## Formal Languages, Automata and

 Codes
## Oleg Gutik



## Lecture 13

### 4.3 Identifying Nonregular Languages

Regular languages can be infinite, as most of our examples have demonstrated. The fact that regular languages are associated with automata that have finite memory, however, imposes some limits on the structure of a regular language. Some narrow restrictions must be obeyed if regularity is to hold. Intuition tells us that a language is regular only if, in processing any string, the information that has to be remembered at any stage is strictly limited. This is true, but has to be shown precisely to be used in any meaningful way. There are several ways in which this can be done.

## Using the Pigeonhole Principle

The term "pigeonhole principle" is used by mathematicians to refer to the following simple observation. If we put n objects into m boxes (pigeonholes), and if $n>m$ then at least one box must have more than one item in it. This is such an obvious fact that it is surprising how many deep results can be obtained from it.

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Suppose $L$ is regular. Then some DFA $M=\left(Q,\{a, b\}, \delta, q_{0}, F\right)$ exists for it. Now look at $\delta^{*}\left(q_{0}, a^{i}\right)$ for $i=1,2,3, \ldots$. Since there are an unlimited number of $i$ 's, but only a finite number of states in $M$, the Pigeonhole Principle tells us that there must be some state, say $q$, such that
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From this we can conclude that

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\delta^{*}\left(q_{0}, a^{m} b^{n}\right) & =\delta^{*}\left(\delta^{*}\left(q_{0}, a^{m}\right), b^{n}\right)= \\
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This contradicts the original assumption that $M$ accepts $a^{m} b^{n}$ only if $n=m$, and leads us to conclude that $L$ cannot be regular.

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\end{aligned}
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with $n \neq m$. But since $M$ accepts $a^{n} b^{n}$ we must have

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\delta^{*}\left(q, b^{n}\right)=q_{f} \in F .
$$

From this we can conclude that

$$
\begin{aligned}
\delta^{*}\left(q_{0}, a^{m} b^{n}\right) & =\delta^{*}\left(\delta^{*}\left(q_{0}, a^{m}\right), b^{n}\right)= \\
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This contradicts the original assumption that $M$ accepts $a^{m} b^{n}$ only if $n=m$, and leads us to conclude that $L$ cannot be regular.

## Example 4.6

Is the language $L=\left\{a^{n} b^{n}: n \geqslant 0\right\}$ regular? The answer is no, as we show using a proof by contradiction.
Suppose $L$ is regular. Then some DFA $M=\left(Q,\{a, b\}, \delta, q_{0}, F\right)$ exists for it. Now look at $\delta^{*}\left(q_{0}, a^{i}\right)$ for $i=1,2,3, \ldots$. Since there are an unlimited number of $i$ 's, but only a finite number of states in $M$, the Pigeonhole Principle tells us that there must be some state, say $q$, such that
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### 4.3 Identifying Nonregular Languages

In this argument, the Pigeonhole Principle is just a way of stating unambiguously what we mean when we say that a finite automaton has a limited memory. To accept all $a^{n} b^{n i}$, an automaton would have to differentiate between all prefixes $a^{n}$ and $a^{m}$. But since there are only a finite number of internal states with which to do this, there are some $n$ and $m$ for which the distinction cannot be made.

In order to use this type of argument in a variety of situations, it is convenient to codify it as a general theorem. There are several ways to do this; the one we give here is perhaps the most famous one.

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If, for some language $L$, there is even one string $w$ that does not have this property, $L$ cannot be regular. This observation can be formally stated as a theorem called the Pumping Lemma.
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## Theorem 4.8

Proof. If $L$ is regular the there exists a DFA that recognizes it. Let such a DFA have states labeled $q_{0}, q_{1}, q_{2}, \ldots, q_{n}$. Now take a string $w$ in $L$ such that $|w| \geqslant m=n+1$. Since $L$ is assumed to be infinite, this can always be done. Consider the set of states the automaton goes through as it processes $w$, say $q_{0}, q_{i}, q_{j}, \ldots, q_{f}$.

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Let $L$ be an infinite regular language. Then there exists some positive integer $m$ such that any $w \in L$ with $|w| \geqslant m$ can be decomposed as
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w=x y z
$$

and

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|x y| \leqslant m
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such that

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|y| \geqslant 1
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\begin{equation*}
w_{i}=x y^{i} z \tag{1}
\end{equation*}
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is also in $L$ for all $i=0,1,2, \ldots$.
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### 4.3 Identifying Nonregular Languages

Since this sequence has exactly $|w|+1$ entries, at least one state must be repeated, and such a repetition must start no later than the $n$th move. Thus, the sequence must look like

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q_{0}, q_{i}, q_{j}, \ldots, q_{r}, \ldots, q_{r}, \ldots, q_{f}
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indicating there must be substrings $x, y, z$ of $w$ such that

$$
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\delta^{*}\left(q_{0}, x\right) & =q_{r}, \\
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with $|x y| \leqslant n+1=m$ and $|y| \geqslant 1$. From this it immediately follows that

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and so on, completing the proof of the theorem.
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In applying the Pumping Lemma, we must keep in mind what the theorem says. We are guaranteed the existence of an $m$ as well as the decomposition $x y z$, but we do not know what they are. We cannot claim that we have reached a contradiction just because the Pumping Lemma is violated for some specific values of $m$ or $x y z$. On the other hand, the Pumping Lemma holds for every $w \in L$ and every $i$. Therefore, if the Pumping Lemma is violated even for one $w$ or $i$, then the language cannot be regular.
The correct argument can be visualized as a game we play against an opponent. Our goal is to win the game by establishing a contradiction of the Pumping Lemma, while the opponent tries to foil us. There are four moves in the game.

