LNU
Spring 2017

Introduction to Dynamic Economic Models
Instructor: Mykola Babiak
Date: May 2, 2017

Homework 1

Rules: You can work in groups of 3-4 people but everyone has to submit his/her own solution. Please specify the names of your group members. Deadline for the homework is 5th of May. Late submissions will give you zero points

Problem 1 There is a centralized economy with one good that can be consumed and invested. The social planner maximizes expected value of the discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_T^{1+\chi}}{1+\chi} \right) \qquad \sigma > 0, \quad \chi > 0$$

where C and N are consumption and working hours. The budget constraint and technology are given by:

$$K_{t+1} = Y_t + (1 - \delta(u_t))K_t - C_t$$
$$Y_t = A_t(K_t u_t)^\alpha N_t^{1-\alpha}$$
$$A_t = A_t^\rho v_t$$

with $K(0)$ and $A(0)$ given.

Notice that the depreciation rate is endogenous and is given by $\delta(u_t) = \mu u_t^\theta$ where $\mu > 0$ and $\theta > 1$ are parameters and u_t is the capital depreciation rate chosen by social planner. A_t is autocorrelated productivity shock and v_t is the i.i.d. random variable such that $E[\ln v_t] = 0$.

1. Identify the state and control variables(s). Write Bellman equation equation for the problem.
2. Derive First Order Conditions and the Envelope Theorem condition(s).
3. Plug E.T. into F.O.C. and derive the Euler equation for the problem.
4. Log-linearize equations of the model: capital law of motion, FOC's, Euler equation, productivity shock law of motion
5. Assume $A_t = \bar{A}$ for all times t. Derive marginal product of capital and the real interest rate in the steady state. Interpret your answer.

Problem 2 Consider social planner that seeks to maximize lifetime utility of a representative agent:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to

$$c_t + \frac{b_{t+1}}{R} + k_{t+1} = w_t(1-\tau)l_t + r_t k_t + (1-\delta)k_t + b_t, \quad R > 0,$$

where c_t is consumption, b_t is money holdings (bonds), k_t is capital, l_t is labour input. Every period household allocates it's resources between consumption, money and capital; household receives wage w (taxed by rate τ) from work, and rent from capital r it allocates to firms. Household takes tax τ, wage w_t and rent r_t as given.

There is a continuum of firms, that produce single output using Cobb-Douglas production function of the form:

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha},$$

where $A_t = A_{t-1}^{\rho} v_t, \quad \mathbb{E}\log(v_t) = 0$. Firms pay wage and capital rent to the household. Market for bonds clears.

1. Clearly identify state and control variables (pay attention to all given conditions).
2. Write down the Bellman equation for the problem (that is, write the problem in a recursive form).
3. Derive First Order Conditions and Envelope Theorem Conditions.
4. Derive the Euler Equation for the problem.
5. Solve firm's maximization problem to get the wage and the capital rent.
6. Assume that utility function is of the form:

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\phi}}{1+\phi}.$$

   Rewrite Euler Equation, FOCs and resource constraint using the above functional form for u and firm's FOC.
7. Log-linearize the above equations using Taylor series approximation.

Extra: Use Uhlig's method to log-linearize equations from 6. Compare two methods, comment.

Problem 3 There are two consumers in the economy. The first one has utility function

$$U_1(c_t^1) = \sum_{t=0}^{\infty} \beta^t c_t^1$$

and endowed with $y_t^1 = \mu$ in every period. The second consumer has utility function

$$U_2(c_t^2) = \sum_{t=0}^{\infty} \beta^t \ln c_t^2$$

and endowed with $y_t^2 = 0$ if t is even and $y_t^2 = \alpha$ if t is odd, $\alpha = \mu(1 + \frac{1}{\beta})$.

1. Define and find Arrow-Debreu equilibrium of this economy (zero-time trading).
2. Given Arrow-Debreu equilibrium prices, what is the wealth of each consumer in every period?
3. Compute the time-0 prices of three risk-free discount bonds, in particular, those promising to pay one unit of time-j consumption for $j = 0, 1, 2, t$ respectively.