Formal Languages, Automata and Codes

Oleg Gutik

Lecture 10

A third way of describing regular languages is by means of certain grammars. Grammars are often an alternative way of specifying languages. Whenever we define a language family through an automaton or in some other way, we are interested in knowing what kind of grammar we can associate with the family. First, we look at grammars that generate regular languages.

Right- and Left-Linear Grammars

Definition 3.3

A grammar $G = (V, T, S, P)$ is said to be *right-linear* if all productions are of the form

$$A \to xB,$$
$$A \to x,$$

where $A, B \in V$, and $x \in T^*$. A grammar is said to be *left-linear* if all productions are of the form

$$A \to Bx,$$
$$A \to x.$$

A *regular grammar* is one that is either right-linear or left-linear.

Note that in a regular grammar, at most one variable appears on the right side of any production. Furthermore, that variable must consistently be either the rightmost or leftmost symbol of the right side of any production.

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Example 3.13

The grammar $G_1 = (\{S\}, \{a, b\}, S, P_1)$, with P_1 given as

$$S \to abS|a$$

is right-linear. The grammar $G_2 = (\{S, S_1, S_2\}, \{a, b\}, S, P_2)$, with productions

$$S \to S_1ab,$$

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$$S_2 \to a,$$

is left-linear. Both G_1 and G_2 are regular grammars.

The sequence

$$S \Rightarrow abS \Rightarrow ababS \Rightarrow ababa$$

is a derivation with G_1. From this single instance it is easy to conjecture that $L(G_1)$ is the language denoted by the regular expression $r = (ab)^*a$. In a similar way, we can see that $L(G_2)$ is the regular language $L(aab(ab)^*)$.

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Example 3.14

The grammar $G = (\{S, A, B\}, \{a, b\}, S, P)$ with productions

$$S \to A,$$
$$A \to aB|\lambda,$$
$$B \to Ab,$$

is not regular. Although every production is either in right-linear or left-linear form, the grammar itself is neither right-linear nor left-linear, and therefore is not regular. The grammar is an example of a linear grammar. A linear grammar is a grammar in which at most one variable can occur on the right side of any production, without restriction on the position of this variable. Clearly, a regular grammar is always linear, but not all linear grammars are regular.

Our next goal will be to show that regular grammars are associated with regular languages and that for every regular language there is a regular grammar. Thus, regular grammars are another way of talking about regular languages.

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Our next goal will be to show that regular grammars are associated with regular languages and that for every regular language there is a regular grammar. Thus, regular grammars are another way of talking about regular languages.

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The grammar $G = (\{S, A, B\}, \{a, b\}, S, P)$ with productions

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$$ab \cdots cD \Rightarrow ab \cdots cdE,$$

arrived at by using a production $D \to dE$. The corresponding NFA can imitate this step by going from state D to state E when a symbol d is encountered. In this scheme, the state of the automaton corresponds to the variable in the sentential form, while the part of the string already processed is identical to the terminal prefix of the sentential form. This simple idea is the basis for the following theorem.

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Theorem 3.3

Let $G = (V, T, S, P)$ be a right-linear grammar. Then $L(G)$ is a regular language.

Proof. We assume that $V = V_0, V_1, \ldots$, that $S = V_0$, and that we have productions of the form $V_0 \to v_1 V_i$, $V_i \to v_2 V_j$, ... or $V_n \to v_l$, ... If w is a string in $L(G)$, then because of the form of the productions

$$\begin{aligned} V_0 &\Rightarrow v_1 V_i \Rightarrow \\ &\Rightarrow v_1 v_2 V_j \overset{*}{\Rightarrow} \\ &\overset{*}{\Rightarrow} v_1 v_2 \cdots v_k V_n \Rightarrow \\ &\Rightarrow v_1 v_2 \cdots v_k v_l = w. \end{aligned} \tag{1}$$

The automaton to be constructed will reproduce the derivation by consuming each of these v's in turn. The initial state of the automaton will be labeled V_0, and for each variable V_i there will be a nonfinal state labeled V_i. For each production

$$V_i \to a_1 a_2 \cdots a_m V_j,$$

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Proof. We assume that $V = V_0, V_1, \ldots$, that $S = V_0$, and that we have productions of the form $V_0 \to v_1 V_i$, $V_i \to v_2 V_j$, ... or $V_n \to v_l$, ... If w is a string in $L(G)$, then because of the form of the productions

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the corresponding transition of the automaton will be

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where V_f is a final state. The intermediate states that are needed to do this are of no concern and can be given arbitrary labels. The general scheme is shown in the following Figure.

Represents $V_i \to a_1 a_2 \cdots a_m V_j$

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The complete automaton is assembled from such individual parts.

