LNU
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Introduction to Dynamic Economic Models
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Exercise Session 9

Problem 1 (Solving the Growth Model and Tax Reform) Consider the following economy populated by a government, a representative firm and a representative household. There is no uncertainty, and the economy and the representative household, firm and government within it last forever. The government sets sequences for taxes, $\{z_t\}_{t=0}^{+\infty}$ where $z_t = (g_t; \tau_{c,t}; \tau_{n,t}; \tau_{k,t}; \tau_{i,t}; \tau_{h,t})$ and g_t is government consumption, the rest are the rates of tax on consumption, tax on labor income, tax on capital income, investment tax credit, and lump-sum tax, respectively. They are possibly time-varying. The preferences of the household are ordered by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0; 1)$ and u is strictly concave, increasing, and twice continuously differentiable. The feasibility condition in the economy is

$$g_t + c_t + k_{t+1} = F(k_t, n_t) + (1 - \delta)k_t$$

where k_t is the stock of capital owned by the household at the beginning of time t and $\delta \in (0; 1)$ is a depreciation rate. At time 0; there are complete markets for dated commodities. The household faces the budget constraint:

$$\sum_{t=0}^{\infty} \Big[q_t \Big\{(1+\tau_{c,t})c_t + (1-\tau_{i,t})(k_{t+1} - (1-\delta)k_t)\Big\}\Big] \leq \sum_{t=0}^{\infty} \Big[(1-\tau_{k,t})r_t k_t + (1-\tau_{n,t})w_t n_t - q_t \tau_{h,t}\Big]$$

where we assume that the household inelastically supplies one unit of labor, and q_t is the price of date t consumption goods, r_t is the rental rate of date t capital, and w_t is the wage rate of date t labor.

A production firm rents labor and capital. The production function is $F(k_t; n_t)$; where $F_1; F_2 > 0; F_{11}; F_{22} < 0$. The value of the firm is

$$\sum_{t=0}^{\infty} \Big[q_t F(k_t, n_t) - w_t n_t - r_t k_t\Big]$$

where k_t is the firm's capital-labor ratio and n_t is the amount of labor it hires. The government sets g_t exogenously and must set taxes to satisfy the budget constraint:

$$\sum_{t=0}^{\infty} [q_t \tau_{c,t} c_t - \tau_{i,t}(k_{t+1} - (1-\delta)k_t) + \tau_{k,t} r_t k_t + \tau_{n,t} w_t n_t + q_t \tau_{h,t}] = \sum_{t=0}^{\infty} q_t g_t.$$

If this condition is satis ed it means that the government policy is *a budget-feasible government policy.*

(a) Write down the definition of a competitive equilibrium for this economy including the household's and the firm's problems. [Hint: A competitive equilibrium with distortionary taxes is (i) a budget-feasible policy government policy, (ii) a feasible allocation, i.e. a sequence $\{c_t; i_t; k_t; n_t\}_{t=0}^{\infty}$; (iii) a price system $\{q_t; r_t; w_t\}$ such that given the price system and the government policy, the allocation solves the representative household's problem and the firms' problem.]

(b) Get the FOC's for the household and the firm and write down the model equations for the evolution of c_t and k_t.

(c) Now the Euler equation and the resource constraint form two equations: $G(c_{t+1}; c_t; k_{t+1}) = 0$ and $H(k_{t+1}; k_t; c_t) = 0$, respectively.

(d) Suppose that initially the economy is such that there is no government consumption and, thus, no taxation:

$$g_t = \tau_{c,t} = \tau_{n,t} = \tau_{k,t} = \tau_{i,t} = \tau_{h,t} = 0, \quad t < 0.$$

Now, at time $t = 0$ the government starts to consume $g_1 > 0$ and announces that it will adjusts its consumption at time $T > 0$ to the amount, $g_2, 0 < g_2 < g_1$, i. e.

$$g_t = \begin{cases} 0, & t < 0 \\ g_1, & 0 \leq t < T \\ g_2, & T \geq 0 \end{cases}$$

The government expenditures are financed only by lump-sum taxes. Using the model equations given by G and H from (c) above and the government policies, $\{z_t\}_{t=0}^{\infty} = 0$, announced, analyze qualitatively the behavior of the model variables, consumption and capital over time in the phase space $(c_t; k_t)$ and draw the evolution of these variables and additionally, of the wage rate and the real interest rate across time.

(e) In the economy depicted in (d), prove that the timing of lump-sum taxes is irrelevant.

Assume that the economy has technology

$$f(k_t) = k_t^{\alpha}$$

and preferences

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

and is characterized by the following parameter values:

$$\alpha = 0.33; \quad \delta = 0.2; \quad \gamma = 2; \quad \beta = 0.95; \quad g = 0.2.$$

(f) How will the evolution of the economy in (d) change when a sudden unexpected earthquake, which destroys 50% of all physical capital, hits the economy at the same time as the future government policy is announced as in (d) above. Explain the economic intuition behind the optimal behavior. Compare it to the regime when there is no change in government policy.

(g) Consider the following experiment. The economy is initially at steady state when the government suddenly announces that starting at $T = 10$ there will be introduced the investment tax credit, $\tau_{i,t} = \tau_i = 0.1$ for $t \geq T = 10$. Analyze and explain **qualitatively** the behavior of the economy in the phase space $(c_t; k_t)$ of the growth model including the resulting time path of the model variables. Use your implementation of the shooting method in MATLAB to simulate the time paths of $c; k; R; r/q$, and w/q.

(h) Consider another experiment. Assume that initially the economy from (c) is at its steady state, i.e. $k_0 = k$. Suddenly and unexpectedly, the government starts to consume permanently $g_t = g$ but at the same time a court decides that lump-sum taxes are illegal and that starting at time 0; expenditures must be financed by using the consumption tax $\tau_{c,t}$. Policy advisor 1 proposes to find a constant consumption tax that satisfies the budget constraint, and to impose it from time 0 onward. What will be the effect on the steady-state value of k and c under this policy? Try to get as far as you can in analyzing the transition path from the old steady state to the new one

(i) Policy advisor number 2 proposes the following alternative policy. Instead of imposing the increase in $\tau_{c,t}$ suddenly, he proposes to ease the pain by postponing the increase for 10 years. Thus, he/she proposes to set $\tau_{c,t}$ for $t = 0; ...; 9$; then to set $\tau_{c,t} = \tau_c$ for $t \geq 10$. Please compute the steady-state level of capital associated with this policy. Can you say anything about the transition path to the new steady-state of k_t under this policy? Try to get as far as you can in analyzing the transition path from the old steady state to the new one.