

Практичне заняття № 19
Контрольна робота №6

Варіант № 1

1.

$$u_{xx} + u_{yy} = 3e^{2x}y,$$
$$\begin{cases} u|_{x=0} = 0, & u_x|_{x=2} = 0, \\ u_y|_{y=0} = \sin 2x, & u|_{y=4} = 3. \end{cases}$$

2.

$$\Delta u = 3r^{3/2} \cos 3\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=0} < \infty, \\ u|_{r=4} = \frac{4}{\sqrt{\pi}} \cos 2\theta + \frac{4}{\sqrt{\pi}} \sin 6\theta. \end{cases}$$

Варіант № 2

1.

$$u_{xx} + u_{yy} = 2e^x y,$$
$$\begin{cases} u_x|_{x=0} = 0, & u|_{x=3} = 0, \\ u_y|_{y=0} = \cos 2x, & u|_{y=5} = e^x. \end{cases}$$

2.

$$\Delta u = 3r^{7/4} \cos \theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=3} = \frac{3}{\sqrt{\pi}} \sin 2\theta, \\ u|_{r=5} = \frac{5}{\sqrt{\pi}} \cos \theta. \end{cases}$$

Варіант № 3

1.

$$u_{xx} + u_{yy} = 2y \sin 2x,$$
$$\begin{cases} u|_{x=0} = 2y, & u_x|_{x=2} = 3y, \\ u_y|_{y=0} = 0, & u|_{y=4} = 0. \end{cases}$$

2.

$$\Delta u = 3r^{-7/4},$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=2} = \frac{6}{\sqrt{\pi}} \cos 5\theta + \frac{5}{\sqrt{\pi}} \sin 7\theta, \\ u|_{r=+\infty} < \infty. \end{cases}$$

Вариант № 4

1.

$$u_{xx} + u_{yy} = 3xy,$$
$$\begin{cases} u|_{x=0} = 0, & u_x|_{x=4} = 0, \\ u|_{y=0} = \cos 2x, & u_y|_{y=3} = 6. \end{cases}$$

2.

$$\Delta u = 3r^{5/2} \cos 3\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=0} < \infty, \\ u|_{r=4} = \frac{4}{\sqrt{\pi}} \cos 3\theta + \frac{4}{\sqrt{\pi}} \sin 7\theta. \end{cases}$$

Вариант № 5

1.

$$u_{xx} + u_{yy} = 2y \sin x,$$
$$\begin{cases} u_x|_{x=0} = \cos y, & u|_{x=5} = 3y, \\ u_y|_{y=0} = 0, & u|_{y=4} = 0. \end{cases}$$

2.

$$\Delta u = 3r^{9/4} \cos 2\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=4} = \frac{3}{\sqrt{\pi}} \sin 3\theta, \\ u|_{r=6} = \frac{5}{\sqrt{\pi}} \cos 2\theta. \end{cases}$$

Вариант № 6

1.

$$u_{xx} + u_{yy} = 2x \cos y,$$
$$\begin{cases} u|_{x=0} = 0, & u|_{x=5} = 0, \\ u_y|_{y=0} = 2e^x, & u|_{y=5} = 3x. \end{cases}$$

2.

$$\Delta u = 3r^{-9/4},$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=6} = \frac{6}{\sqrt{\pi}} \cos 5\theta + \frac{5}{\sqrt{\pi}} \sin 7\theta, \\ u|_{r=+\infty} < \infty. \end{cases}$$

Вариант № 7

1.

$$u_{xx} + u_{yy} = 3xe^y,$$
$$\begin{cases} u|_{x=0} = 0, & u_x|_{x=3} = 0, \\ u_y|_{y=0} = \cos 3x, & u_y|_{y=5} = 6. \end{cases}$$

2.

$$\Delta u = 3r^{7/2} \cos \theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=0} < \infty, \\ u|_{r=2} = \frac{4}{\sqrt{\pi}} \cos 4\theta + \frac{4}{\sqrt{\pi}} \sin 8\theta. \end{cases}$$

Вариант № 8

1.

$$u_{xx} + u_{yy} = 3y \sin 2x,$$
$$\begin{cases} u_x|_{x=0} = \cos y, & u|_{x=4} = 3y, \\ u_y|_{y=0} = 2x, & u|_{y=5} = \cos x. \end{cases}$$

2.

$$\Delta u = 3r^{11/4} \cos 3\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=1} = \frac{3}{\sqrt{\pi}} \sin 2\theta, \\ u|_{r=4} = \frac{5}{\sqrt{\pi}} \cos 3\theta. \end{cases}$$

Вариант № 9

1.

$$u_{xx} + u_{yy} = 2x \cos 2y,$$
$$\begin{cases} u|_{x=0} = 0, & u|_{x=3} = 0, \\ u_y|_{y=0} = 2 \cos x, & u_y|_{y=6} = 3x. \end{cases}$$

2.

$$\Delta u = 3r^{-11/4} \cos 2\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=3} = \frac{6}{\sqrt{\pi}} \cos 3\theta + \frac{5}{\sqrt{\pi}} \sin 8\theta, \\ u|_{r=+\infty} < \infty. \end{cases}$$

Вариант № 10

1.

$$u_{xx} + u_{yy} = 3e^{2x} \cos y,$$
$$\begin{cases} u_x|_{x=0} = 2y, & u|_{x=2} = 2 \cos y, \\ u_y|_{y=0} = 0, & u|_{y=4} = 0. \end{cases}$$

2.

$$\Delta u = 3r^{7/2} \cos 2\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=0} < \infty, \\ u|_{r=4} = \frac{5}{\sqrt{\pi}} \cos 3\theta + \frac{4}{\sqrt{\pi}} \sin 4\theta. \end{cases}$$

Вариант № 11

1.

