Macro Topics: Introduction to Matlab
Fall 2016
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Homework #1

Suggested Solutions

Problem 1. *Consider the following problem of optimal harvesting of a natural resource. A firm begins with a given stock $y_0 > 0$ of a natural resource. In each period $t = 1, 2, ...T$ of a finite horizon, the firm must decide how much of the resource to sell on the market that period. If the firm decides to sell x_t units of the resource, it receives a profit of $\pi(x_t)$ at that period, where $\pi : \mathbb{R}_+ \to \mathbb{R}$. The amount of $y_t - x_t$ of the resource left unharvested grows to an available amount of $f(y_t - x_t)$ at the beginning of the next period, where $f : \mathbb{R}_+ \to \mathbb{R}$ (negative stock is not allowed). The firm wishes to choose strategy that will maximize the sum of its profits over the model's T–period horizon.*

(1) Describe precisely the state space and the (feasible) action space. How does the transtion law look like?

(2) Assuming $\pi(x) = \log(x)$ and $f(z) = z$, solve for the firm's optimal strategy using backward induction for $T = 3$.

(3) Conisder concretization of the problem from part (2): $y_0 = 30$, $x_t = \{1, 2...10\}$, $T = 5$. Solve the problem using matlab.

Hint:

(a) You want to create a function for period reward function and function for transition law.

(b) Compute the reward function for all possible choices in all possible states (this is gonna be matrix 31×10) in the last period.

(c) Tell matlab to choose the best choice for any given state (function max *can return the index of your choice).*

(d) Move one period backwards, perform the same with taking the values of reward–to–go from previous part as given).

Note: *In the last period, assume that anything not harvested is lost.*

Solution:

(1) The state space in this formulation is given by y_t and feasible action space is given by x_t or in particular period $x_t \in [0, y_t]$. Transition law is $y_{t+1} = f(y_t - x_t)$.

(2) We can solve it by the same algorythm as in previous exercise to obtain:

$$J(y_3) = \pi(x_3) = \log(x_3),$$

$$y_3 - x_3 = 0 \ \Rightarrow \ x_3 = y_3, \ \ J(y_3) = \log(y_3);$$

$$J(y_2) = \pi(x_2) + J(y_3) = \log(x_2) + \log(y_2 - x_2),$$

$$\max_{x_2} J(y_2) \quad \Rightarrow \quad x_2 = \frac{y_2}{2}, \quad J(y_2) = 2\log(\frac{y_2}{2});$$

$$J(y_1) = \pi(x_1) + J(y_2) = \log(x_1) + 2\log(\frac{y_1 - x_1}{2}),$$

$$\max_{x_1} J(y_1) \quad \Rightarrow \quad x_1 = \frac{y_1}{3}, \quad J(y_1) = 3\log(\frac{y_1}{3}).$$

(3) See separate Matlab file.