Formal Languages, Automata and Codes

Oleg Gutik

Lecture 7

2.4 Reduction of the Number of States in Finite Automata

Any DFA defines a unique language, but the converse is not true. For a given language, there are many DFA's that accept it. There may be a considerable difference in the number of states of such equivalent automata. In terms of the questions we have considered so far, all solutions are equally satisfactory, but if the results are to be applied in a practical setting, there may be reasons for preferring one over another.

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The two DFA's depicted in (a) and (b) are equivalent, as a few test strings will quickly reveal.

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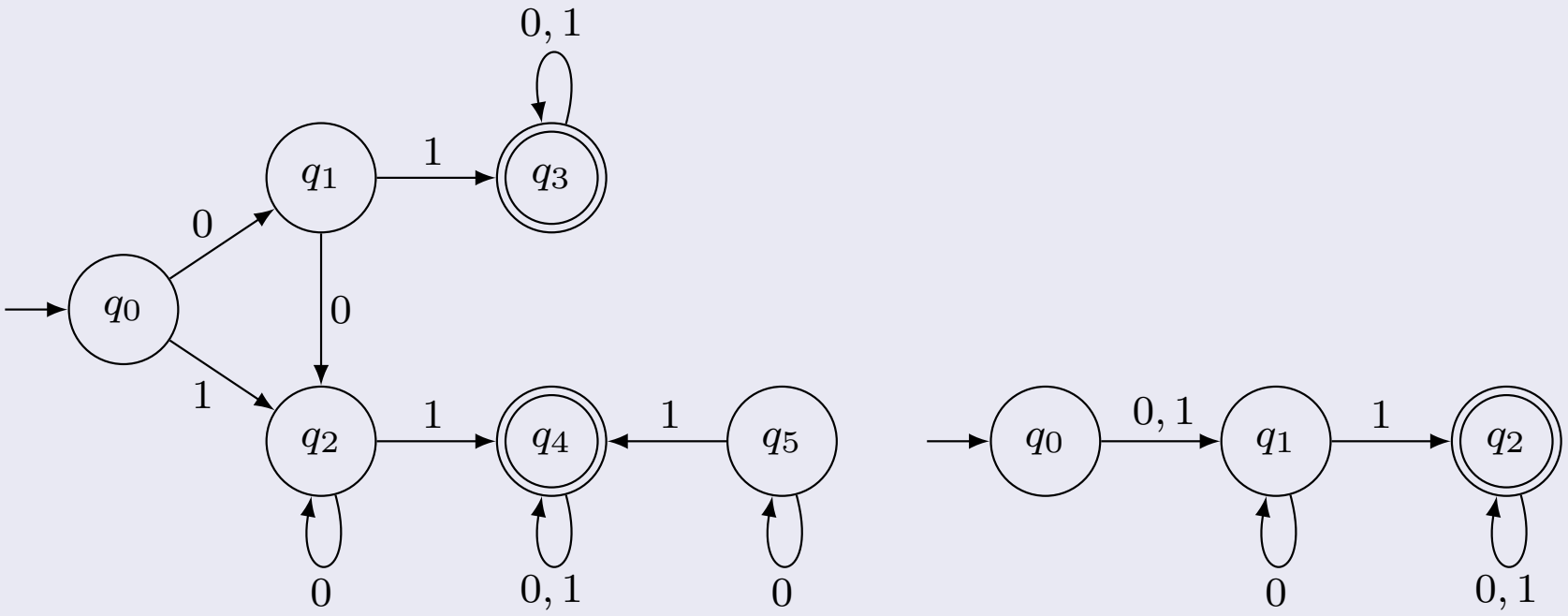

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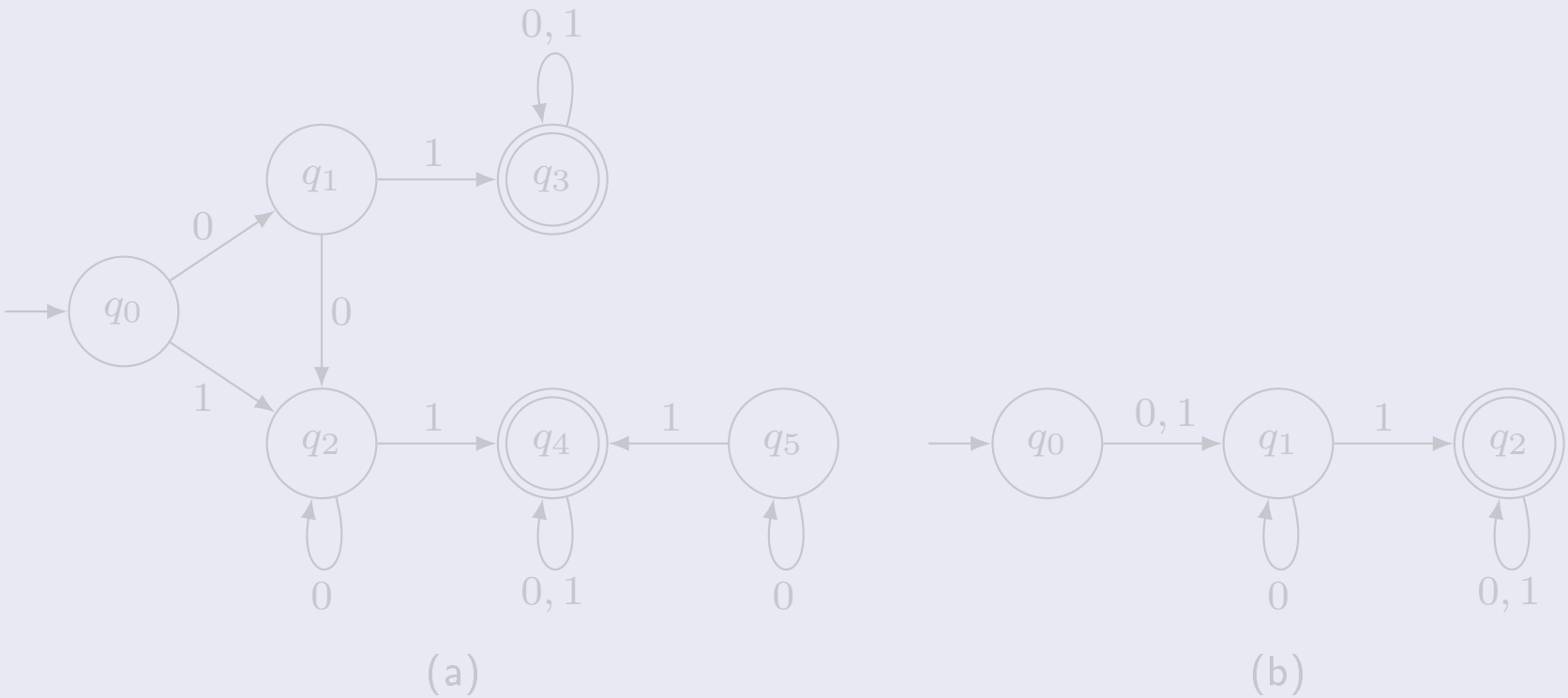

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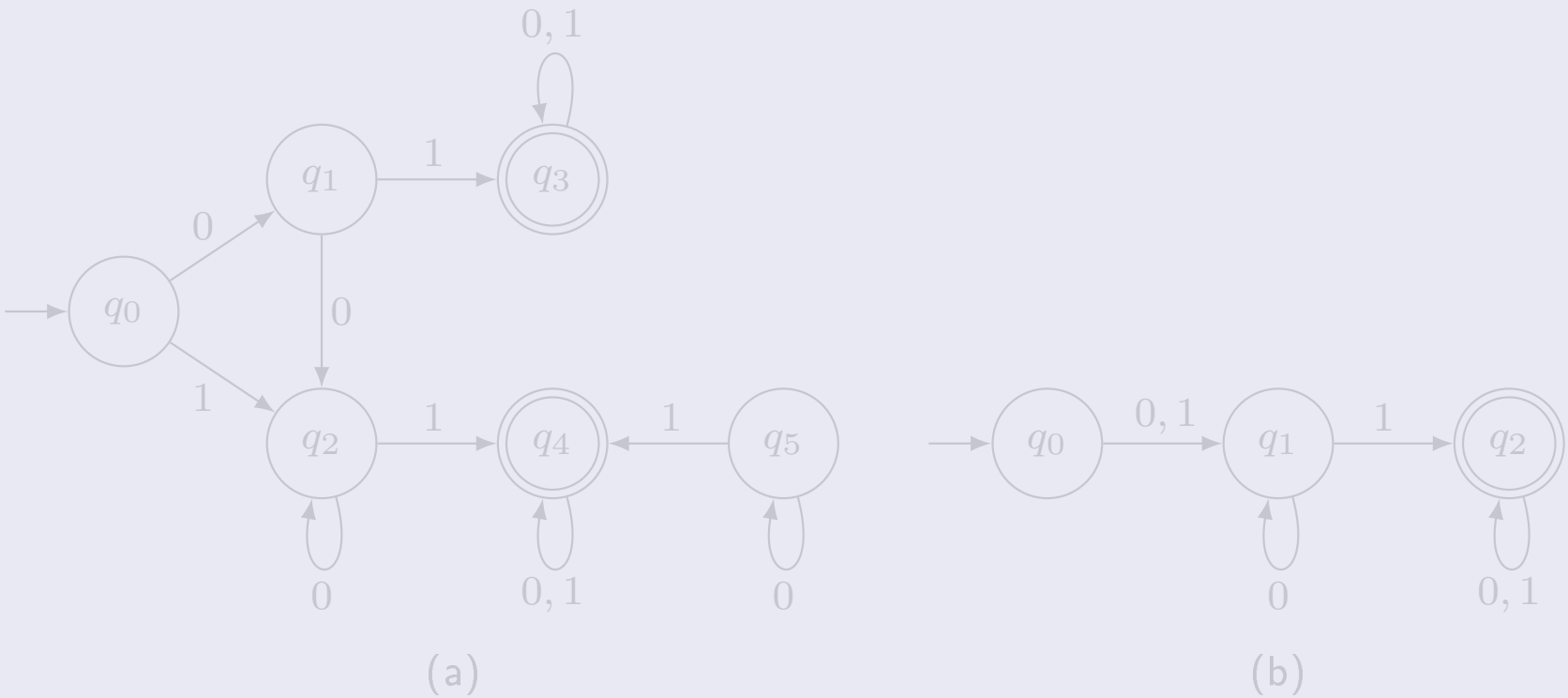

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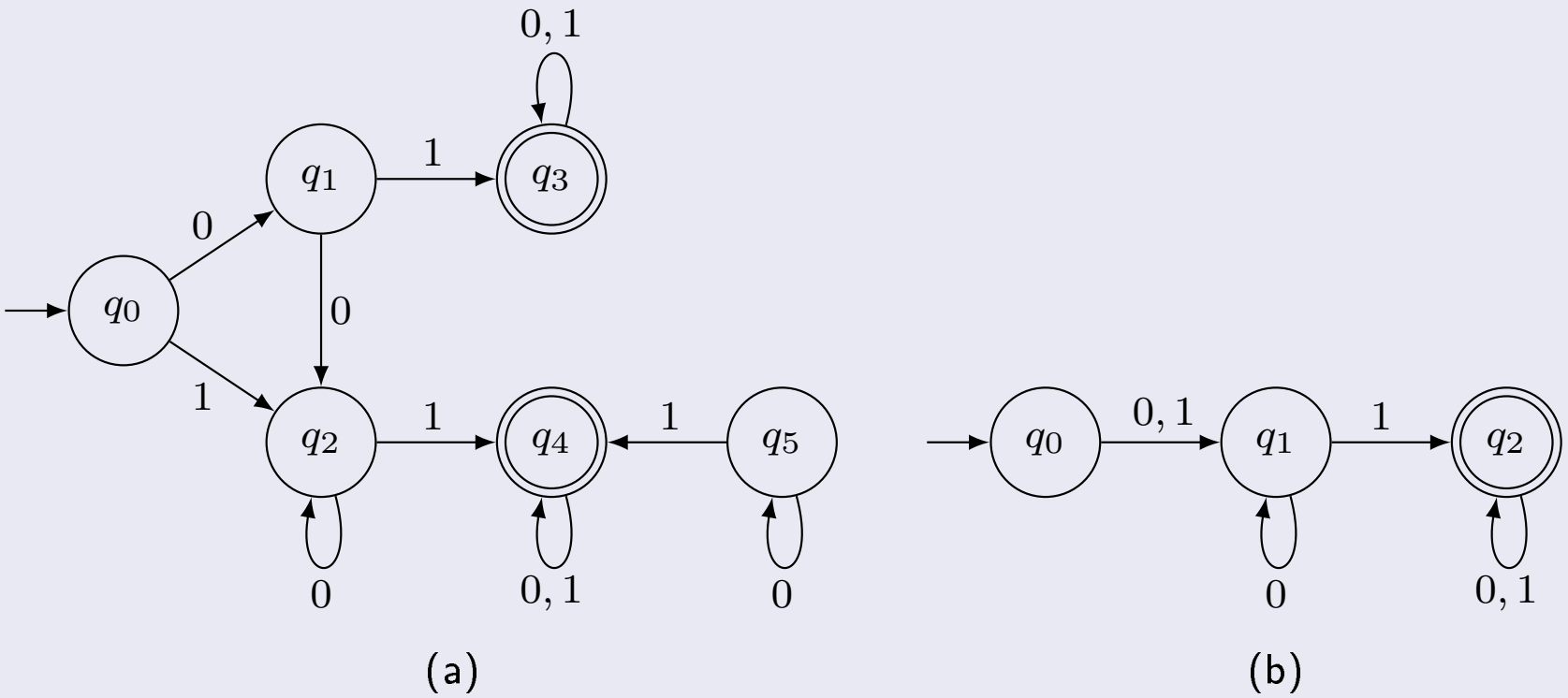

