LNU
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Introduction to Dynamic Economic Models
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Exercise Session 8

Problem 1 (Solving the Growth Model and Tax Reform) Consider the following economy populated by a government, a representative firm and a representative household. There is no uncertainty, and the economy and the representative household, firm and government within it last forever. The government sets sequences for taxes, $\{z_t\}_{t=0}^{+\infty}$ where $z_t = (g_t; \tau_{c,t}; \tau_{n,t}; \tau_{k,t}; \tau_{i,t}; \tau_{h,t})$ and g_t is government consumption, the rest are the rates of tax on consumption, tax on labor income, tax on capital income, investment tax credit, and lump-sum tax, respectively. They are possibly time-varying. The preferences of the household are ordered by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0;1)$ and u is strictly concave, increasing, and twice continuously differentiable. The feasibility condition in the economy is

$$g_t + c_t + k_{t+1} = F(k_t, n_t) + (1-\delta)k_t$$

where k_t is the stock of capital owned by the household at the beginning of time t and $\delta \in (0;1)$ is a depreciation rate. At time 0; there are complete markets for dated commodities. The household faces the budget constraint:

$$\sum_{t=0}^{\infty} \left[q_t \Big\{ (1+\tau_{c,t})c_t + (1-\tau_{i,t})(k_{t+1} - (1-\delta)k_t) \Big\} \right] \leq \sum_{t=0}^{\infty} \Big[(1-\tau_{k,t})r_t k_t + (1-\tau_{n,t})w_t n_t - q_t \tau_{h,t} \Big]$$

where we assume that the household inelastically supplies one unit of labor, and q_t is the price of date t consumption goods, r_t is the rental rate of date t capital, and w_t is the wage rate of date t labor.

A production firm rents labor and capital. The production function is $F(k_t; n_t)$; where $F_1; F_2 > 0; F_{11}; F_{22} < 0$. The value of the firm is

$$\sum_{t=0}^{\infty} \Big[q_t F(k_t, n_t) - w_t n_t - r_t k_t \Big]$$

where k_t is the firm's capital-labor ratio and n_t is the amount of labor it hires. The government sets g_t exogenously and must set taxes to satisfy the budget constraint:

$$\sum_{t=0}^{\infty} [q_t \tau_{c,t} c_t - \tau_{i,t}(k_{t+1} - (1-\delta)k_t) + \tau_{k,t} r_t k_t + \tau_{n,t} w_t n_t + q_t \tau_{h,t}] = \sum_{t=0}^{\infty} q_t g_t.$$

If this condition is satis ed it means that the government policy is *a budget-feasible government policy.*

(a) Write down the definition of a competitive equilibrium for this economy including the household's and the firm's problems. [Hint: A competitive equilibrium with distortionary taxes is (i) a budget-feasible policy government policy, (ii) a feasible allocation, i.e. a sequence $\{c_t; i_t; k_t; n_t\}_{t=0}^{\infty}$; (iii) a price system $\{q_t; r_t; w_t\}$ such that given the price system and the government policy, the allocation solves the representative household's problem and the firms' problem.]

(b) Get the FOC's for the household and the firm and write down the model equations for the evolution of c_t and k_t.

(c) Now the Euler equation and the resource constraint form two equations: $G(c_{t+1}; c_t; k_{t+1}) = 0$ and $H(k_{t+1}; k_t; c_t) = 0$, respectively.

(d) Suppose that the government consumes a constant amount $g_t = g > 0, t > 0$ and historically it had unlimited access to lump-sum taxes and availed itself ofthem. Thus, for a long time the economy had $g_t = g > 0; \tau_{c,t} = \tau_{n,t} = \tau_{k,t} = \tau_{i,t} = 0; t > 0$. Suppose that this situation had been expected to go on forever. Tell how to nd the steady-state capital-labor ratio for this economy.

(e) In the economy depicted in (d), prove that the timing of lump-sum taxes is irrelevant.

(f) Assume that the economy from (d) has technology $f(k) = k?\alpha$ and preferences

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

and is characterized by the following parameter values:

$$\alpha = 0.33; \delta = 0.2; \gamma = 2; \beta = 0.95; g = 0.2.$$

Assume further that an earthquake happened suddenly in such an economy which destroyed 50% of all physical capital. Using the shooting method, compute the recovery of this economy. [Write down a MATLAB program based on the algorithm for shooting method and use it for the model simulation.]