) > 0.$$

- The reservation wage equation becomes

$$\bar{w} - c = \beta(\mathbb{E}w - c) + \beta g(\bar{w}).$$

Alternative Characterization of the Reservation Wage



Mean Preserving Spread in F

- Consider a class of distributions $F(w,r)$ with the same mean:

$$\int_0^B w dF(w,r_1) = \int_0^B w dF(w,r_2) \quad \forall r_1, r_2 \in \mathbb{R}.$$

- Integrating by parts,

$$\int_0^B w dF(w,r) = B - \int_0^B F(w,r) dw.$$

- Thus, if all the distributions have the same mean,

$$\int_0^B F(w,r_1) dw = \int_0^B F(w,r_2) dw \quad \forall r_1, r_2 \in \mathbb{R}. \tag{3}$$

Mean Preserving Spread in F

- Two distributions r_1 and r_2 satisfy a single crossing property if they cross only once, i.e there exists $\hat{w} \in [0, B]$ such that

$$F(w, r_1) \leq F(w, r_2) \quad \forall w \geq \hat{w},$$

$$F(w, r_1) \geq F(w, r_2) \quad \forall w \leq \hat{w}.$$

- Using a single crossing property and equation (3),

$$\int_0^w F(w, r_2) dw \geq \int_0^w F(w, r_1) dw \quad \forall r_1, r_2 \in \mathbb{R}.$$

- If this condition holds, then the distribution r_2 is a mean preserving spread of r_1.

Mean Preserving Spread in F

- With r continuous, one can say that an increase in r constitutes a mean preserving spread if

$$\int_0^B F_r(w,r)dw = 0,$$

$$\int_0^w F_r(w,r)dw \geq 0.$$

- Recall that the reservation wage is given by

$$\bar{w} - c = \beta(\mathbb{E}w - c) + \beta g(\bar{w}, r),$$

where

$$g(w,r) = \int_0^w F(w',r)dw'.$$

- Thus, a mean preserving spread increases the reservation wage.

Extension: What If People Can Be Fired?

- Suppose now that people can be fired each period with probability $\delta \in [0, 1]$.
- A worker who is fired becomes unemployed for one period before drawing a new wage.
- The value function:

$$V(w) = \max\{w + \beta(1-\delta)V(w) + \beta\delta(c + \beta\mathbb{E}V), c + \beta\mathbb{E}V\},$$

where $\mathbb{E}V = \int_0^B V(w')dF(w')$.

Extension: What If People Can Be Fired?

- Rearranging, one obtains

$$\frac{\bar{w}}{1-\beta} = c + \beta \mathbb{E} V. \tag{4}$$

- Solving for the expected value function,

$$\mathbb{E}V = \frac{1}{1-\beta}\left\{\frac{\delta\beta}{1+\delta\beta}c + \frac{1}{1+\delta\beta}\left[\bar{w} + \int_{\bar{w}}^{B}(w' - \bar{w})dF(w')\right]\right\}.$$

- Substitute to (4) to obtain

$$\bar{w} - c = \frac{\beta}{1-\beta(1-\delta)}\int_{\bar{w}}^{B}(w' - \bar{w})dF(w').$$

- An increase in δ decreases the reservation wage $\bar{w}$. As if the discount factor decreases.

Aggregate Unemployment Rate

- The aggregate unemployment rate in period t, U_t, satisfies

$$U_{t+1} = \delta(1 - U_t) + F(\bar{w})U_t.$$

- In steady state

$$U = \frac{\delta}{\delta + 1 - F(\bar{w})}.$$

- Since the mean employment duration is $d_E = 1/\delta$ and mean unemployment duration is $d_U = 1/(1-F(\bar{w}))$, one can write the steady state unemployment rate as

$$U = \frac{d_U}{d_U + d_E}.$$

Distribution of Wages

- Let $G(w)$ be the c.d.f. of wages (not wage offers!)
- Clearly, $G(\bar{w}) = 0$.
- In the steady state flows in and out of the pool of employed people with wage less than w must be equal:

$$U(F(w) - F(\bar{w})) = (1 - U)\delta G(w).$$

- Rearranging,

$$G(w) = \frac{U}{1-U} \frac{F(w) - F(\bar{w})}{\delta}.$$

- Using the expression for U,

$$G(w) = \frac{F(w) - F(\bar{w})}{1 - F(\bar{w})}.$$

- It follows that $G(B) = 1$ and $G(w) \leq F(w)$.