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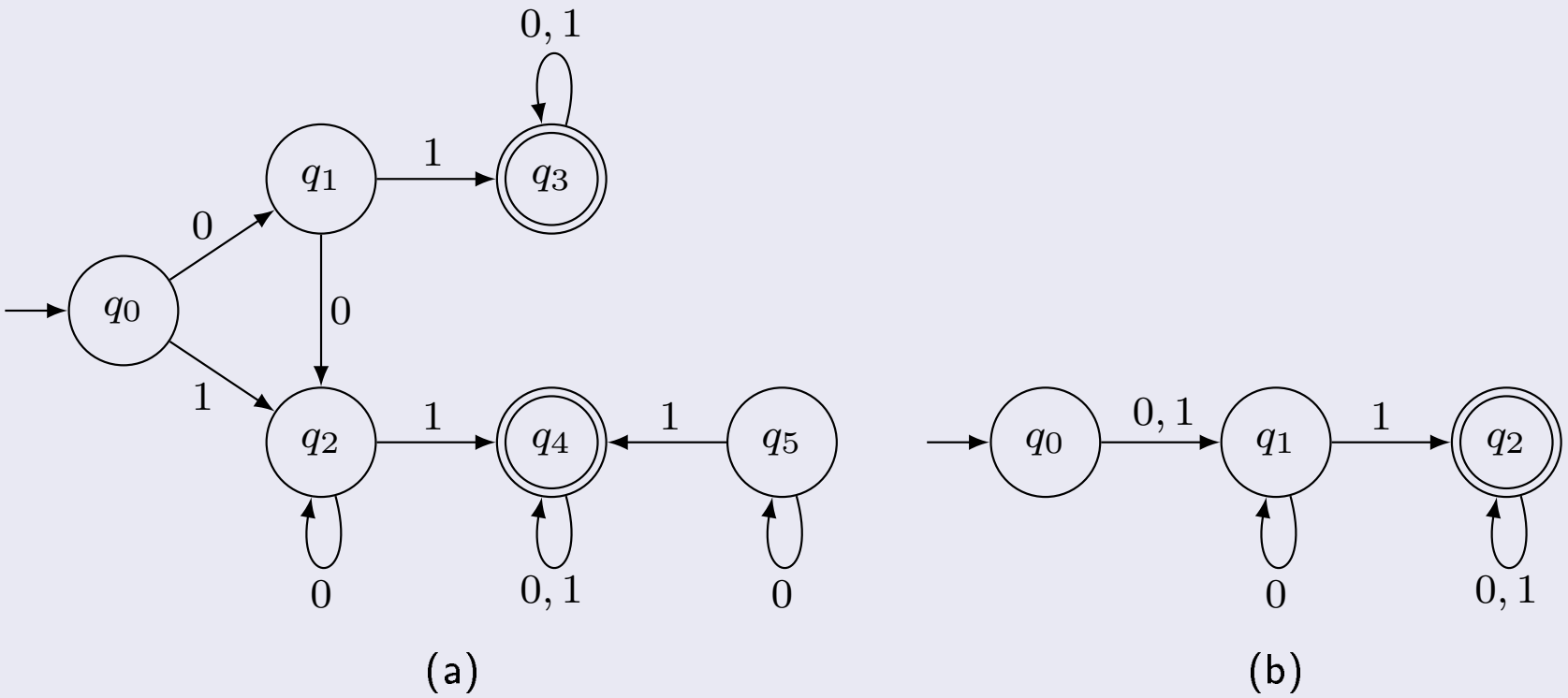

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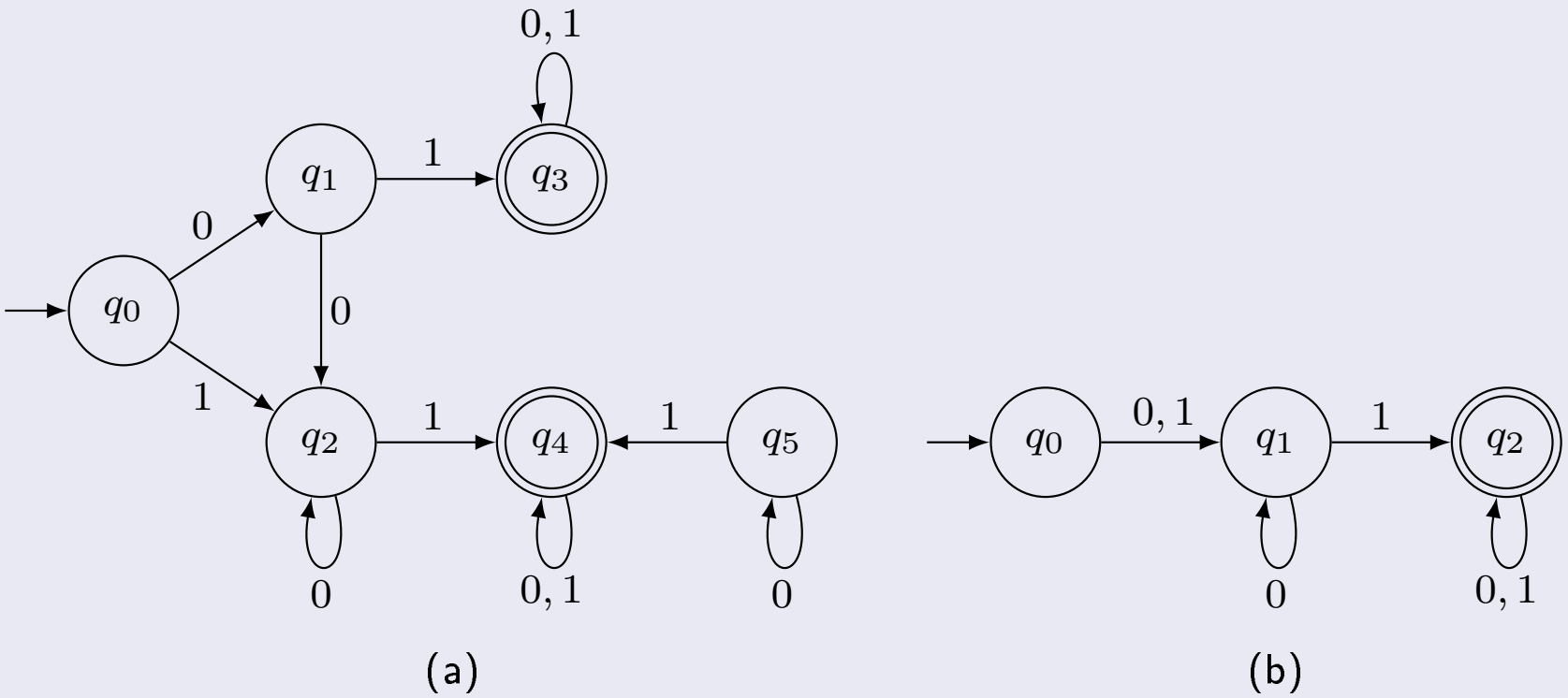

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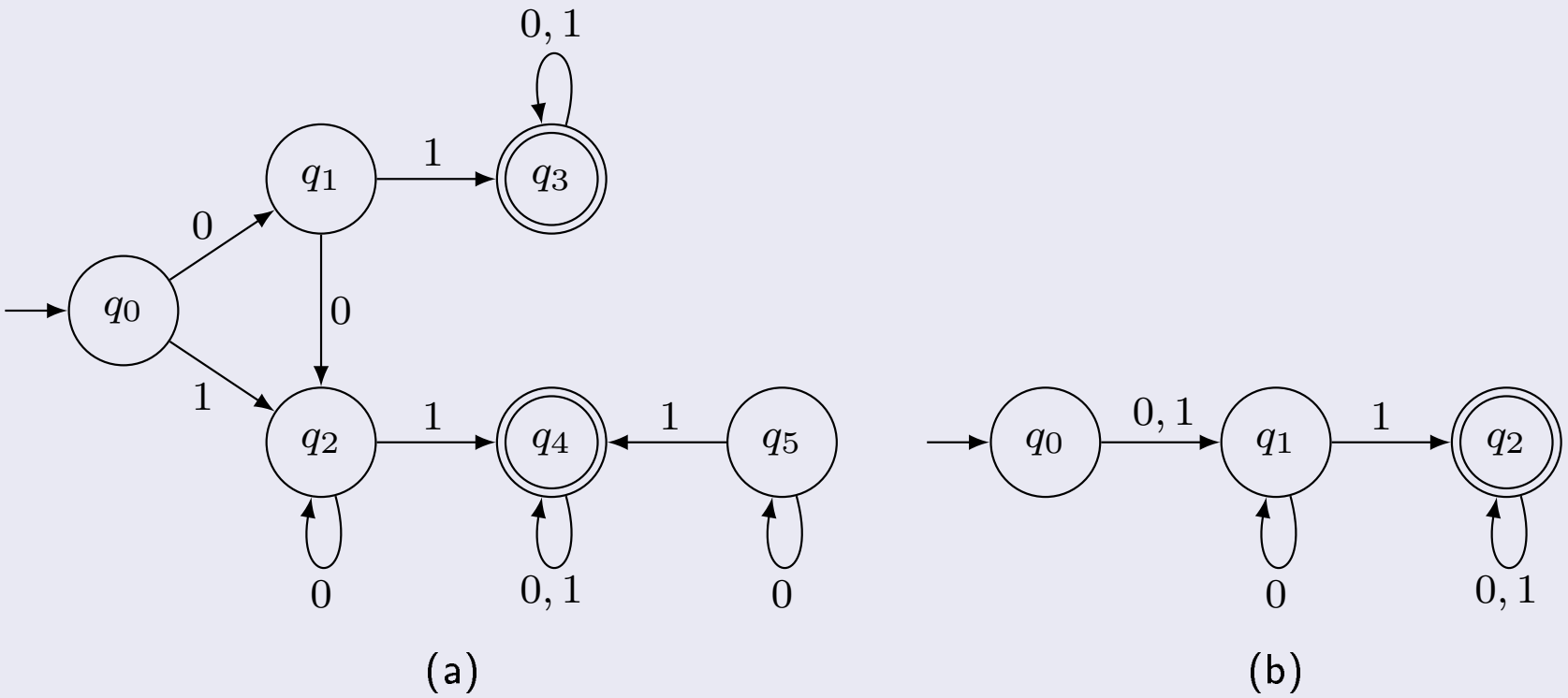

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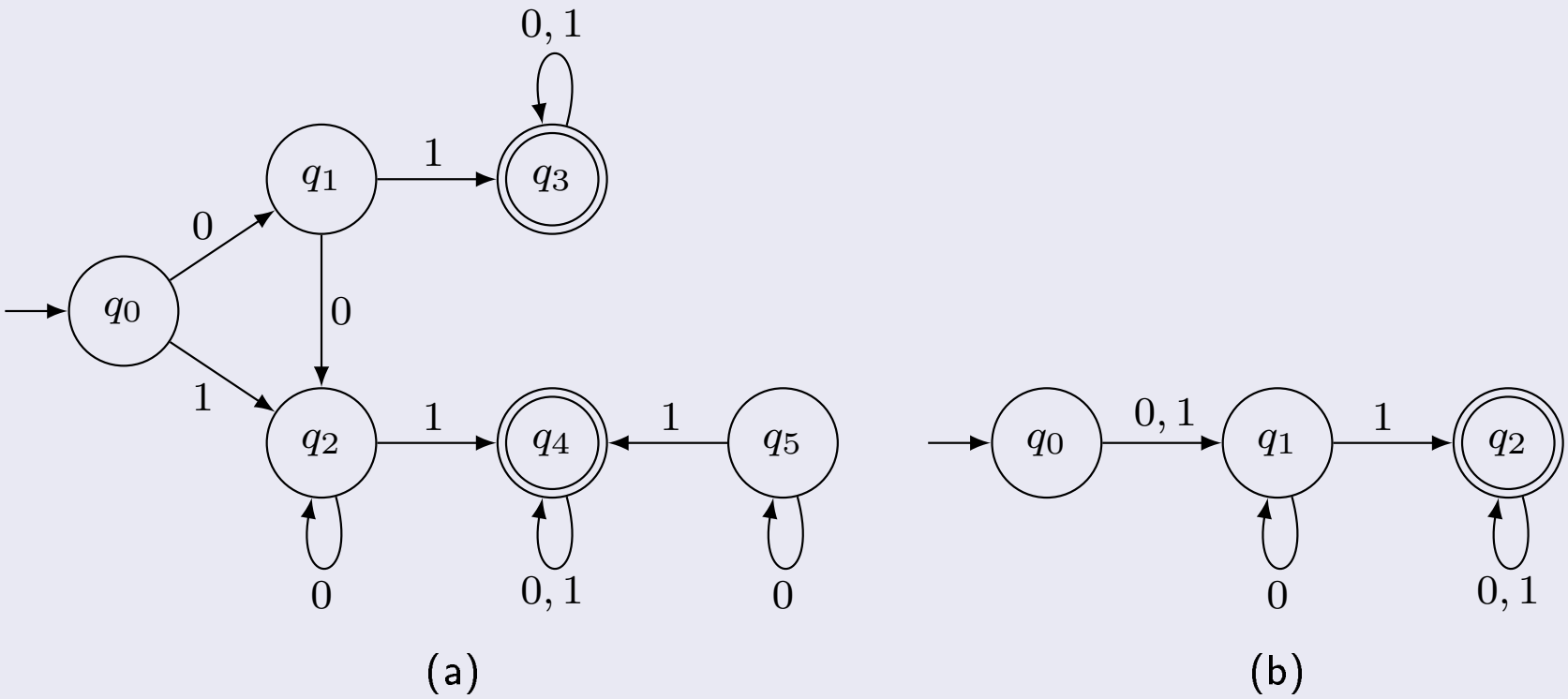

