

LNU  
Spring 2017

Introduction to Dynamic Economic Models  
Instructor: Mykola Babiak  
Date: April 28, 2017

## Exercise Session 6

**Problem 1** An economy consists of two infinitely lived consumers named  $i = 1, 2$ . There is one nonstorable consumption good. Consumer  $i$  consumes  $c_t^i$  at time  $t$ . Consumer  $i$  ranks consumption streams by

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i),$$

where  $\beta \in (0, 1)$  and  $u(c)$  is increasing, strictly concave, and twice continuously differentiable. Consumer 1 is endowed with a stream of the consumption good  $y_t^1 = \{1, 0, 0, 1, 0, 0, 1, \dots\}$ . Consumer 2 is endowed with a stream of the consumption good  $y_t^2 = \{0, 1, 1, 0, 1, 1, 0, \dots\}$ . Assume that there are complete markets with time-0 trading.

1. Define competitive equilibrium with time 0 trading. Be careful to include definitions of all objects of which a competitive equilibrium is composed.
2. Compute a competitive equilibrium allocation with time 0 trading.
3. Suppose that one of the consumers markets a derivative asset that promises to pay 0.05 units of consumption each period. What would the price of that asset be?

**Problem 2** A pure endowment economy consists of two types of consumers. Consumers of type 1 order consumption streams of the one good according to

$$\sum_{t=0}^{\infty} \beta^t \log(c_t^1)$$

and consumers of type 2 order consumption streams according to

$$\sum_{t=0}^{\infty} \beta^t c_t^2$$

where  $c_t^i \geq 0$  is the consumption of a type  $i$  consumer and  $\beta \in (0, 1)$  is a common discount factor. The consumption good is tradable but nonstorable. There are equal numbers of the two types of consumer. The consumer of type 1 is endowed with the consumption sequence

$$y_t^1 = \begin{cases} 0 & \text{if } t = 0, 3, 6, \dots \\ \alpha & \text{if } t = 1, 4, 7, \dots \\ \mu & \text{if } t = 2, 5, 8, \dots \end{cases}$$

where  $\alpha = \mu \left(1 + \frac{1}{\beta}\right)$  and  $\mu > 0$ . The consumer of type 2 is endowed with the consumption sequence

$$y_t^2 = \mu > 0.$$

1. Define competitive equilibrium with time 0 trading. Be careful to include definitions of all objects of which a competitive equilibrium is composed.
2. Compute a competitive equilibrium allocation with time 0 trading.
3. Compute the time  $t$  wealths of the two types of consumers using the competitive equilibrium prices.