General Equilibrium Considerations

- So far, we have taken $F(w)$ as exogenous (partial equilibrium).
- Ways to endogenize F?
- Easy way: assume that there is exogenous heterogeneity in productivities (fishermen looking for lakes).
- Here we want to look at what happens if firms are homogeneous.
- Bad news: there will be no equilibrium dispersion in wages!
  - No firm will offer a wage smaller than $\bar{w}$: such wage will not be accepted.
  - No firm will offer a wage strictly greater than $\bar{w}$: they are losing profits.
  - Thus, all firms will offer wage $\bar{w}$.

Equilibrium Wage Dispersion

- Here we present a modification of the model that will have an equilibrium wage dispersion.
- Burdett, K., & Mortensen, D. T. (1998). Wage differentials, employer size, and unemployment. *International Economic Review, 39*(2), 257-273.
- We will introduce on the job search: workers searching while working.
- With probability α a worker is given a chance to draw a new wage offer from a given distribution F.
- Note: we will endogenize F later.

Equilibrium Wage Dispersion

- Value functions for the employed ($W(w)$) and the unemployed (U):

$$U = c + \beta \int_0^B \max[U, W(w')] dF(w'),$$

$$W(w) = w + \beta\delta U + \beta\alpha \int_0^B \max[W(w'), W(w)] dF(w') +$$

$$+ \beta(1 - \alpha - \delta) W(w).$$

- Note a different definition of the value function: U is taken before the current wage offer is revealed, while V was taken after w is revealed.

Equilibrium Wage Dispersion

- The reservation wage for the unemployed:

$$U = W(\bar{w}).$$

- Solving for the reservation wage,

$$\bar{w} - c = \beta(1-\alpha)\int_{\bar{w}}^{B}[W(w') - U]dF(w').$$

- Integrate by parts,

$$\int_{\bar{w}}^{B}[W(w') - U]dF(w') = \int_{\bar{w}}^{B} W'(w')[1 - F(w')]dw'.$$

- Using the derivative W' and combining,

$$\bar{w} - c = \int_{\bar{w}}^{B} \frac{\beta(1-\alpha)[1 - F(w')]}{1 - \beta(1-\delta) + \beta\alpha[1 - F(w')]}dw'.$$

- Previous result recovered when $\alpha = 0$.

Equilibrium Wage Dispersion

Steady State Unemployment Rate and Wage Distribution

- Steady state unemployment rate is the same as before:

$$U = \frac{\delta}{\delta + 1 - F(\bar{w})}.$$

- Equilibrium distribution of wages on $[\bar{w}, B]$: equate flows in the pool of employed people with wage less than w to the flows out of the pool:

$$(1-U)[\delta + \alpha(1 - F(w))]G(w) = U[F(w) - F(\bar{w})].$$

- Rearranging and eliminating U,

$$G(w) = \frac{1}{1 - F(\bar{w})} \frac{F(w) - F(\bar{w})}{1 + \kappa[1 - F(w)]},$$

where $\kappa = \dfrac{\alpha}{\delta}$.

Equilibrium Wage Dispersion

Workers Per Wage Offered

- Assume that F is continuous (no mass points).
- For each wage offered (for each firm offering a given wage), the number of workers accepting a job is

$$L(w) = \frac{G'(w)}{F'(w)}(1-U) = \frac{1+\kappa(1-F(\bar{w}))}{\delta+1-F(\bar{w})}\frac{1}{(1+\kappa(1-F(w)))^2}$$

  for $w \in [\bar{w}, B]$ and zero otherwise.
- L is increasing in w:

$$L'(w) = \frac{2(1+\kappa(1-F(\bar{w})))}{\delta+1-F(\bar{w})}\frac{\kappa F'(w)}{(1+\kappa(1-F(w)))^3} > 0.$$

Equilibrium Wage Dispersion

Firm Behavior

- All the firms are ex-ante identical: they have productivity p such that $p > c$.
- They choose a wage offer to maximize steady state profits π:

$$\pi = \max_{w \in [0,B]} (p - w)L(w).$$

- Trade-off: higher wages yield less revenue per worker, but attract more people.
- In equilibrium, all wages that are offered must yield the same profits:

$$(p - w)L(w) = \pi \text{ for all } w \text{ on support of } F,$$

$$(p - w)L(w) \leq \pi \text{ otherwise.}$$

Equilibrium Wage Dispersion

Firm Behavior

- No firm will offer a wage $w \leq \bar{w}$, because such wage will not get accepted ($L(w) = 0$).
- $F(w)$ is continuous:
  - Suppose not: there is a mass point at some wage $\hat{w}$. Then $L(w)$ is discontinuous at $\hat{w}$.
  - Consider a firm offering a wage $\hat{w} + \varepsilon$.
  - Such wage offer yields only a small decrease in profits per worker, but attracts discontinuously more workers.
  - Hence, offering $\hat{w}$ is not optimal and $\hat{w}$ cannot be on support of F.
- This rules out a single market wage as an equilibrium outcome!