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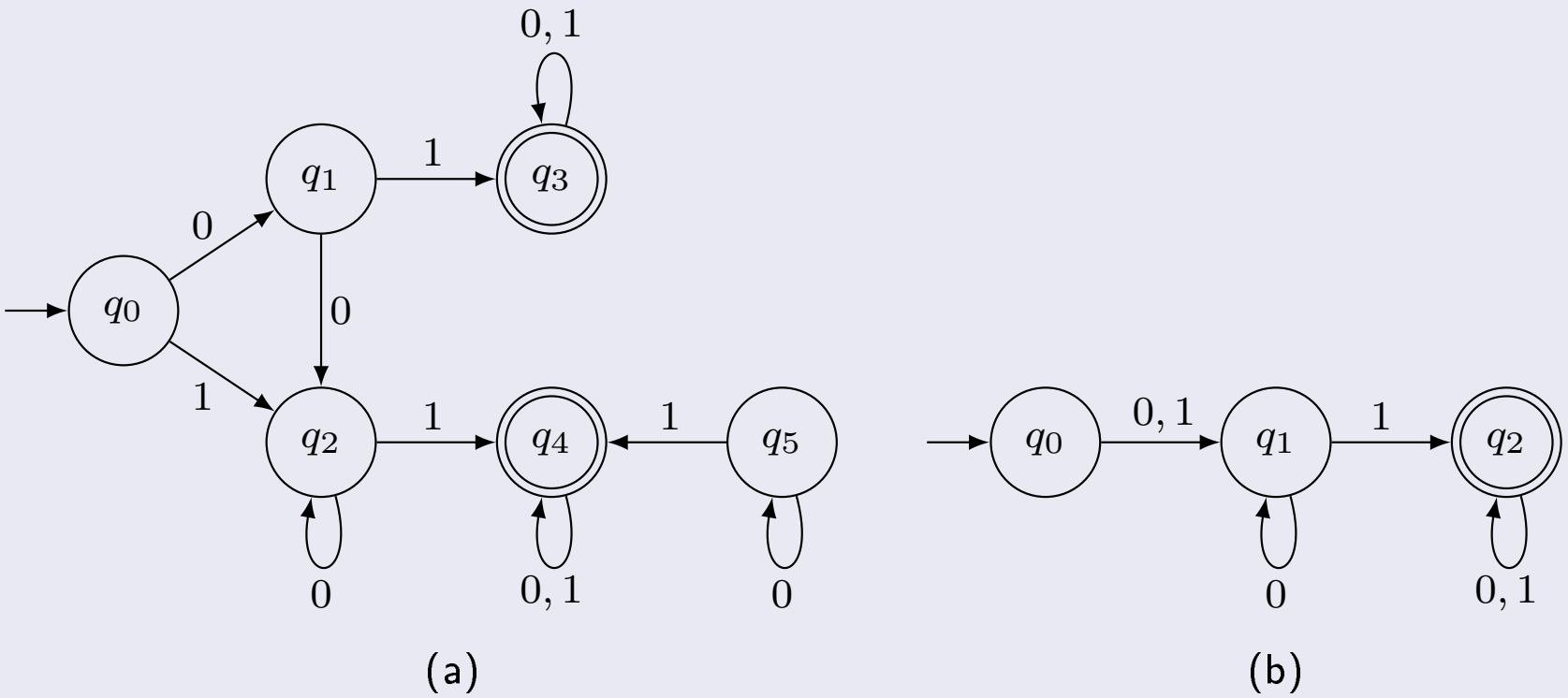

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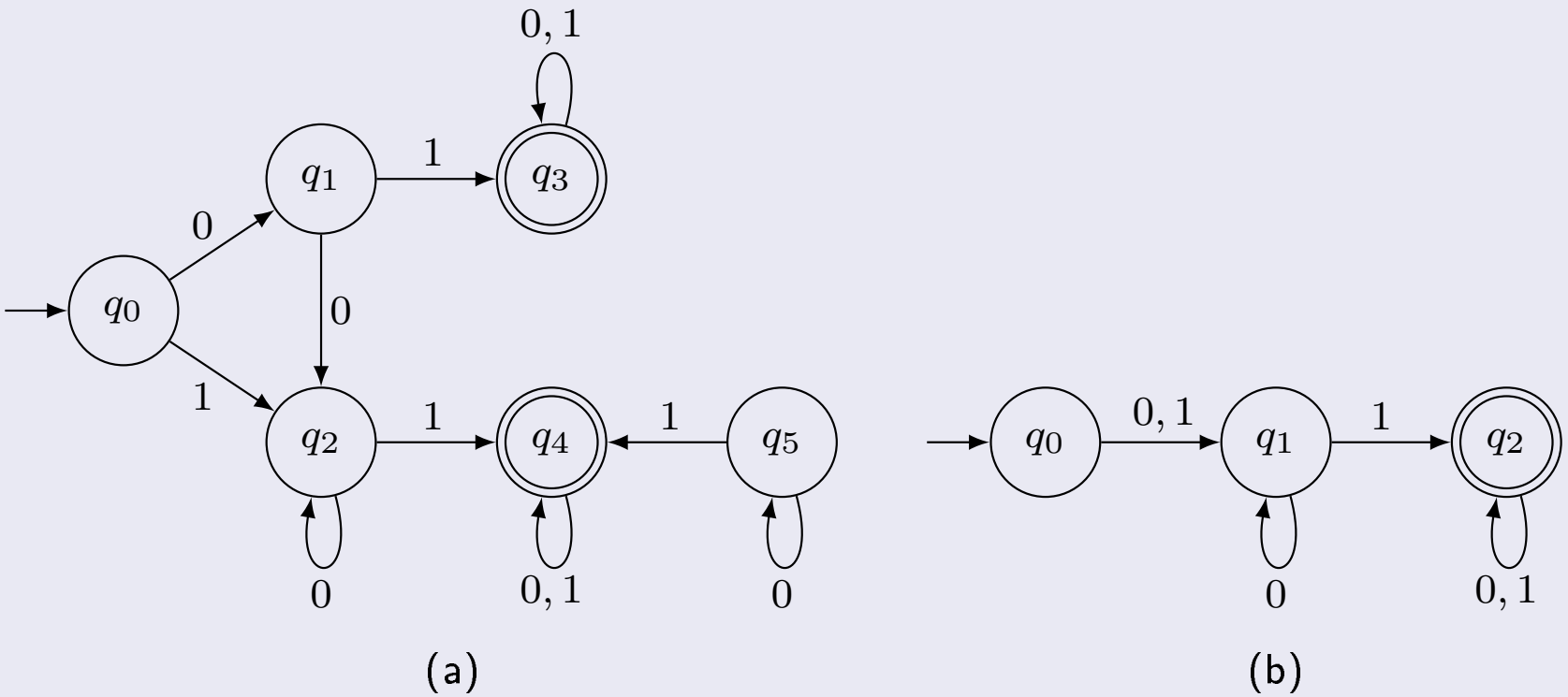

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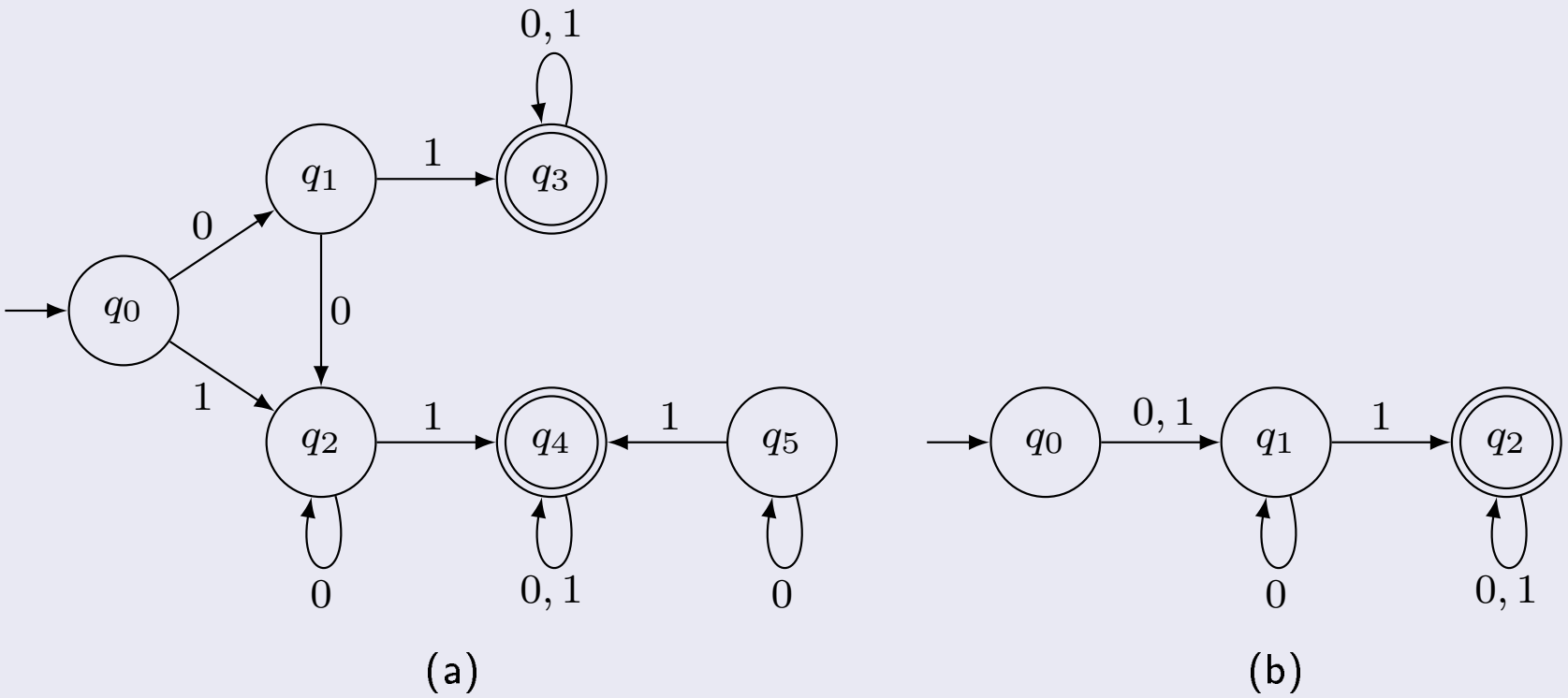

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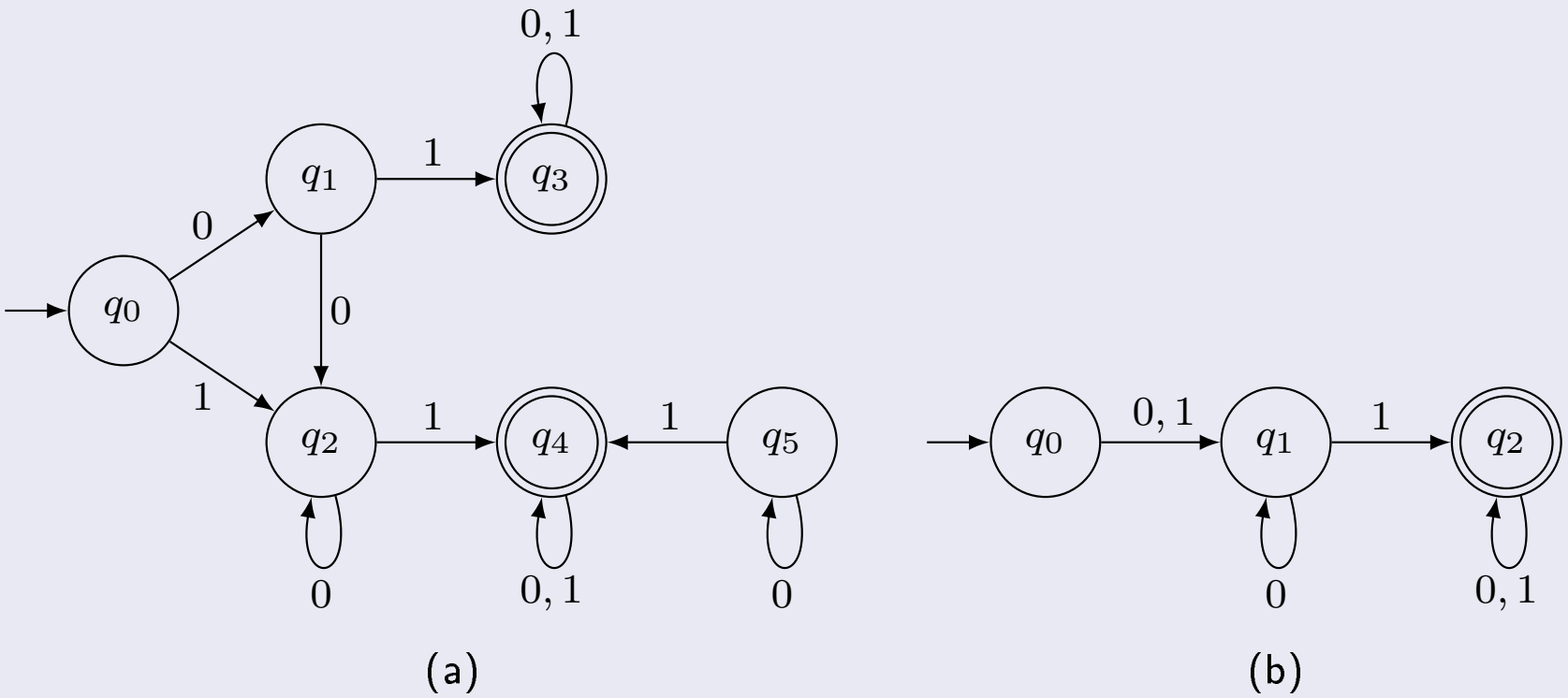

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From a strictly theoretical point of view, there is little reason for preferring the automaton in Figure (b) over that in Figure (a).