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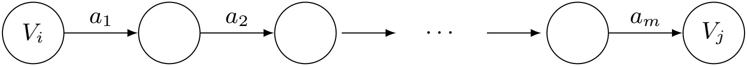

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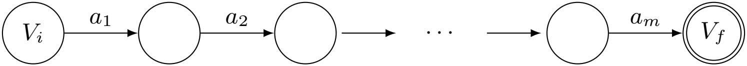

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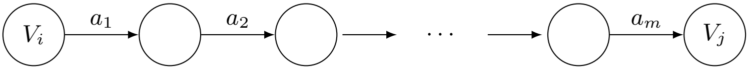

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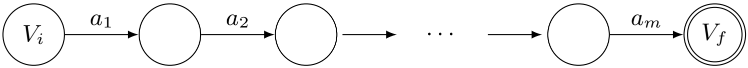

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Suppose now that $w \in L(G)$ so that (1) is satisfied. In the NFA there is, by construction, a path from V_0 to V_i labeled v_1, a path from V_i to V_j labeled v_2, and so on, so that clearly

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Conversely, assume that w is accepted by M. Because of the way in which M was constructed, to accept w the automaton has to pass through a sequence of states $V_0, V_i, \ldots$ to V_f, using paths labeled $v_1, v_2, \ldots$. Therefore, w must have the form

$$w = v_1 v_2 \cdots v_k v_l$$

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$$V_0 \Rightarrow v_1 V_i \Rightarrow v_1 v_2 V_j \stackrel{*}{\Rightarrow} v_1 v_2 \cdots v_k V_k \Rightarrow v_1 v_2 \cdots v_k v_l$$

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$$w = v_1 v_2 \cdots v_k v_l$$

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Example 3.15

Construct a finite automaton that accepts the language generated by the grammar

$$V_0 \to aV_1,$$
$$V_1 \to abV_0|b,$$

where V_0 is the start variable. We start the transition graph with vertices V_0, V_1, and V_f. The first production rule creates an edge labeled a between V_0 and V_1. For the second rule, we need to introduce an additional vertex so that there is a path labeled ab between V_1 and V_0. Finally, we need to add an edge labeled b between V_1 and V_f, giving the automaton shown in the following Figure.

The language generated by the grammar and accepted by the automaton is the regular language $L((aab)^*ab)$.

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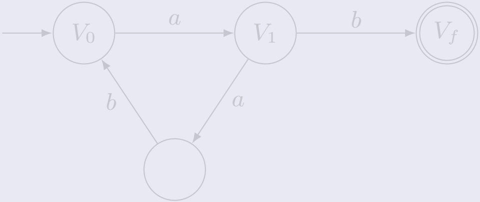

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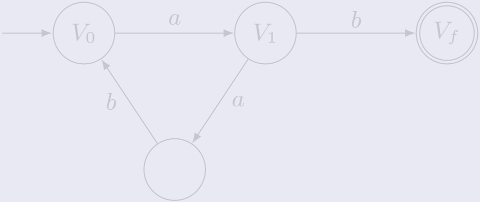

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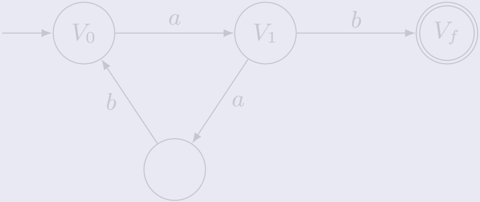

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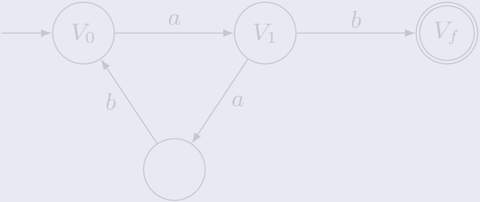

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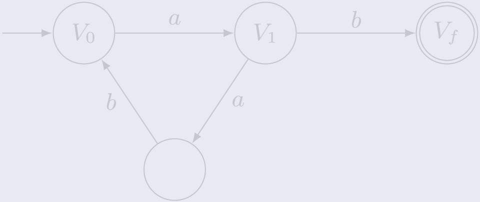

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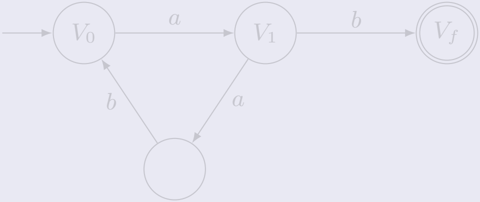

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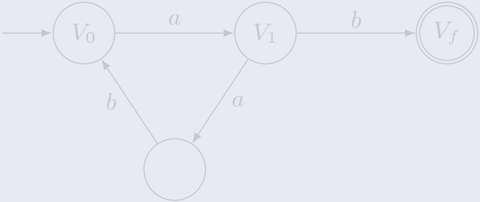

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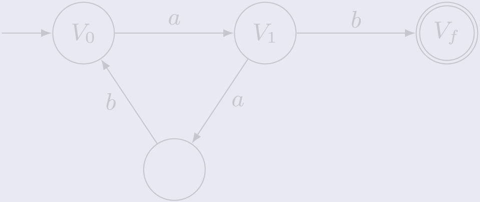