Equilibrium Wage Dispersion

Equilibrium Profits

- Let $A \geq \bar{w}$ be the lowest wage offered $(F(A) = 0)$. The number of worker per firm at A is

$$L(w) = \frac{1}{(1+\kappa)(1+\delta)} \quad w \in [\bar{w}, A].$$

- Implies that $A = \bar{w}$.
- The equilibrium profits must be

$$\pi = \frac{p - \bar{w}}{(1+\kappa)(1+\delta)} \text{ for all } w \text{ on support of } F.$$

- Solving for the equilibrium distribution of the wage offers,

$$F(w) = \frac{1+\kappa}{\kappa}\left(1 - \sqrt{\frac{p-w}{p-\bar{w}}}\right).$$

Equilibrium Wage Dispersion

Equilibrium Profits

- We know that the lowest wage offered is $\bar{w}$. What is the highest wage offered B?

$$F(B) = 1.$$

- Substitute into the equilibrium equation for F:

$$B = p - \frac{p - \bar{w}}{(1+\kappa)^2}.$$

Equilibrium Wage Dispersion

Special Cases

- Limiting case 1: $\kappa = \frac{\alpha}{\delta}$ goes to zero.
  - The highest wage offered B goes to the reservation wage $\bar{w}$.
  - The reservation wage goes to c.
  - No equilibrium wage dispersion (Diamond (1971) paradox).
  - Why? Since worker cannot move to higher paying jobs, they are equally likely to stay at any job offering $w \geq \bar{w}$. That is, $L(w)$ is independent of w for such jobs. No trade-off for firms, and they all offer the lowest possible wage $\bar{w}$.
- Limiting case 2: $\kappa = \frac{\alpha}{\delta}$ goes to infinity.
  - The highest wage offered B goes to productivity p.
  - No profits in equilibrium.
  - The only wage offered is $w = p$.
  - Competitive solution.

Equilibrium Wage Dispersion

Additional Closed Form Solution Results

- The reservation wage:

$$\bar{w} = \frac{\alpha(1+\kappa)^2 c + (1-\alpha)\kappa^2 p}{\alpha(1+\kappa)^2 + (1-\alpha)\kappa^2},$$

$$B = \frac{\alpha c + (2\alpha + \kappa)\kappa p}{\alpha(1+\kappa)^2 + (1-\alpha)\kappa^2}.$$

Matching and Bargaining

- How are matches between firms and workers formed or dissolved?
- We need to address two questions:
  - How do workers and firms meet?
  - How are wages determined?
- Rogerson, R., Shimer, R., & Wright, R. (2005). Search-theoretic models of the labor market: A survey. *Journal of economic literature, 43*(4), 959-988.
- Based on Diamond (1982), Mortensen (1982), Pissarides (1990): Nobel Prize in 2010.

Matching and Bargaining

How Do Workers and Firms Meet?

- Suppose there are v vacancies posted by the firms and u unemployed workers.
- The number of matches m is given by a matching function:

$$m = M(u, v).$$

- Assumptions:
  - $M(0, v) = M(u, 0) = 0,$
  - $M(ku, kv) = kM(u, v)$ (constant returns to scale).
- Common specification: $M(u, v) = u^{\alpha} v^{1-\alpha}$.

Matching and Bargaining

How Do Workers and Firms Meet?

- The arrival rate (probability of finding a vacancy) for unemployed workers is

$$\alpha_w = \frac{M(u,v)}{u} = M\left(1, \frac{v}{u}\right).$$

- The arrival rate (probability of finding a worker) for employers is

$$\alpha_e = \frac{M(u,v)}{v} = M\left(\frac{u}{v}, 1\right).$$

- Because of CRS, the arrival rates depend only on the vacancy to unemployment ratio $\gamma = \frac{v}{u}$:

$$\alpha_w = \gamma m(\gamma), \quad \alpha_e = m(\gamma),$$

where $m(\gamma) = M(1/\gamma, 1)$.

- The ratio γ is called market tightness.

Matching and Bargaining

How Are Wages Determined?

- If a worker and a firm meet, they bargain over the wage.
- If a worker agrees on a wage, he produces y. The worker gets w and the firm gets $\pi = y - w$ until the match ends (with probability λ).
- Let U be the worker's value of being unemployed and $W(w)$ be the value of having a wage w.
- Similarly, let V be the value of having a vacancy, and $J(\pi)$ be the expected discounted profits (value of filled job).
- Wages are set by a generalized Nash bargaining solution:

$$\max_{w}(W(w) - U)^{\theta}(J(y - w) - V)^{1-\theta}.$$

- θ denotes bargaining power of a worker.