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However, in terms of simplicity, the second alternative is clearly preferable. Representation of an automaton for the purpose of computation requires space proportional to the number of states. For storage efficiency, it is desirable to reduce the number of states as far as possible. We now describe an algorithm that accomplishes this.

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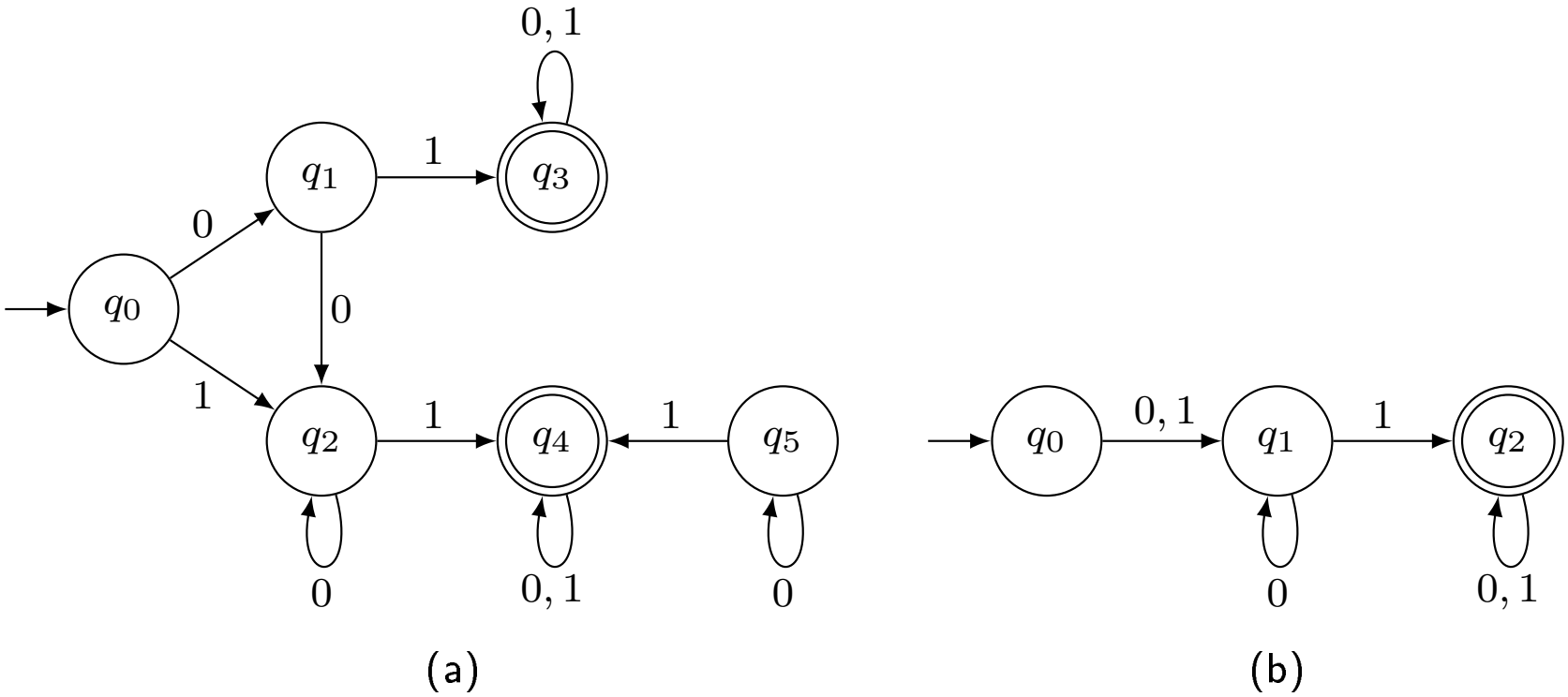

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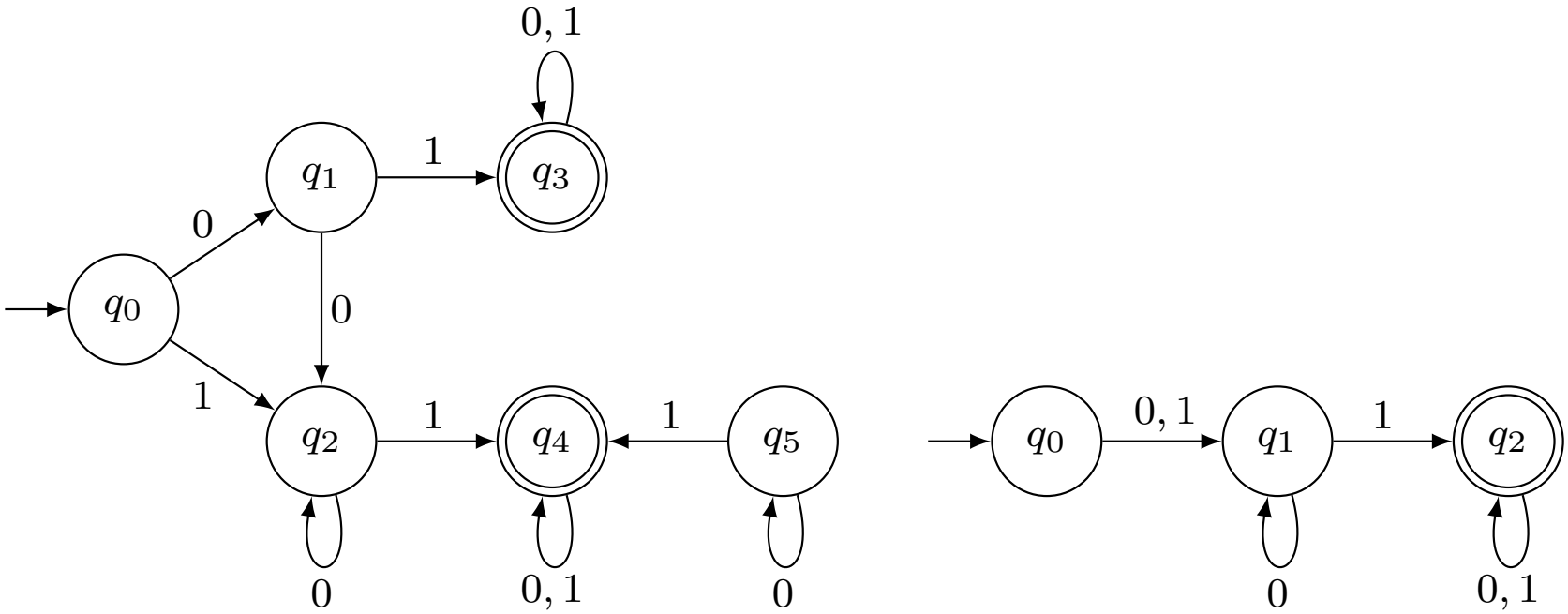

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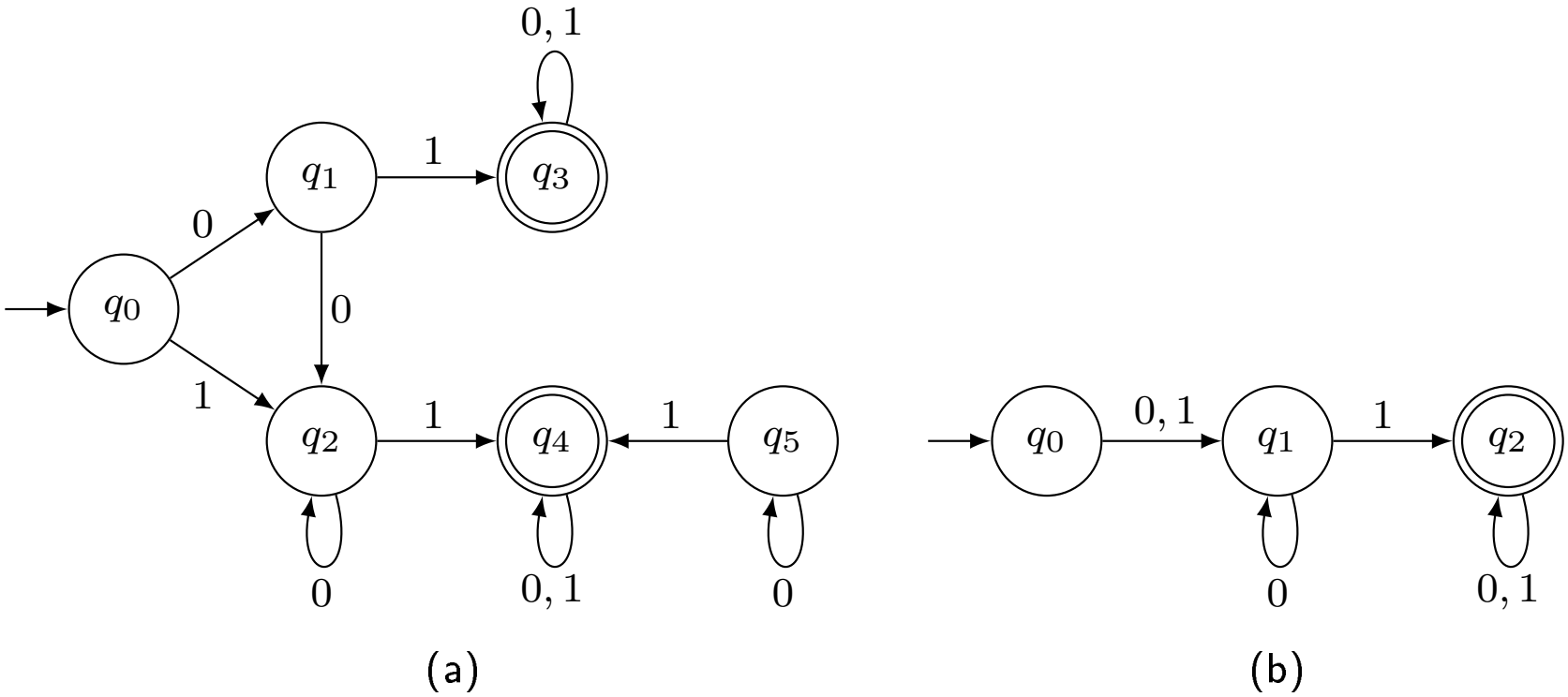

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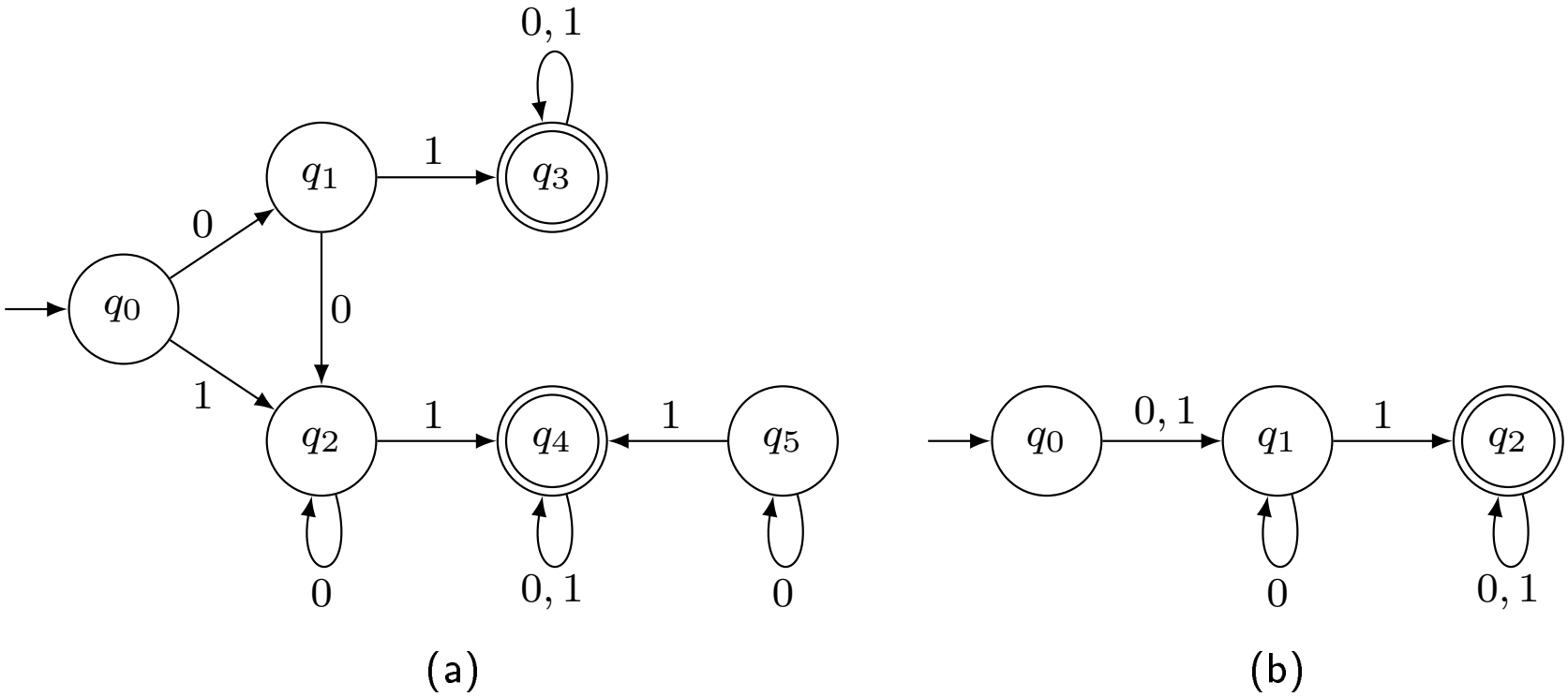

