LNU
Spring 2017

Introduction to Dynamic Economic Models
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Exercise Session 7

Problem 1 Consider an economy populated by N identical infinitely-lived households with his/her lifetime utility based on consuming a single good

$$\mathsf{E}_t \sum_{j=0}^{\infty} \beta^j \frac{c_{t+j}^{1-\theta}}{1-\theta}$$

with $0 < \beta < 1$ and $\theta > 0$. To finance future consumption, the agent can transfer its wealth, $A_t > 0$, over time by investing into bond and equity holdings. One-period bonds earn a risk-free real gross interest rate R_t, measured in units of time $t+1$ consumption good per time t consumption good. A share of equity/tree entitles the owner to its stochastic dividend/fruit stream y_t and p_t be the share price in period t net of the period's dividend. At the beginning of period t, each tree yields nonstorable fruit or dividends in the amount y_t which is assumed to be governed by a Markov process, $y_t = s_t$, where s_t is the state of the economy. So any agent faces the following budget constraint

$$c_t + R_t^{-1} L_t + p_t M_t \leq A_t.$$

1. Write down the formula for the next period's wealth A_{t+1}.
2. Write down the household problem in this economy. What are the state and control/decision variables?
3. Write down the household's Bellman equation.
4. Derive the first-order conditions and the Euler equations. Interpret the optimality of the Euler equations economically.
5. Let us normalize the size of the economy by $N = 1$ and work with only a representative agent. Let us determine the general equilibrium of the economy, i.e. all the markets clear at the equilibrium prices. What these conditions imply for the trade in bonds and trees? What are the implications for the Euler conditions? Write down the changed Euler equations.

6. Assume now a similar economy which nevertheless differs in the following: (i) the economy is opened up so that their agents can borrow/lend to/from foreigners at the fixed gross interest rate $R > 1$, $R\beta = 1$, and (ii) all the trees are genetically modified in such sense that all of them are now identical and the delivery of the fruit is deterministic and growing according to $y_t = \lambda^t$, where $1 < \lambda < R$ so that $\sum_{t=0}^{\infty} \beta^t y_t < \infty$ holds. There is no international trade in trees. How will the Euler equations from 5 look like now?

7. Assume again the economy from 6. Derive the formula for the tree price as a function of its dividend stream?

8. Assume again the economy from 6 and assume further that the trade in the trees has prohibited.

   (a) Write down the household budget constraint and the next period wealth under this situation. Assume now additionally that there is a borrowing constraint $L_t \geq \bar{L}_t$ where $\bar{L}_t$ is either a borrowing limit given by $\bar{L}_t = 0$ in the case of the no-borrowing constraint or given by $\bar{L}_t = \tilde{L}_t$ in the case of the natural-borrowing constraint.

   (b) Explain the meaning if the natural-borrowing constraint? Write down the maximization problem of a representative household and its Lagrangian, including a borrowing constraint. What are the control/decision variables? Derive the FOC's. Derive the Euler equation. Discuss the situation when the borrowing limit is binding (nonbinding) and it's implication for the behaviour of the household.

   (c) Using the flow budget constraint above, derive the intertemporal budget constraint while imposing the transversatility condition (TVC), $\lim_{j \to \infty} R^{-j} L_{t+j} = 0$. Explain the economic meaning of the derived intertemporal budget constraint and the economic role of TVC.

   (d) Write down the maximization problem of a representative household and its Lagrangian using the intertemporal budget constraint from above instead of the flow budget constraint. What are the control/decision variables? Show that you get exactly the same FOCs as in the case of the flow budget constraint. What are the differences between the Lagrange multipliers for problem (b) and (d).