Matching and Bargaining

Value Functions

- For workers:

$$W(w) = w + \beta(\lambda U + (1-\lambda)W(w)),$$

$$U = b + \beta(\alpha_w W(w) + (1-\alpha_w)U),$$

where b are unemployment benefits.

- For firms:

$$J(\pi) = \pi + \beta(\lambda V + (1-\lambda)J(\pi)),$$

$$V = -k + \beta(\alpha_e J(\pi) + (1-\alpha_e)V),$$

where k are recruiting costs.

- Define reservation wage and profits by

$$W(w^R) = U,$$

$$J(\pi^R) = V.$$

Matching and Bargaining

Solving Nash Bargaining

- First-order condition in w:

$$\theta(J(y-w)-V)W'(w) = (1-\theta)(W(w)-U)J'(y-w).$$

- From the Bellman equations:

$$W'(w) = J'(\pi) = \frac{1}{1-\beta(1-\lambda)}.$$

- Hence,

$$W(w) = U + \theta(J(y-w) - V + W(w) - U).$$

- Rearranging,

$$w = w^R + \theta(\pi - \pi^R + w - w^R).$$

- Split the surplus of the current period!

Matching and Bargaining

Equilibrium

- Zero profit condition for firms:

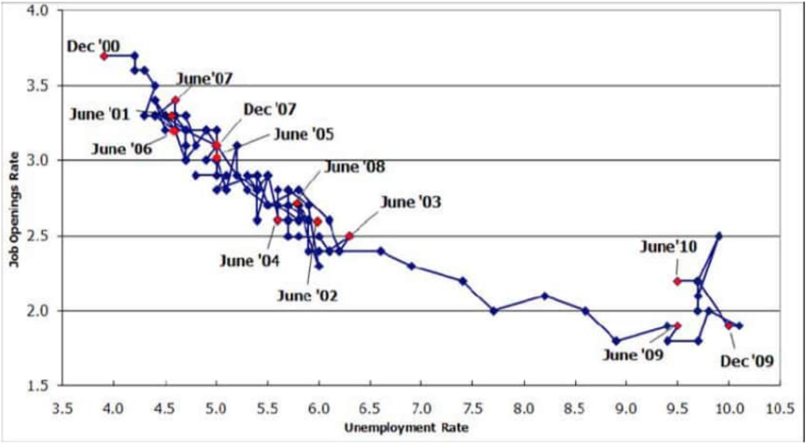
$$V = 0 \quad \Rightarrow \quad \beta \alpha_e J(\pi) = k.$$

- The steady state equilibrium consists of the value functions (J, V, W, U), the wage w and the unemployment and vacancy rates (u, v) such that
  1. The Bellman equations are satisfied.
  2. Wage w satisfies the bargaining solution.
  3. Free entry of firms: $V = 0$.
  4. Steady state unemployment rate is given

$$u = \frac{\lambda}{\lambda + \alpha_w(v/u)}.$$

Matching and Bargaining

Beveridge Curve: U.S. since 2000



Matching and Bargaining

Equilibrium

- Let $S = J - V + W - U$ be match surplus. Then $W - U = \theta S$ and $J = (1 - \theta)S$.
- Solve for U, W and J:

$$W = \frac{w}{1-\beta} - \frac{\beta}{1-\beta}\lambda\theta S,$$

$$U = \frac{b}{1-\beta} + \frac{\beta}{1-\beta}\alpha_w\theta S,$$

$$J = \frac{y-w}{1-\beta} - \frac{\beta}{1-\beta}\lambda(1-\theta)S.$$

- Rearrange:

$$w = (1-\beta)U + (\beta\lambda + 1 - \beta)\theta S,$$

$$y - w = (\beta\lambda + 1 - \beta)(1-\theta)S.$$

Matching and Bargaining

Equilibrium

- Rearrange to eliminate S:

$$w = (1-\beta)U + \theta(y - (1-\beta)U).$$

- The wage is equal to the annuity value of being unemployed plus a fraction θ of one-period surplus.
- Rewrite the value for U as

$$(1-\beta)U = b + \beta\alpha_w\theta S = b + \beta\alpha_w \frac{\theta}{1-\theta} J \ =$$

$$= \ b + \frac{\alpha_w}{\alpha_e}\frac{\theta}{1-\theta}k = b + \gamma\frac{\theta}{1-\theta}k.$$

- Combining,

$$w = b + \theta(y - b + \gamma k).$$

Matching and Bargaining

Remarks

- Comparative statics: An increase in b raises the bargained wage w, decreases job creation, increases unemployment duration.
- Many applications of this framework:
  - Business cycle analysis,
  - Heterogeneous agents or firms,
  - Marriage markets,
  - Economics of crime,
  - Housing search,
  - etc.