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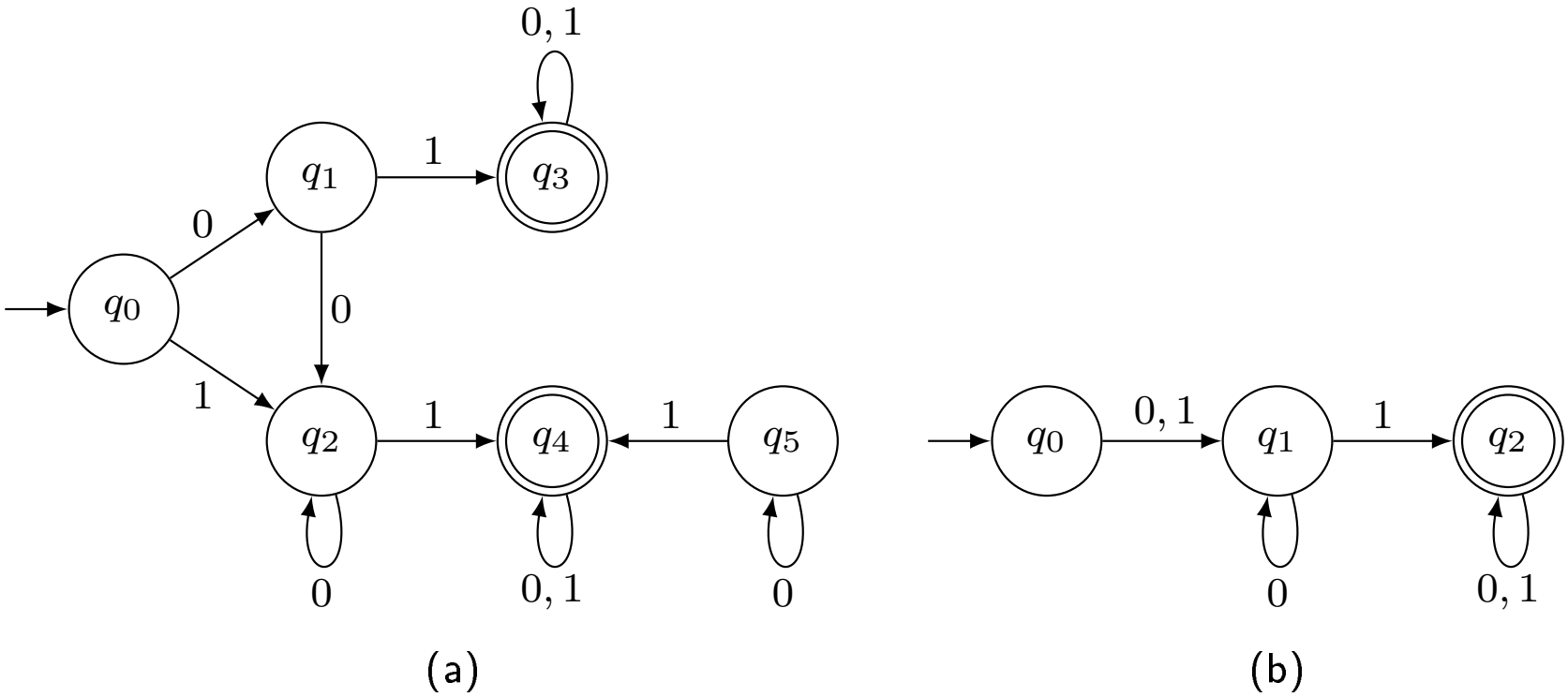

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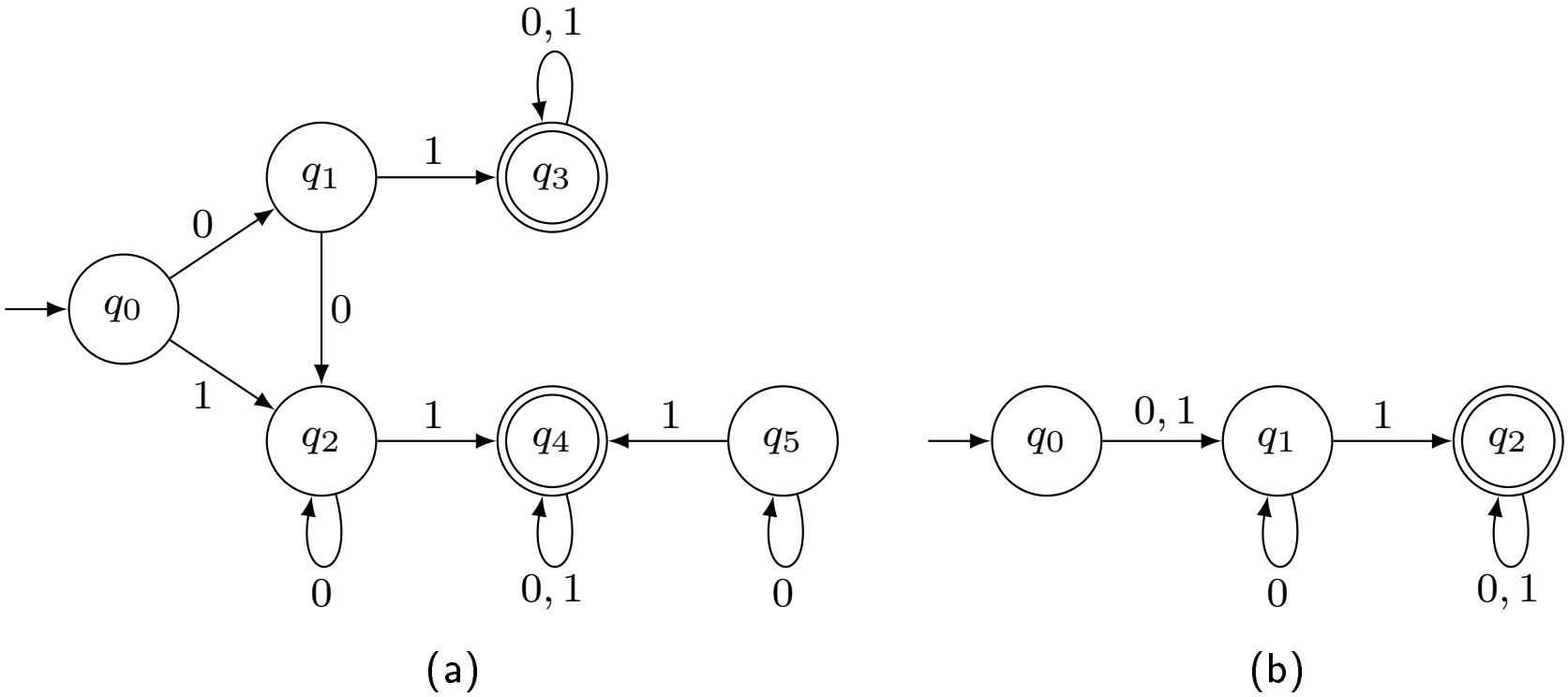

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Definition 2.8

Two states p and q of a DFA are called *indistinguishable* if

$$\delta^*(p, w) \in F \quad \text{implies} \quad \delta^*(q, w) \in F,$$

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for all $w \in \Sigma^*$. If, on the other hand, there exists some string $w \in \Sigma^*$ such that

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or vice versa, then the states p and q are said to be *distinguishable* by a string w.

Clearly, two states are either indistinguishable or distinguishable. Indistinguishability has the properties of an equivalence relation: If p and q are indistinguishable and if q and r are also indistinguishable, then so are p and r, and all three states are indistinguishable.

One method for reducing the states of a DFA is based on finding and combining indistinguishable states. We first describe a method for finding pairs of distinguishable states.

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We claim that this procedure constitutes an algorithm for marking all distinguishable pairs.

Theorem 2.3

The procedure *mark*, applied to any DFA $M = (Q, \Sigma, \delta, q_0, F)$, terminates and determines all pairs of distinguishable states.

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3. Repeat the following step until no previously unmarked pairs are marked. For all pairs (p, q) and all $a \in \Sigma$, compute $\delta(p, a) = pa$ and $\delta(q, a) = qa$. If the pair (pa, qa) is marked as distinguishable, mark (p, q) as distinguishable.

We claim that this procedure constitutes an algorithm for marking all distinguishable pairs.

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The procedure *mark*, applied to any DFA $M = (Q, \Sigma, \delta, q_0, F)$, terminates and determines all pairs of distinguishable states.

Proof. Obviously, the procedure terminates, because there are only a finite number of pairs that can be marked. It is also easy to see that the states of any pair so marked are distinguishable. The only claim that requires elaboration is that the procedure finds all distinguishable pairs.

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Example 2.15

Consider the automaton in the following Figure.

In the second step of procedure mark we partition the state set into final and nonfinal states to get two equivalence classes $\{q_0, q_1, q_3\}$ and $\{q_2, q_4\}$. In the next step, when we compute

$$\delta(q_0, 0) = q_1$$

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we recognize that q_0 and q_1 are distinguishable, so we put them into different sets. So $\{q_0, q_1, q_3\}$ is split into $\{q_0\}$ and $\{q_1, q_3\}$. Also, since $\delta(q_2, 0) = q_3$ and $\delta(q_4, 0) = q_4$, the class $\{q_2, q_4\}$ is split into $\{q_2\}$ and $\{q_4\}$. The rest of the computations show that no further splitting is needed.

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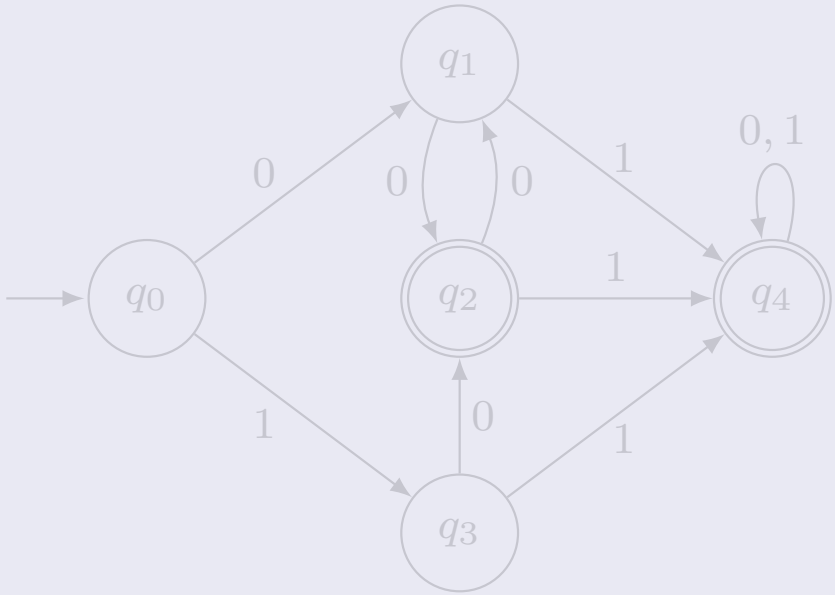

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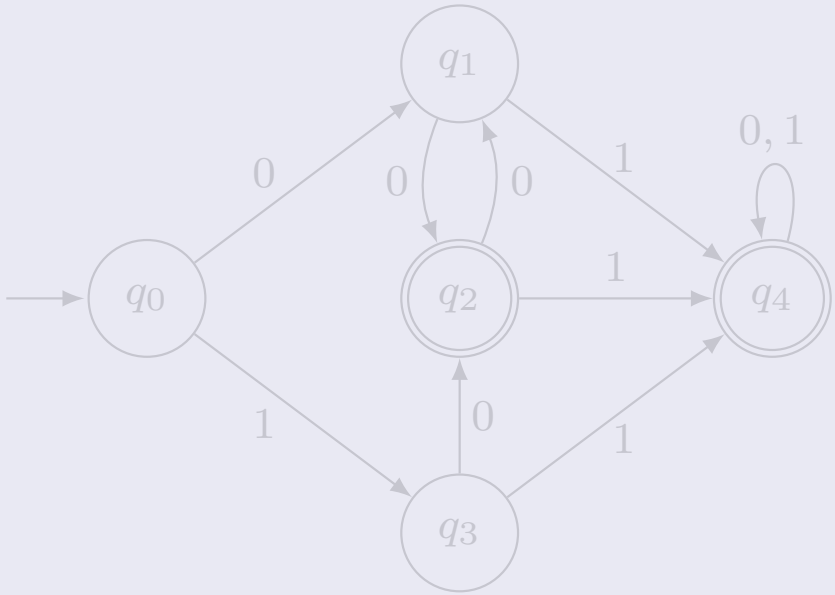