$$u_{xx} + u_{yy} = 2xy,$$
$$\begin{cases} u_x|_{x=0} = 0, & u|_{x=4} = 0, \\ u_y|_{y=0} = \cos 2x, & u|_{y=8} = x. \end{cases}$$

2.

$$\Delta u = 3r^{7/4} \cos 2\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=2} = \frac{5}{\sqrt{\pi}} \sin 3\theta, \\ u|_{r=5} = \frac{6}{\sqrt{\pi}} \cos 2\theta. \end{cases}$$

Вариант № 12

1.

$$u_{xx} + u_{yy} = 7y \sin 3x,$$
$$\begin{cases} u|_{x=0} = 2y, & u_x|_{x=3} = 3y, \\ u_y|_{y=0} = 0, & u|_{y=5} = 0. \end{cases}$$

2.

$$\Delta u = 4r^{-11/4} \cos \theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=2} = \frac{4}{\sqrt{\pi}} \cos 6\theta + \frac{5}{\sqrt{\pi}} \sin 3\theta, \\ u|_{r=+\infty} < \infty. \end{cases}$$

Вариант № 13

1.

$$u_{xx} + u_{yy} = 3xy,$$
$$\begin{cases} u|_{x=0} = 0, & u_x|_{x=4} = 0, \\ u|_{y=0} = \cos 2x, & u_y|_{y=3} = 6. \end{cases}$$

2.

$$\Delta u = 3r^{5/2} \cos 3\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=0} < \infty, \\ u|_{r=4} = \frac{4}{\sqrt{\pi}} \cos 3\theta + \frac{4}{\sqrt{\pi}} \sin 7\theta. \end{cases}$$

Вариант № 14

1.

$$u_{xx} + u_{yy} = 2y \cos 2x,$$
$$\begin{cases} u_x|_{x=0} = \cos y, & u|_{x=6} = 4y, \\ u|_{y=0} = 0, & u|_{y=4} = 0. \end{cases}$$

2.

$$\Delta u = 4r^{9/4} \cos 3\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=3} = \frac{3}{\sqrt{\pi}} \sin 2\theta, \\ u|_{r=6} = \frac{9}{\sqrt{\pi}} \cos 3\theta. \end{cases}$$

Вариант № 15

1.

$$u_{xx} + u_{yy} = 3xy,$$
$$\begin{cases} u|_{x=0} = 0, & u|_{x=4} = 0, \\ u_y|_{y=0} = 2e^x, & u|_{y=3} = 3x. \end{cases}$$

2.

$$\Delta u = 3r^{-9/4},$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=6} = \frac{6}{\sqrt{\pi}} \cos 5\theta + \frac{5}{\sqrt{\pi}} \sin 7\theta, \\ u|_{r=+\infty} < \infty. \end{cases}$$

Вариант № 16

1.

$$u_{xx} + u_{yy} = 3xe^y,$$
$$\begin{cases} u|_{x=0} = 0, & u_x|_{x=3} = 0, \\ u_y|_{y=0} = \cos 3x, & u_y|_{y=5} = 6. \end{cases}$$

2.

$$\Delta u = 4r^{7/2} \cos 2\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=0} < \infty, \\ u|_{r=2} = \frac{4}{\sqrt{\pi}} \cos 5\theta + \frac{4}{\sqrt{\pi}} \sin 2\theta. \end{cases}$$

Вариант № 17

1.

$$u_{xx} + u_{yy} = 2y \sin 3x,$$
$$\begin{cases} u_x|_{x=0} = \cos y, & u|_{x=5} = 3y, \\ u_y|_{y=0} = 0, & u|_{y=5} = 0. \end{cases}$$

2.

$$\Delta u = 3r^{11/4} \cos 3\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=1} = \frac{3}{\sqrt{\pi}} \sin 2\theta, \\ u|_{r=4} = \frac{5}{\sqrt{\pi}} \cos 3\theta. \end{cases}$$

Вариант № 18

1.

$$u_{xx} + u_{yy} = 2x \cos 3y,$$
$$\begin{cases} u|_{x=0} = 0, & u_x|_{x=4} = 0, \\ u_y|_{y=0} = 2 \cos x, & u_y|_{y=6} = 3x. \end{cases}$$

2.

$$\Delta u = 3r^{-13/4} \cos 3\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=3} = \frac{6}{\sqrt{\pi}} \cos 2\theta + \frac{5}{\sqrt{\pi}} \sin 5\theta, \\ u|_{r=+\infty} < \infty. \end{cases}$$

Вариант № 19

1.

$$u_{xx} + u_{yy} = 3x \cos 2y,$$
$$\begin{cases} u|_{x=0} = 0, & u_x|_{x=2} = 0, \\ u_y|_{y=0} = \cos 4x, & u_y|_{y=6} = 5. \end{cases}$$

2.

$$\Delta u = 3r^{9/2} \cos \theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=0} < \infty, \\ u|_{r=2} = \frac{5}{\sqrt{\pi}} \cos 3\theta + \frac{6}{\sqrt{\pi}} \sin 9\theta. \end{cases}$$

Вариант № 20

1.

$$u_{xx} + u_{yy} = 3y \sin 2x,$$
$$\begin{cases} u_x|_{x=0} = 0, & u|_{x=2} = 0, \\ u_y|_{y=0} = 2x, & u_y|_{y=5} = \cos x. \end{cases}$$

2.

$$\Delta u = 3r^{13/4} \cos 2\theta,$$
$$u(r, \theta) = u(r, \theta + 2\pi), \quad \begin{cases} u|_{r=2} = \frac{4}{\sqrt{\pi}} \sin 3\theta, \\ u|_{r=4} = \frac{6}{\sqrt{\pi}} \cos 4\theta. \end{cases}$$