Macro Topics: Introduction to Matlab
Fall 2016
Instructor: Olga Bychkova
Date: 9/12/2016

Exercise Session #5

Suggested Solutions

Problem 1. ***(LS 6.1: Being unemployed with a chance of an offer)***

An unemployed worker samples wage offers on the following terms. Each period, with probability ϕ, $1 > \phi > 0$, she receives no offer (we may regard this as a wage offer of zero forever). With probability $(1-\phi)$ she receives an offer to work for w forever, where w is drawn from a cumulative distribution function $F(w)$. Successive drawings across periods are independently and identically distributed. The worker chooses a strategy to maximize

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t y_t, \quad \textit{where } 0 < \beta < 1,$$

$y_t = w$ if the worker is employed, and $y_t = c$ if the worker is unemployed. Here c is unemployment compensation, and w is the wage at which the worker is employed. Assume that, having once accepted a job offer at wage w, the worker stays in the job forever.

Let $V(w)$ be the expected value of $\sum_{t=0}^{\infty}\beta^t y_t$ for an unemployed worker who has offer w in hand and who behaves optimally. Write Bellman equation for the worker's problem.

Solution: Here the maximization is over the two actions: accept the offer to work forewer at wage w, or reject the current offer and take a chance on drawing a new offer next period.

$$V(w) = \max\left\{\frac{w}{1-\beta},\ c + \phi\beta V(0) + (1-\phi)\beta\int_0^B V(w')dF(w')\right\}.$$

Problem 2. ***(LS 6.2: Two offers per period)***

Consider an unemployed worker who each period can draw two independently and identically distributed wage offers from the cumulative probability distribution function $F(w)$, with $F(0) = 0$, $F(B) = 1$ for $B < 1$. The worker will work forever at the same wage after having once accepted an offer. In the event of unemployment during a period, the worker receives unemployment compensation c. The worker derives a decision rule to maximize $\mathbb{E}\sum_{t=0}^{\infty}\beta^t y_t$, where $y_t = w$ or $y_t = c$, depending on whether she is employed or unemployed.

Let $V(w)$ be the value of $\mathbb{E}\sum_{t=0}^{\infty}\beta^t y_t$ for a currently unemployed worker who has best offer w in hand.

(a) Formulate Bellman equation for the worker's problem.

(b) *Prove that the worker's reservation wage is higher than it would be had the worker faced the same c and been drawing only one offer from the same distribution $F(w)$ each period.*

Solution:

(a) Note that the event $\max\{w_1, w_2\} < w$ is the event $(w_1 < w) \cap (w_2 < w)$. Therefore $prob\{\max(w_1, w_2) < w\} = (F(w))^2$. The worker will evidently limit his choice to the larger of the two offers each period. Bellman equation is therefore

$$V(w) = \max\left\{\frac{w}{1-\beta},\ c + \beta \int_0^B V(w')d(F^2)(w')\right\},$$

where w is the best offer in hand.

(b) The reservation wage satisfies

$$\frac{\bar{w}_2}{1-\beta} = c + \beta \int_0^B V(w')d(F^2)(w').$$

The value function $V(w)$ is given by

$$V(w) = \begin{cases} \frac{\bar{w}_2}{1-\beta} & \text{if } w \leq \bar{w}_2, \\ \frac{w}{1-\beta} & \text{if } w \geq \bar{w}_2. \end{cases}$$

Write the reservation wage equation as follows:

$$\frac{\bar{w}_2}{1-\beta} = c + \beta \int_0^{\bar{w}_2} \frac{\bar{w}_2}{1-\beta} d(F^2)(w') + \beta \int_{\bar{w}_2}^B \frac{w'}{1-\beta} d(F^2)(w').$$

Rearranging,

$$\bar{w}_2 - c = \frac{\beta}{1-\beta} \int_{\bar{w}_2}^B (w' - \bar{w}_2) d(F^2)(w').$$

Using the usual integration by part argument, on obtains the equation:

$$h_2(\bar{w}_2) \equiv (1-\beta)(\bar{w}_2 - c) - \beta \int_{\bar{w}_2}^B (1 - (F(w'))^2) dw' = 0.$$