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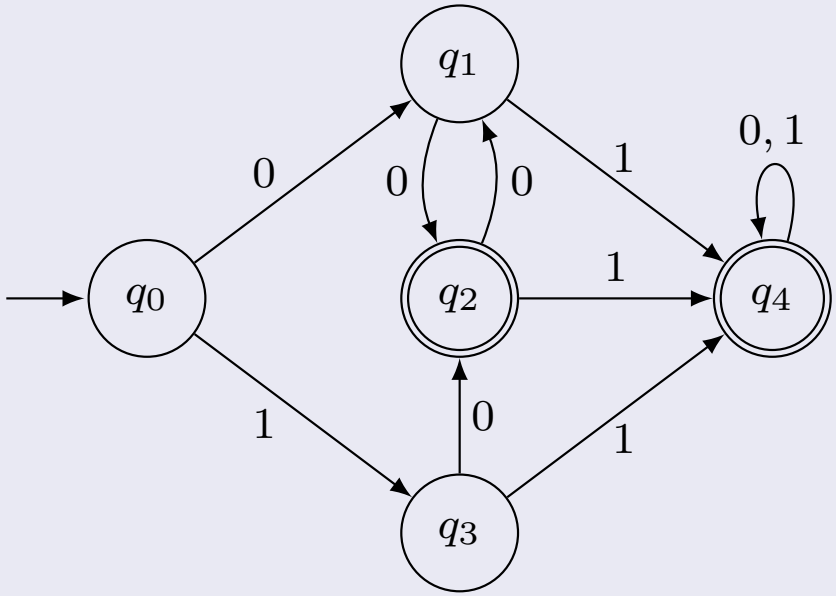

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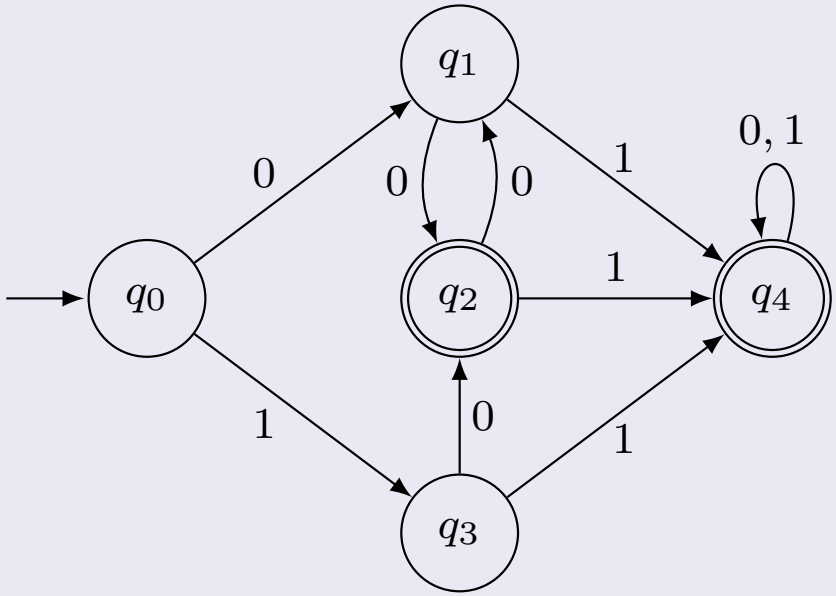

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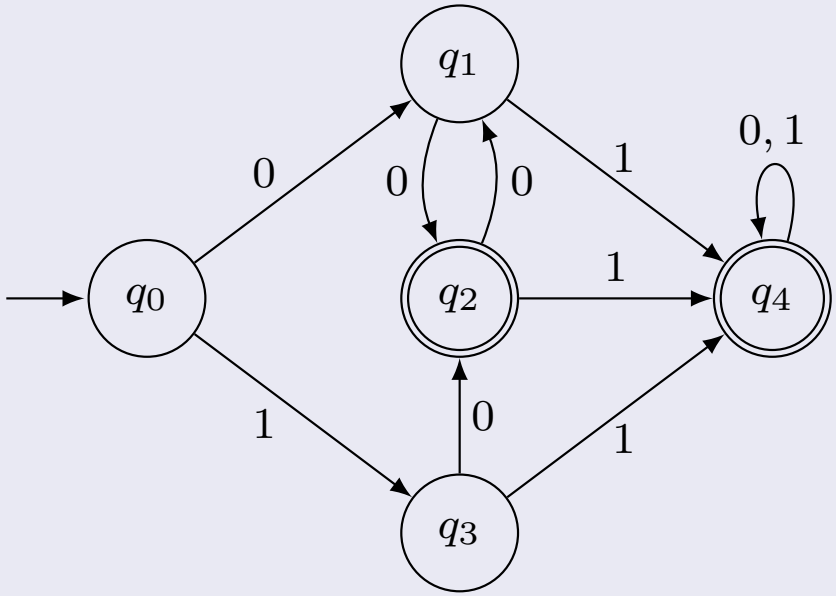

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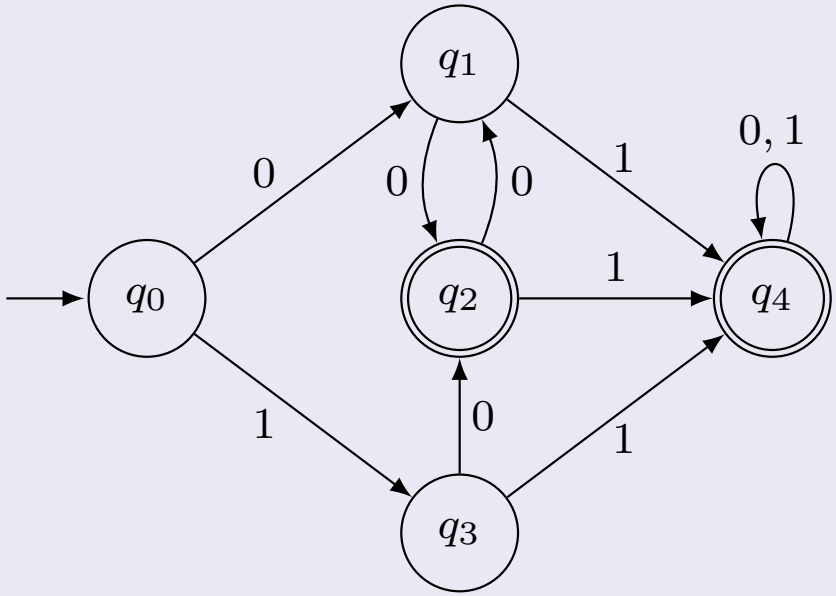

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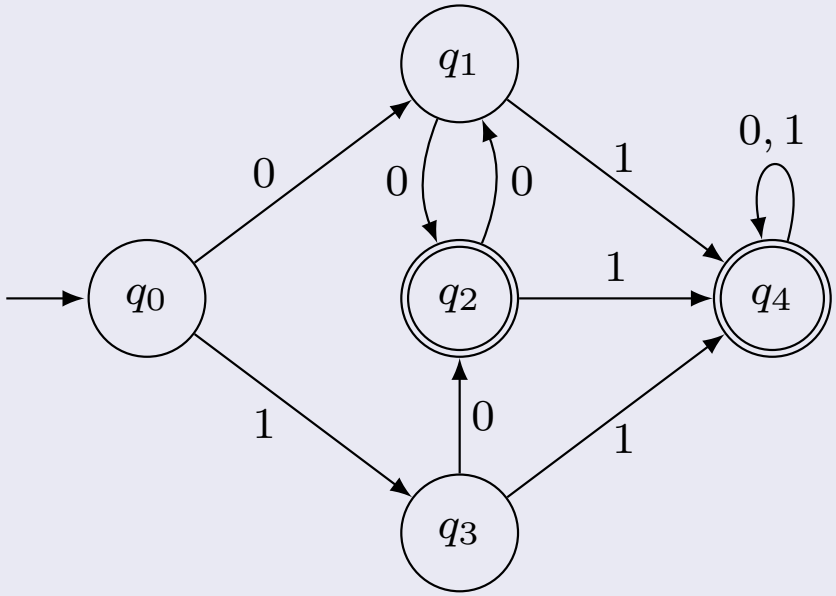

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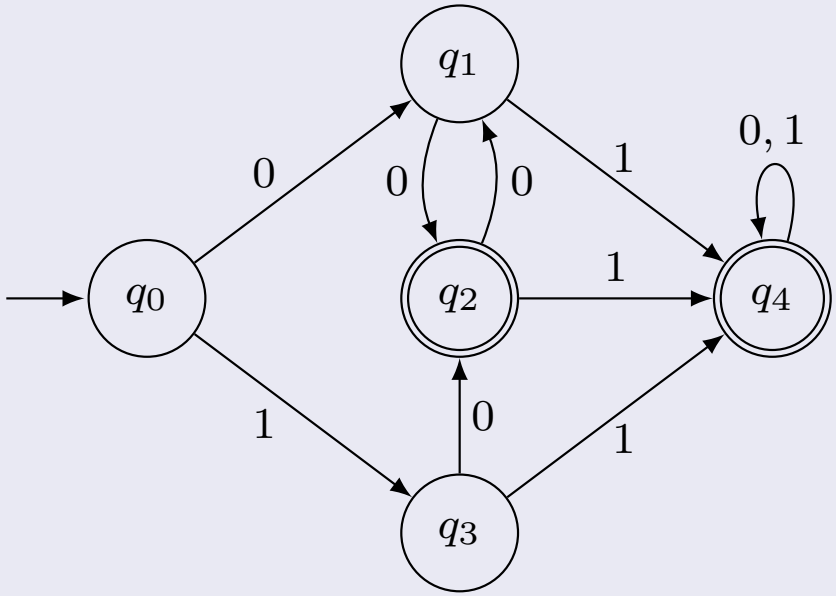

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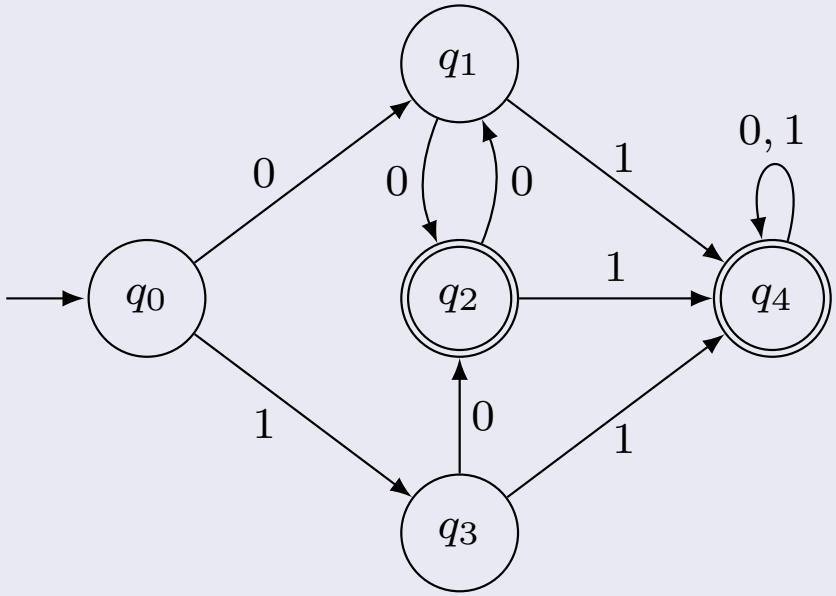

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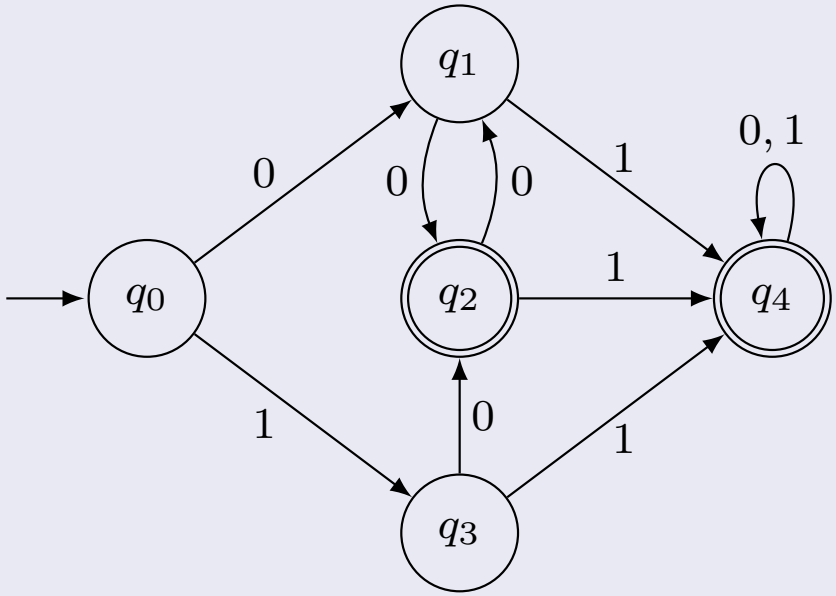

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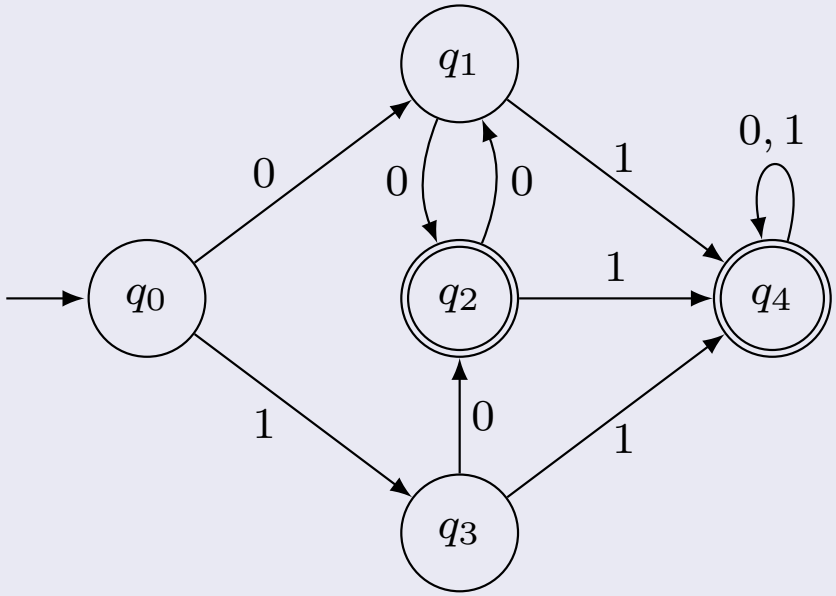

