LNU
Spring 2017

Introduction to Dynamic Economic Models
Instructor: Mykola Babiak
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Homework 2

Rules: You can work in groups of 3-4 people but everyone has to submit his/her own solution. Please specify the names of your group members. Deadline for the homework is 12th of May. Late submissions will give you zero points

Problem 1 (Solving the Growth Model and Tax Reform II) Consider the following economy populated by a government, a representative firm and a representative household. There is no uncertainty, and the economy and the representative household, firm and government within it last forever. The government sets sequences for taxes, $\{z_t\}_{t=0}^{+\infty}$ where $z_t = (g_t; \tau_{c,t}; \tau_{n,t}; \tau_{k,t}; \tau_{i,t}; \tau_{h,t})$ and g_t is government consumption, the rest are the rates of tax on consumption, tax on labor income, tax on capital income, investment tax credit, and lump-sum tax, respectively. They are possibly time-varying. The preferences of the household are ordered by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0; 1)$ and u is strictly concave, increasing, and twice continuously differentiable. The feasibility condition in the economy is

$$g_t + c_t + k_{t+1} = F(k_t, n_t) + (1-\delta)k_t$$

where k_t is the stock of capital owned by the household at the beginning of time t and $\delta \in (0; 1)$ is a depreciation rate. At time 0; there are complete markets for dated commodities. The household faces the budget constraint:

$$\sum_{t=0}^{\infty} \left[q_t \Big\{ (1+\tau_{c,t})c_t + (1-\tau_{i,t})(k_{t+1} - (1-\delta)k_t) \Big\} \right] \leq \sum_{t=0}^{\infty} \left[(1-\tau_{k,t})r_t k_t + (1-\tau_{n,t})w_t n_t - q_t \tau_{h,t} \right]$$

where we assume that the household inelastically supplies one unit of labor, and q_t is the price of date t consumption goods, r_t is the rental rate of date t capital, and w_t is the wage rate of date t labor.

A production firm rents labor and capital. The production function is $F(k_t; n_t)$; where $F_1; F_2 > 0; F_{11}; F_{22} < 0$. The value of the firm is

$$\sum_{t=0}^{\infty} \left[q_t F(k_t, n_t) - w_t n_t - r_t k_t \right]$$

where k_t is the firm's capital-labor ratio and n_t is the amount of labor it hires. The government sets g_t exogenously and must set taxes to satisfy the budget constraint:

$$\sum_{t=0}^{\infty} [q_t \tau_{c,t} c_t - \tau_{i,t}(k_{t+1} - (1-\delta)k_t) + \tau_{k,t} r_t k_t + \tau_{n,t} w_t n_t + q_t \tau_{h,t}] = \sum_{t=0}^{\infty} q_t g_t.$$

Suppose that the government consumes a constant amount $g_t = g > 0, t > 0$ and historically it had unlimited access to lump-sum taxes and availed itself of them. Thus, for a long time the economy had $g_t = g > 0; \tau_{c,t} = \tau_{n,t} = \tau_{k,t} = \tau_{i,t} = 0; t > 0$. Suppose that this situation had been expected to go on forever. Assume further that the economy has technology $f(k) = k^{\alpha}$ and preferences

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

and is characterized by the following parameter values:

$$\alpha = 0.33; \quad \delta = 0.2; \quad \gamma = 2; \quad \beta = 0.95; \quad g = 0.2.$$

(a) Consider the following experiment. The economy is initially at steady state when the government suddenly announces that starting at $T = 10$, there will be introduced the investment tax credit, $\tau_{i,t} = \tau_i = 0.1$ for $t > 10$. Analyze it and explain qualitatively the behavior of the economy in the phase space $(c_t; k_t)$ of the growth model including the resulting time path of the model variables. Use your implementation of the shooting method in MATLAB done before to simulate the time paths of $c; k; R; r/q$ and w/q.

(b) Consider now another experiment. Assume again that initially the economy is at its steady state, i.e. $k_0 = k$ and ($\tau_{i,t} = 0; t > 0$). Suddenly and unexpectedly, a court decision rules that lump-sum taxes will be illegal starting at time 10 and the government will have to finance expenditures using the consumption tax $\tau_{c,t}$ The value of g_t remains constant g, i.e. $\tau_{c,t} = 0$ for $t = 0; ...; 9$, and $\tau_{c,t} = \tau_c$ for $t > 10$. Ammend your program in MATLAB by another loop, in which you search for the right value of τ_c in order to determine the necessary level of the constant consumption tax rate which makes the government policy feasible.