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Because of this choice, and the requirement that $|x y| \leqslant m$, the opponent is restricted in Step 3 to choosing a string $y$ that consists entirely of $a$ 's. In Step 4 , we use $i=0$.
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### 4.3 Identifying Nonregular Languages

A strategy that allows us to win whatever the opponent's choices is tantamount to a proof that the language is not regular. In this, Step 2 is crucial. While we cannot force the opponent to pick a particular decomposition of $w$, we may be able to choose $w$ so that the opponent is very restricted in Step 3, forcing a choice of $x, y$, and $z$ that allows us to produce a violation of the Pumping Lemma on our next move.

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w=a^{2 m},
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which is in $L$. To defeat us, the opponent need only pick

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y=a a .
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Now $w_{i}$ is in $L$ for all $i$, and we lose.
To apply the Pumping Lemma we cannot assume that the opponent will make a wrong move. If, in the case where we pick $w=a^{2 m}$, the opponent were to pick

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### 4.3 Identifying Nonregular Languages

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L=\left\{w \in \Sigma^{*}: n_{a}(w)<n_{b}(w)\right\}
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is not regular.
Sunnose wne are given $m$. Since we have complete freedom in choosing $w$, we
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y=a^{k}, \quad 1 \leqslant k \leqslant m
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We now pump up, using $i=2$. The resulting string

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y=a^{k}, \quad 1 \leqslant k \leqslant m
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### 4.3 Identifying Nonregular Languages

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Let $\Sigma=\{a, b\}$. The language

$$
L=\left\{(a b)^{n} a^{k}: n>k, k \geqslant 0\right\}
$$

is not regular.
Given $m$, we nick as our string

$$
w=(a b)^{m+1} a^{m}
$$

which is in $L$. Since of the constraint $|x y| \leqslant m$, both $x$ and $y$ must be in the part of the string made up of $a b$ 's. The choice of $x$ does not affect the argument, so let us see what can be done with $y$. If our opponent picks $y=a$, we choose $i=0$ and get a string not in $L\left((a b)^{*} a^{*}\right)$. If the opponent picks $y=a b$, then we can choose $i=0$ again. Now we get the string $(a b)^{m} a^{m}$, which is not in $L$. In the same way, we can deal with any possible choice by the opponent, thereby proving our claim.

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### 4.3 Identifying Nonregular Languages

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### 4.3 Identifying Nonregular Languages

## Example 4.11

In some cases, closure properties can be used to relate a given problem to one we have already classified. This may be simpler than a direct application of the Pumping Lemma.

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Let $\Sigma=\{a, b\}$. The language

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If $w=x y z$ is the decomposition, then clearly

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with $1 \leqslant k \leqslant m$. In that case,

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w_{0}=a^{m^{2}-k}
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But $m^{2}-k>(m-1)^{2}$, so that $w_{0}$ cannot be in $L$. Therefore, the language is not regular.

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### 4.3 Identifying Nonregular Languages

## Example 4.12

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Let $\Sigma=\{a, b, c\}$. The language

$$
L=\left\{a^{n} b^{k} c^{n+k}: n \geqslant 0, k \geqslant 0\right\}
$$

is not regular.
It is not difficult to apply the Pumping Lemma directly, but it is even easier to use closure under homomorphism. Take

$$
h(a)=a, \quad h(b)=a, \quad h(c)=c,
$$

then

$$
\begin{aligned}
h(L) & =\left\{a^{n+k} c^{n+k}: n+k \geqslant 0\right\}= \\
& =\left\{a^{i} c^{i}: i \geqslant 0\right\} .
\end{aligned}
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But we know this language is not regular; therefore, $L$ cannot be regular either.

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### 4.3 Identifying Nonregular Languages

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i=1+\frac{m m!}{k}
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One mistake is to try using the Pumping Lemma to show that a language is regular. Even if you can show that no string in a language $L$ can ever be pumped out, you cannot conclude that $L$ is regular. The Pumping Lemma can only be used to prove that a language is not regular.
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## Thank You for attention!


[^0]:    Using the Pigeonhole Principle
    The term "pigeonhole principle" is used by mathematicians to refer to the following simple observation. If we put $n$ objects into $m$ boxes (pigeonholes) and if $n>m$ then at least one box must have more than one item in it. This is such an obvious fact that it is surprising how many deep results can be obtained from it

[^1]:    Example 4.7

[^2]:    (a) We try to pick $i$ in such a way that the pumped string $w_{2}$

[^3]:    Because of this choice, and the requirement that $|x y| \leqslant m$, the opponent is restricted in Step 3 to choosing a string $y$ that consists entirely of $a$ 's. In Step 4 , we use $i=0$. The string obtained in this fashion has fewer $a$ 's on the left than on the right and so cannot be of the form $w w^{R}$. Therefore, $L$ is not regular.

[^4]:    regular

[^5]:    assumes that the opponent is so accommodating is automatically incorrect.

[^6]:    Pumping Lemma

[^7]:    is not regular. An argument that starts with "Given $m$, let $w=a^{m} \ldots$," is incorrect because $m$ is not necessarily prime. To avoid this pitfall, we need to start with something like "Given $m$, let $w=a^{M}$, where $M$ is a prime number larger than $m$."

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