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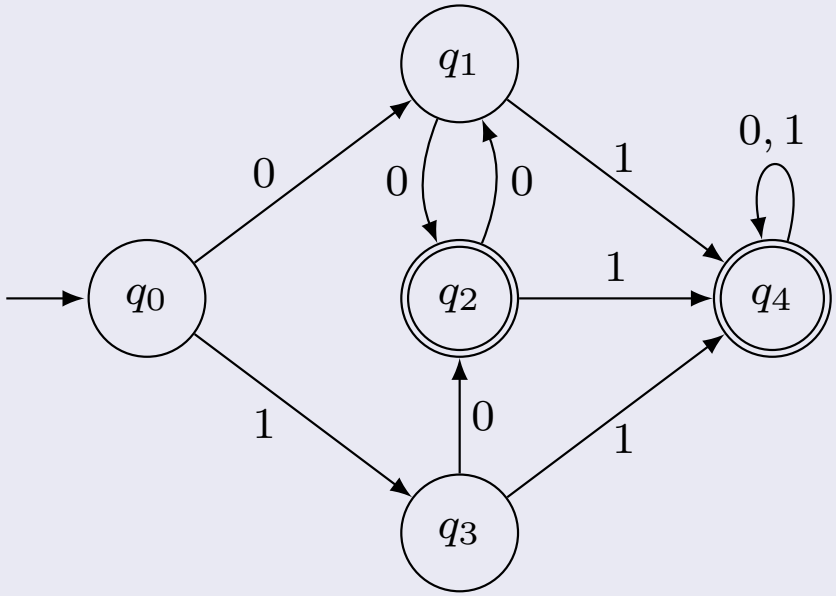

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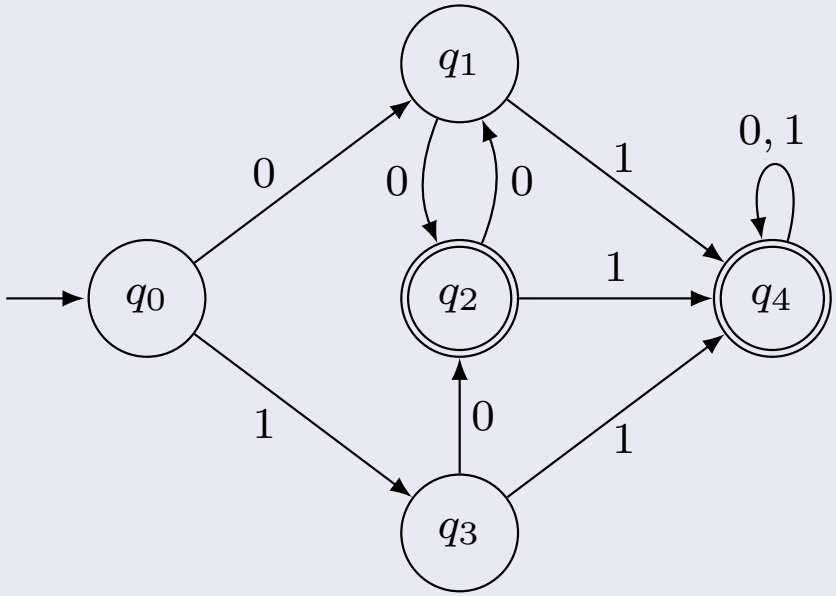

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2.4 Reduction of the Number of States in Finite Automata

Once the indistinguishability classes are found, the construction of the minimal DFA is straightforward.

Procedure: reduce

Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$, we construct a reduced DFA $\widehat{M} = (\widehat{Q}, \Sigma, \widehat{\delta}, \widehat{q_0}, \widehat{F})$ as follows.

1. Use procedure *mark* to generate the equivalence classes, say $\{q_i, q_j, \ldots, q_k\}$, as described.
2. For each set $\{q_i, q_j, \ldots, q_k\}$ of such indistinguishable states, create a state labeled by $ij \cdots k$ for $\widehat{M}$.
3. For each transition rule of M of the form

$$\delta(q_r, a) = q_s$$

find the sets to which q_r and q_s belong. If $q_r \in \{q_i, q_j, \ldots, q_k\}$ and $q_s \in \{q_l, q_m, \ldots, q_n\}$, add to $\widehat{\delta}$ a rule

$$\widehat{\delta}(ij \cdots k, a) = lm \cdots n$$

4. The initial state $\widehat{q_0}$ is that state of $\widehat{M}$ whose label includes the symbol 0.
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find the sets to which q_p and q_s belong. If $q_s \in \{q_t, q_u, \ldots, q_v\}$ and $q_p \in \{q_r, q_w, \ldots, q_x\}$, add to $\widehat{\delta}$ a rule

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4. The initial state $\widehat{q_0}$ is that state of $\widehat{M}$ whose label includes the symbol 0.
5. $\widehat{F}$ is the set of all the states whose label contains i such that $q_i \in F$.

Once the indistinguishability classes are found, the construction of the minimal DFA is straightforward.

Procedure:* *reduce

Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$, we construct a reduced DFA $\widehat{M} = (\widehat{Q}, \Sigma, \widehat{\delta}, \widehat{q_0}, \widehat{F})$ as follows.

1. Use procedure *mark* to generate the equivalence classes, say $\{q_i, q_j, \ldots, q_k\}$, as described.
2. For each set $\{q_i, q_j, \ldots, q_k\}$ of such indistinguishable states, create a state labeled by $ij \cdots k$ for $\widehat{M}$.
3. For each transition rule of M of the form
$$\delta(q_r, a) = q_p,$$
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Continuing with Example 2.15,

we create the states in the following Figure.

Since, for example, $\delta(q_1, 0) = q_2$, there is an edge labeled 0 from state 13 to state 2. The rest of the transitions are easily found, giving the minimal DFA in the Figure.

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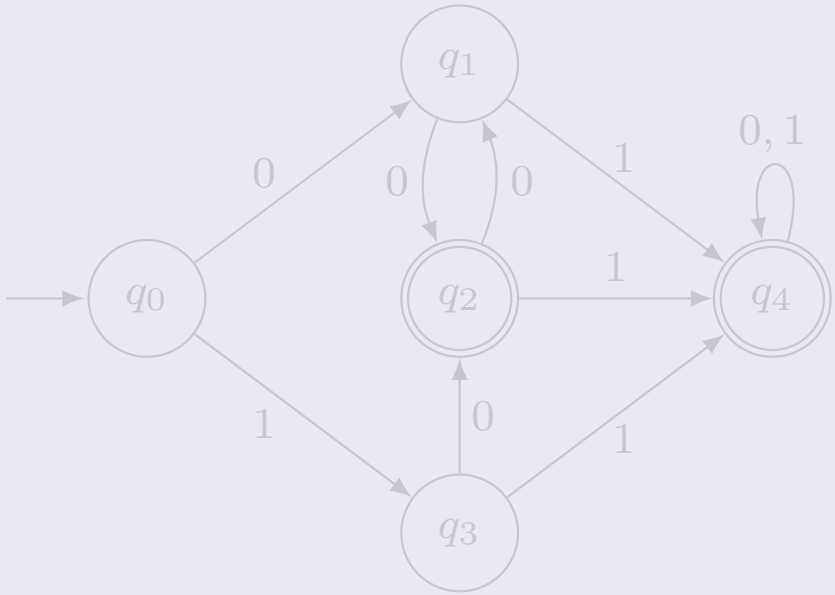

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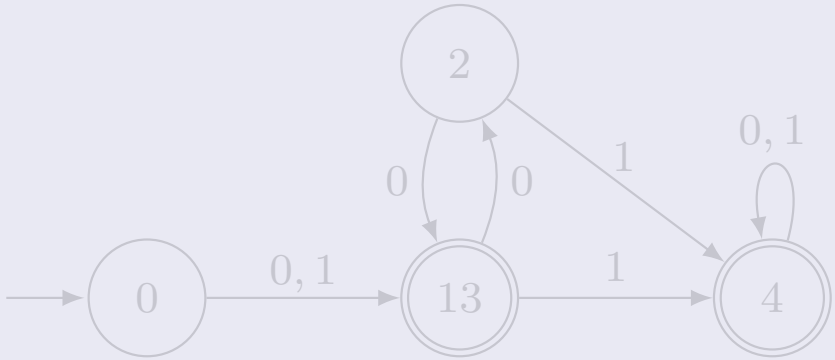

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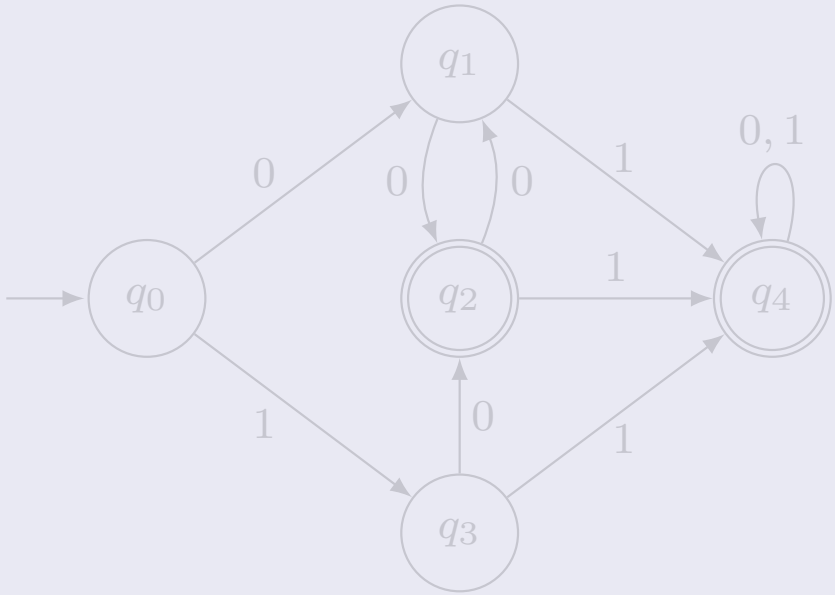

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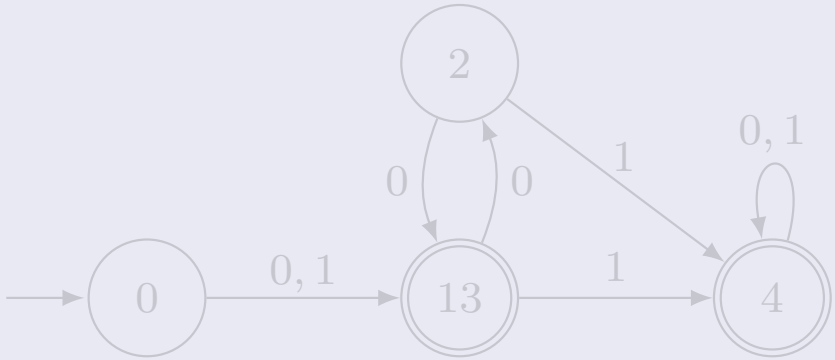

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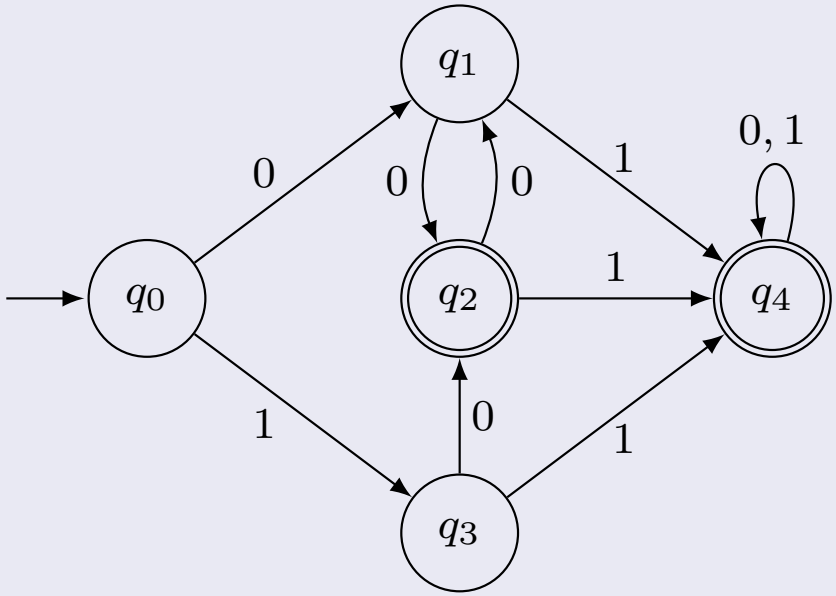

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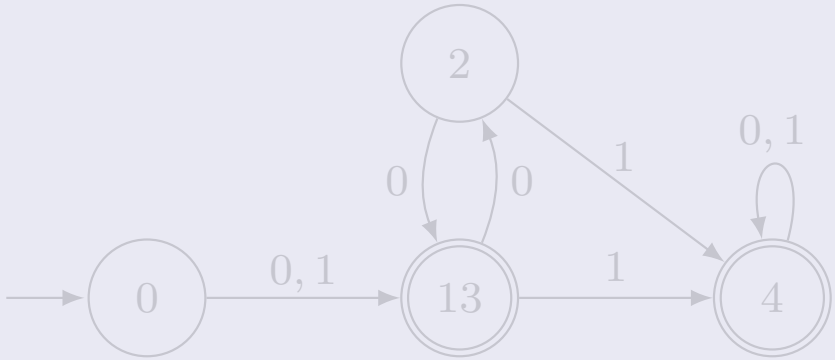

