

# Formal Languages, Automata and Codes

Oleg Gutik

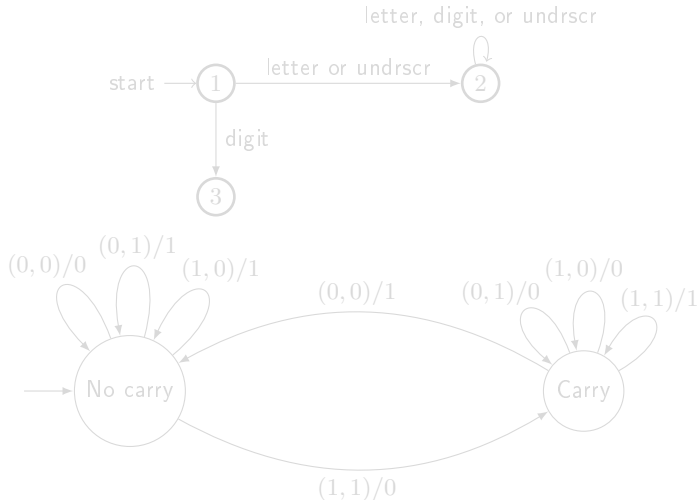


## Lecture 4

## 2.1 Deterministic Finite Accepters: Deterministic Accepters and Transition Graphs

The first type of automaton we study in detail are finite accepters that are deterministic in their operation. We start with a precise formal definition of deterministic accepters.