When the worker is given only one offer, the reservation wage solves:

$$h_1(\bar{w}_1) \equiv (1-\beta)(\bar{w}_1 - c) - \beta \int_{\bar{w}_1}^B (1 - F(w')) dw' = 0.$$

Since $(F(w))^2 \leq F(w)$, we have $h_2(w) \leq h_1(w)$. Therefore:

$$0 = h_1(\bar{w}_1) = h_2(\bar{w}_2) \leq h_1(\bar{w}_2).$$

Since h_1 is increasing it follows that

$$\bar{w}_1 \leq \bar{w}_2.$$

The intuition underlying this result is as follows: the worker could always choose to ignore the second offer. This policy, possibly suboptimal, would leave the worker with a decision problem that is formally identical to the standard one–offer problem. The value of the objective function of the true problem is at least as high as the value of the objective function under the artificially restricted problem. Because the resrvation wage has the property of equating the value of accepting the job, $w/(1-\beta)$, with the value of rejecting, $c + \beta \mathbb{E} V(w')$, a higher value of $\mathbb{E} V(w')$, which results in the two–offer case, requires a higher reservation wage.

Problem 3. ***(LS 6.6: Mortensen externality)***

Two parties to a match (say, worker and firm) jointly draw a match parameter θ from a c.d.f. $F(\theta)$. Once matched, they stay matched forever, each one deriving a benefit of θ per period from the match. Each unmatched pair of agents can influence the number of offers received in a period in the following way. The worker receives n offers per period, where $n = f(c_1 + c_2)$ and c_1 represents the resources the worker devotes to searching and c_2 represents the resources the typical firm devotes to searching. Symmetrically, the representative firm receives n offers per period where $n = f(c_1 + c_2)$. (We shall define the situation so that firms and workers have the same reservation θ so that there is never unrequited love.) Both c_1 and c_2 must be chosen at the beginning of the period, prior to searching during the period. Firms and workers have the same preferences, given by the expected present value of the match parameter θ, net of search costs. The discount factor β is the same for worker and firm.

(a) Consider a Nash equilibrium in which party i chooses c_i, taking c_j, $j \neq i$, as given. Let Q_i be the value for an unmatched agent of type i before the level of c_i has been chosen. Formulate Bellman equation for agents of type 1 and 2.

(b) Consider the social planning problem of choosing c_1 and c_2 sequentially so as to maximize the criterion of λ times the utility of agent 1 plus $(1-\lambda)$ times the utility of agent 2, $0 < \lambda < 1$. Let $Q(\lambda)$ be the value for this problem for two unmatched agents before c_1 and c_2 have been chosen. Formulate Bellman equation for this problem.

(c) Comparing the results in (a) and (b), argue that, in the Nash equilibrium, the optimal amount of resources has not been devoted to search.

Solution:

(a) $Q_1 = \max_{c_1} \int \max\left\{ \dfrac{\theta}{1-\beta} - c_1,\ -c_1 + \beta Q_1 \right\} d(F^n)(\theta)$, subject to $n = f(c_1 + c_2)$, c_2 given.

$Q_2 = \max_{c_2} \int \max\left\{ \dfrac{\theta}{1-\beta} - c_2,\ -c_2 + \beta Q_2 \right\} d(F^n)(\theta)$, subject to $n = f(c_1 + c_2)$, c_1 given.

(b) $$Q(\lambda) = \max_{c_1, c_2} \int \max\{\lambda \left(\frac{\theta}{1-\beta} - c_1 \right) + (1-\lambda) \left(\frac{\theta}{1-\beta} - c_2 \right), \\ -\lambda c_1 - (1-\lambda) c_2 + \beta Q(\lambda)\} d(F^n)(\theta), \text{ subject to } n = f(c_1 + c_2).$$

(c) The Nash equilibrium is a (c_1, c_2) pair that solves the two functional equations in (a). In general, this (c_1, c_2) pair will not solve the functional equation in (b) because each agent in (a) neglects the effects of his choice of c_j on the welfare of the other agent. In general, there will be too little search in the Nash equilibrium if $f(c_1 + c_2)$ is increasing in $(c_1 + c_2)$.