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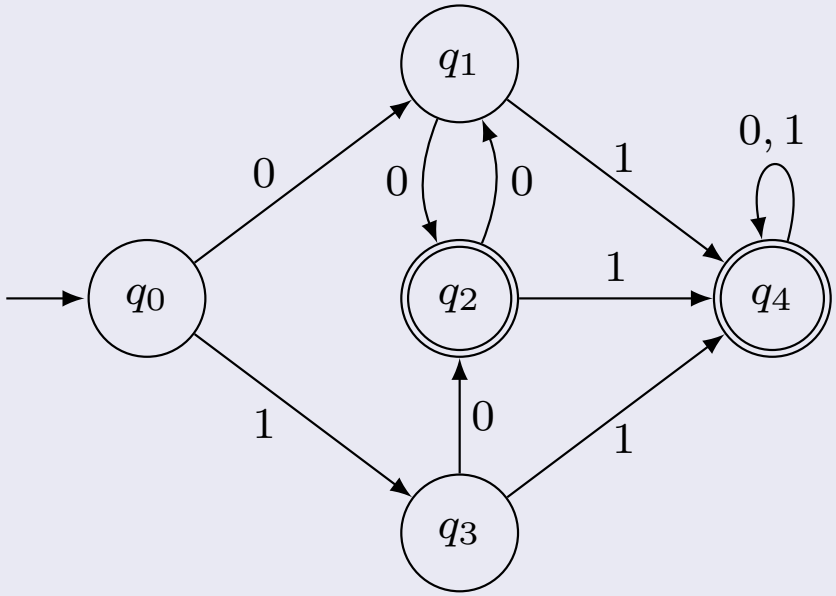

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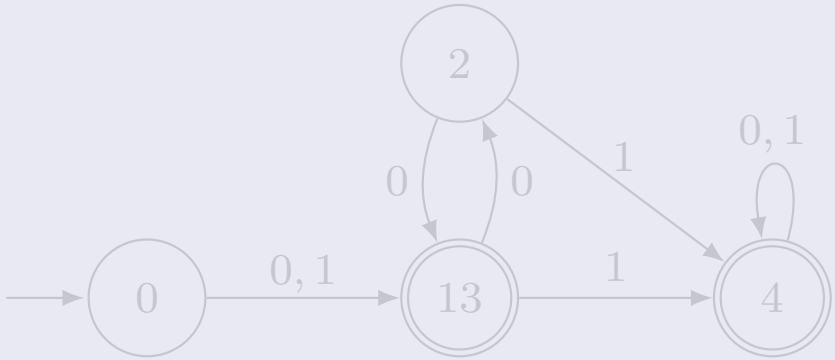

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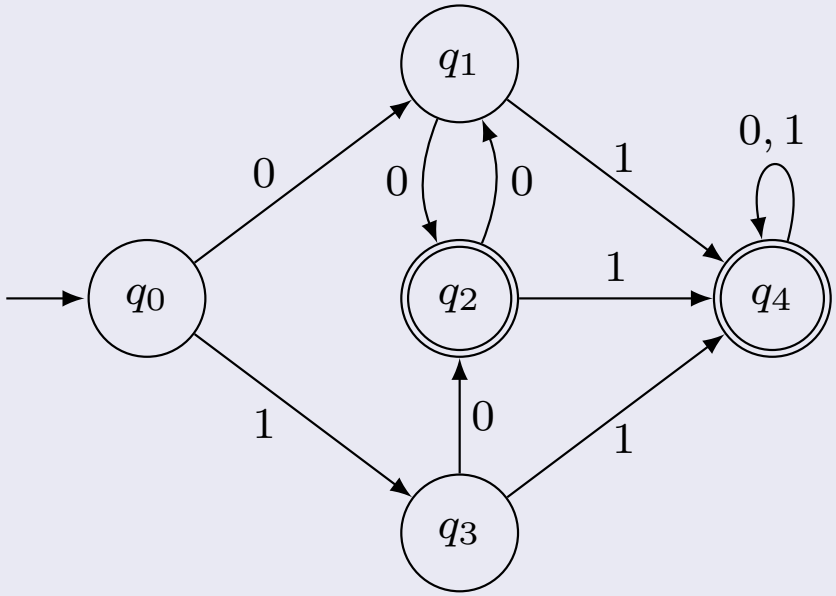

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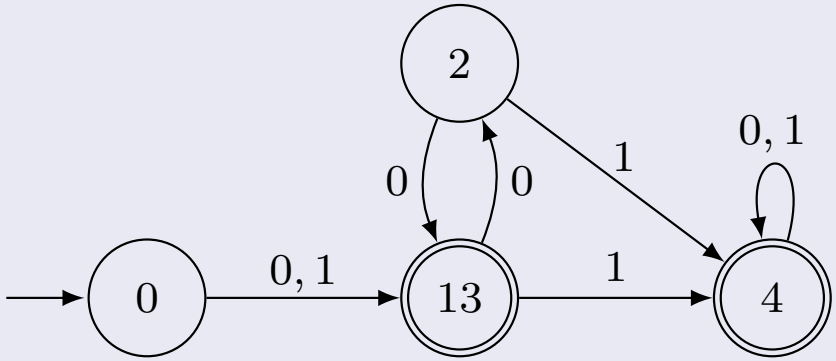

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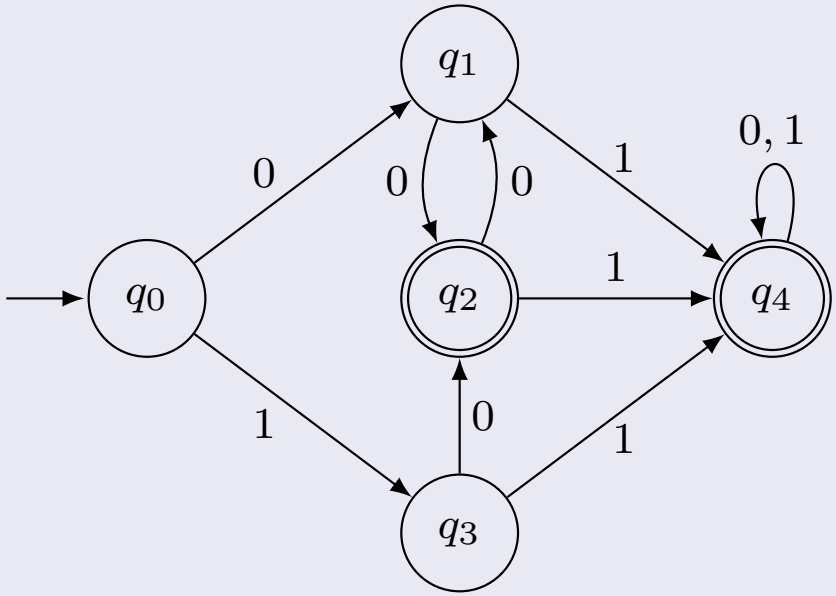

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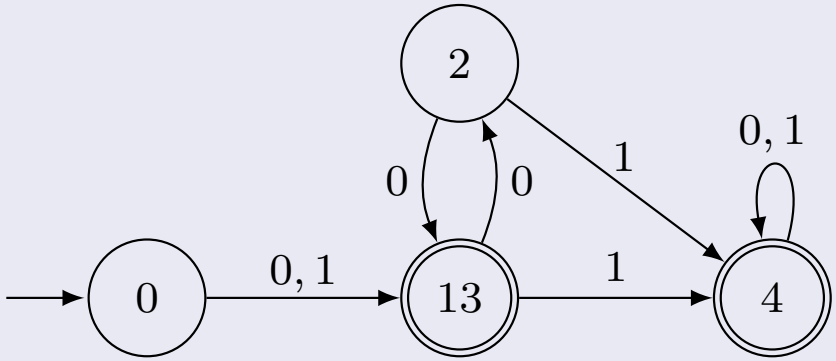

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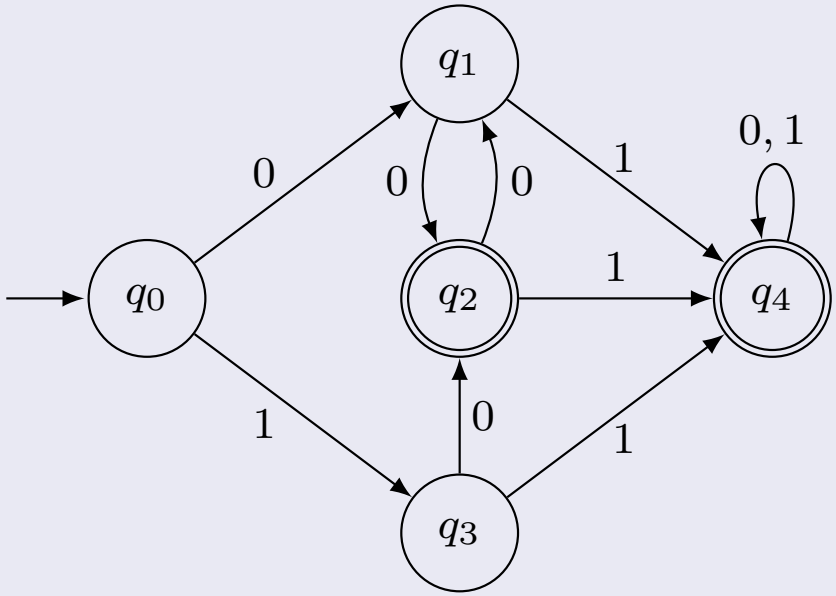

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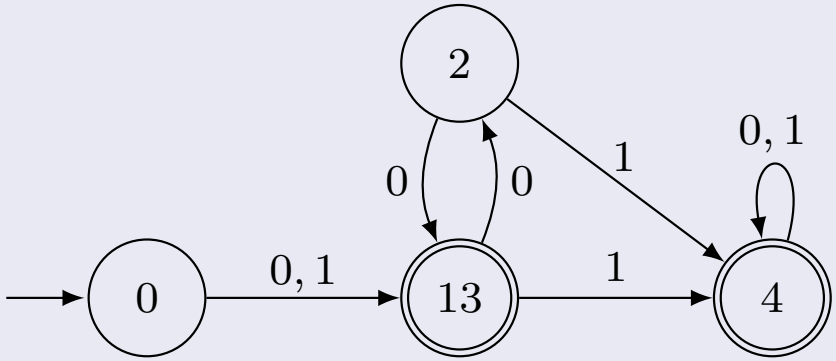

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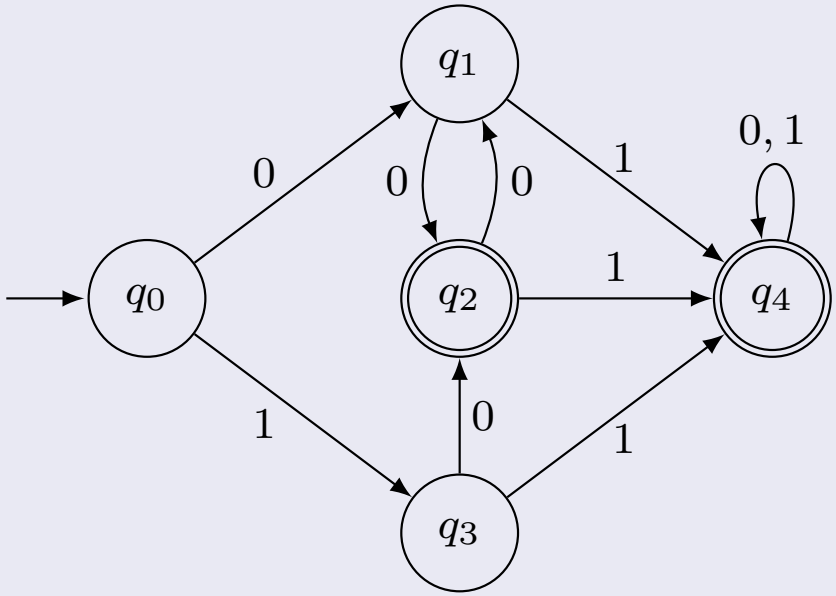

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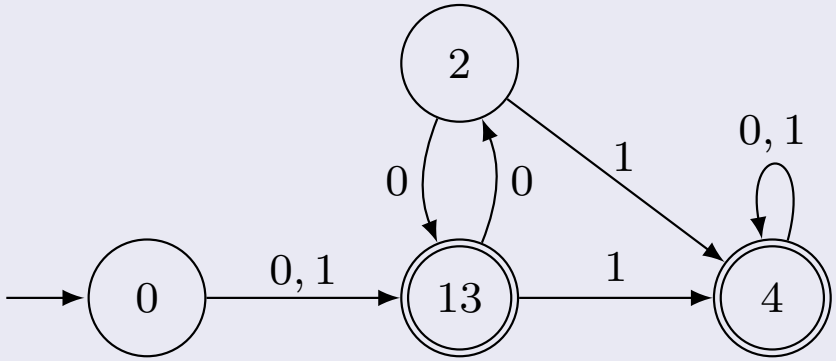

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Given any DFA M, application of the procedure reduce yields another DFA $\widehat{M}$ such that

$$L(M) = L(\widehat{M}).$$

Furthermore, $\widehat{M}$ is minimal in the sense that there is no other DFA with a smaller number of states that also accepts $L(M)$.

Proof. There are two parts. The first is to show that the DFA created by reduce is equivalent to the original DFA. This is relatively easy and we can use inductive arguments similar to those used in establishing the equivalence of DFA's and NFA's. All we have to do is to show that $\delta^*(q_i, w) = q_j$ if and only if the label of $\widehat{\delta}^*(q_i, w)$ is of the form $\ldots j \ldots$. We shall leave this as an exercise.

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