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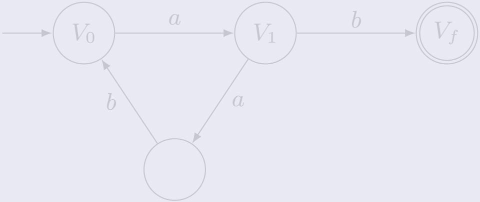

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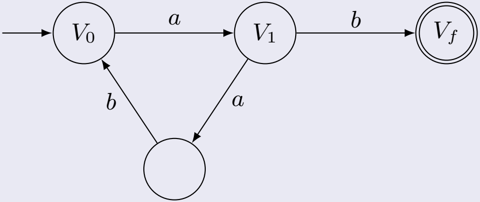

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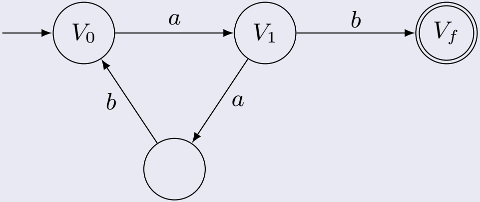

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Right-Linear Grammars for Regular Languages

To show that every regular language can be generated by some right-linear grammar, we start from the DFA for the language and reverse the construction shown in Theorem 3.3. The states of the DFA now become the variables of the grammar, and the symbols causing the transitions become the terminals in the productions.

Theorem 3.4

If L is a regular language on the alphabet Σ, then there exists a right-linear grammar $G = (V, \Sigma, S, P)$ such that $L = L(G)$.

Proof. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA that accepts L. We assume that $Q = \{q_0, q_1, \ldots, q_n\}$ and $\Sigma = \{a_1, a_2, \ldots, a_m\}$. Construct the right-linear grammar $G = (V, \Sigma, S, P)$ with

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In addition, if q_k is in F, we add to P the production

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To show that every regular language can be generated by some right-linear grammar, we start from the DFA for the language and reverse the construction shown in Theorem 3.3. The states of the DFA now become the variables of the grammar, and the symbols causing the transitions become the terminals in the productions.

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We first show that G defined in this way can generate every string in L. Consider $w \in L$ of the form

$$w = a_i a_j \cdots a_k a_l.$$

For M to accept this string it must make moves via

$$\delta(q_0, a_i) = q_p,$$

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By construction, the grammar will have one production for each of these δ's. Therefore, we can make the derivation

$$\begin{aligned} q_0 \Rightarrow a_i q_p \Rightarrow a_i a_j q_r \overset{*}{\Rightarrow} a_i a_j \cdots a_k q_t \Rightarrow \\ \Rightarrow a_i a_j \cdots a_k a_l q_f \Rightarrow a_i a_j \cdots a_k a_l, \end{aligned} \tag{4}$$

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We first show that G defined in this way can generate every string in L. Consider $w \in L$ of the form

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For the purpose of constructing a grammar, it is useful to note that the restriction that M be a DFA is not essential to the proof of Theorem 3.4. With minor modification, the same construction can be used if M is an NFA.

Example 3.16

Construct a right-linear grammar for $L(aab^*a)$. The transition function for an NFA, together with the corresponding grammar productions, is given in the following Figure.

$\delta(q_0, a) = \{q_1\}$	$q_0 \to aq_1$
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3.3 Regular Grammars

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Equivalence of Regular Languages and Regular Grammars

The previous two theorems establish the connection between regular languages and right-linear grammars. One can make a similar connection between regular languages and left-linear grammars, thereby showing the complete equivalence of regular grammars and regular languages.

Theorem 3.5

A language L is regular if and only if there exists a left-linear grammar G such that $L = L(G)$.

Proof. We only outline the main idea. Given any left-linear grammar with productions of the form

$$A \to Bv,$$

or

$$A \to v,$$

we construct from it a right-linear grammar $\widehat{G}$ by replacing every such production of G with

$$A \to v^R B,$$

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respectively. A few examples will make it clear quickly that $L(G) = (L(\widehat{G}))^R$. Next, we use the fact, which tells us that the reverse of any regular language is also regular. Since $\widehat{G}$ is right-linear, $L(\widehat{G})$ is regular. But then so are $(L(\widehat{G}))^R$ and $L(G)$. ∎

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Putting Theorems 3.4 and 3.5 together, we arrive at the equivalence of regular languages and regular grammars.

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A language L is regular if and only if there exists a regular grammar G such that $L = L(G)$.

We now have several ways of describing regular languages: DFA's, NFA's, regular expressions, and regular grammars. While in some instances one or the other of these may be most suitable, they are all equally powerful. Each gives a complete and unambiguous definition of a regular language. The connection between all these concepts is established by the four theorems in this lecture, as shown in the following Figure.

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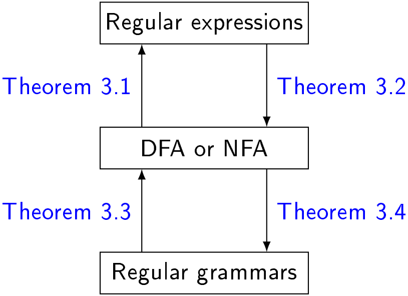

Thank You for attention!