

LNU
Spring 2017

Introduction to Dynamic Economic Models
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Exercise Session 2

Problem 1 (Guess-and-Verify Method: Lucky Guess) Consider the problem of consumer who seeks to solve

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (\log c_t + \gamma \log c_{t-1}), \quad 0 < \gamma < 1$$

subject to the following constraints:

$$\begin{aligned} k_{t+1} + c_t &\leq Ak_t^\alpha, \quad A > 0, \quad \alpha \in (0, 1), \\ c_t > 0, \quad k_t > 0, \quad k_0, c_{-1} &\text{ given.} \end{aligned}$$

1. Clearly identify state and control variable(s). Set up the Bellman equation for the problem (that is write the problem in the recursive form). *Hint:* Value function for the consumer will be the function of two variables.
2. Guessing that the value function is of the form:

$$V(k_t, c_{t-1}) = E + F \log k_t + G \log c_{t-1},$$

derive constants E, F and G . Calculate the optimal policy given your value function.

Problem 2 (Guess-and-Verify Method: Failure) Consider the household that seeks to maximize his lifetime utility

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

subject to the following constraint:

$$k_{t+1} + c_t = Ak_t^\alpha + (1 - \delta)k_t, \quad 0 < \delta < 1.$$

Show that guessing that policy function is

- constant, that is $V(k_t) = C$,
- linear in state (excluding constant), that is $V(k_t) = Ck_t$

does not work.

Problem 3 (Value Function Iteration: Analytical Solution) A planner chooses a sequence $\{c_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

subject to a given value for k_0 and a transition law

$$k_{t+1} + c_t = Ak_t^\alpha,$$

where $A > 0$, $\alpha \in (0, 1)$, $\beta \in (0, 1)$. Solve this problem using value function iterations.