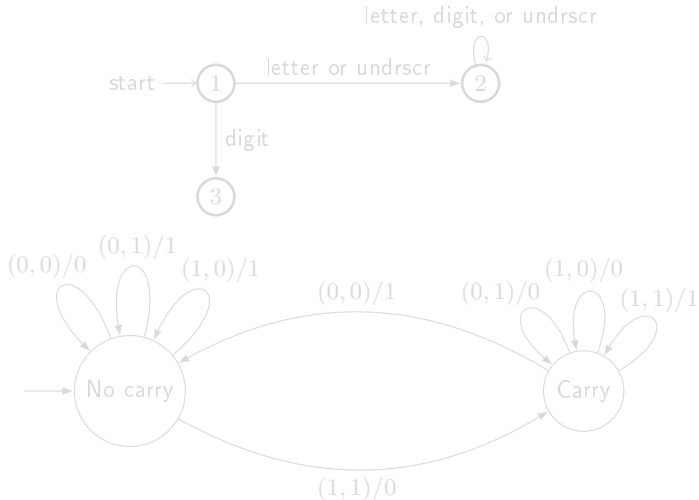
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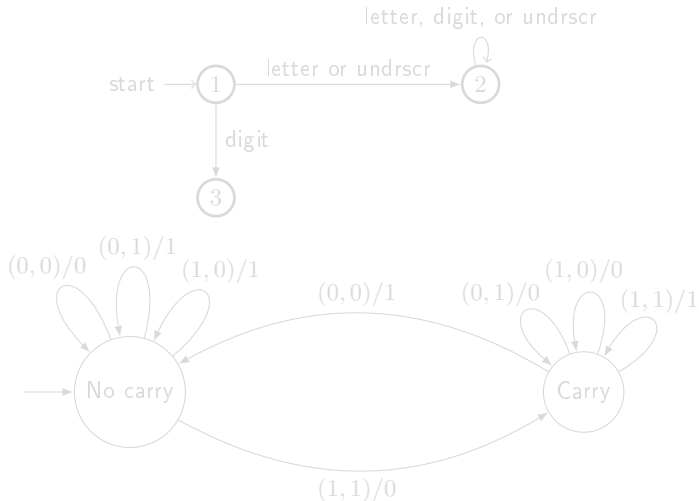
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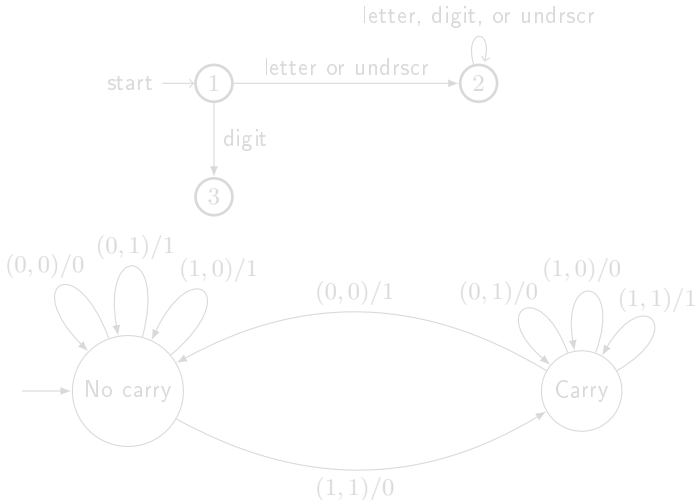
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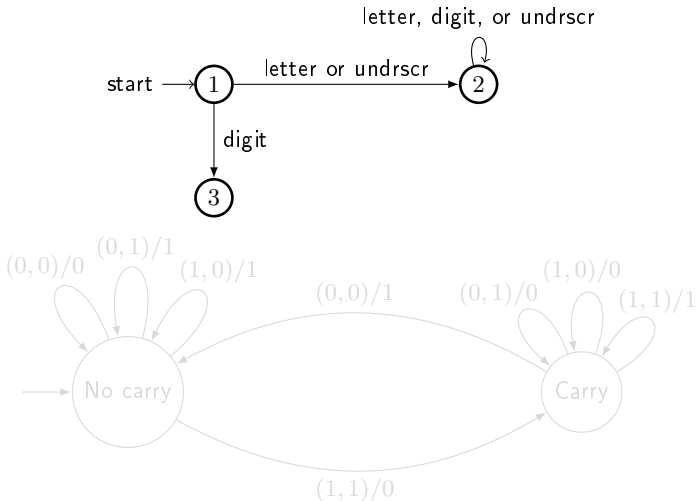
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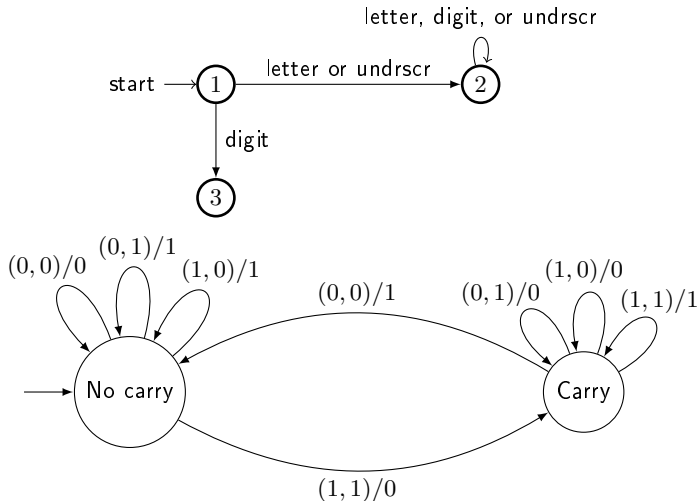
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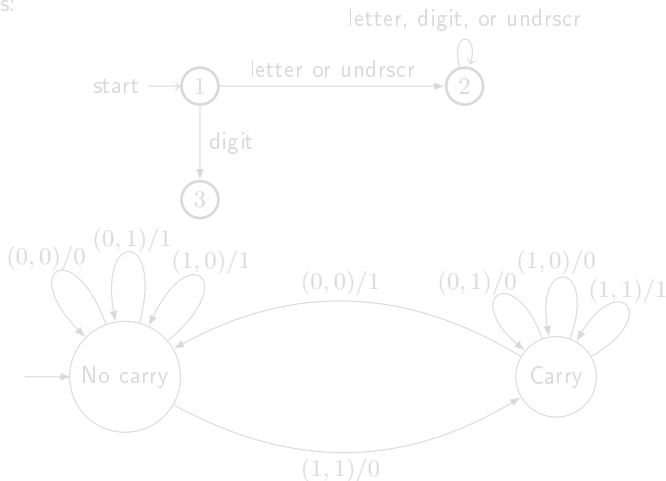
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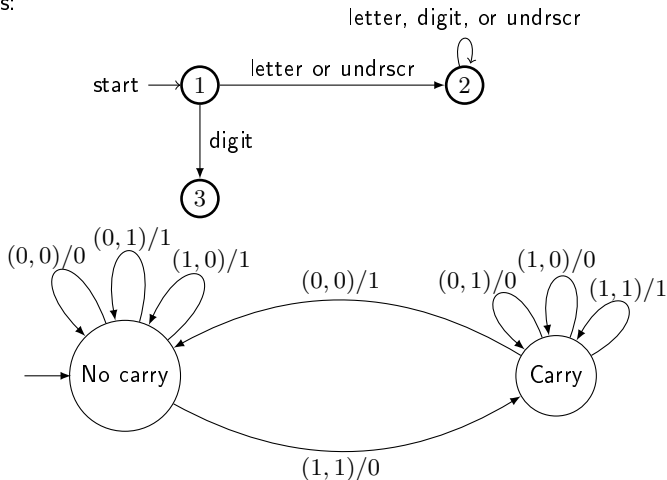


- Both have a finite number of internal states.
- Both process an input string, consisting of a sequence of symbols.
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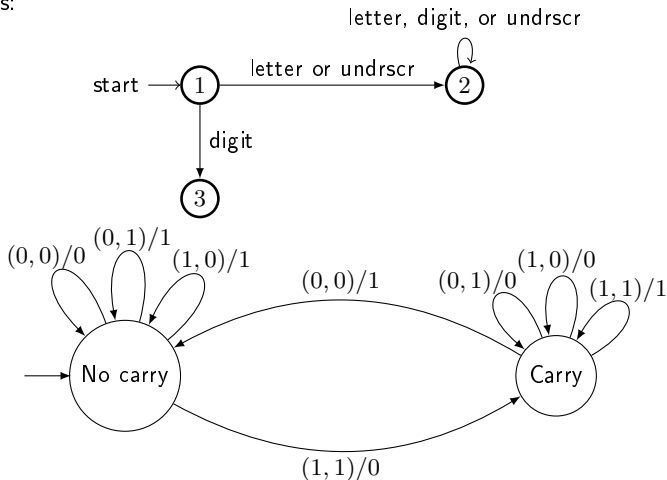
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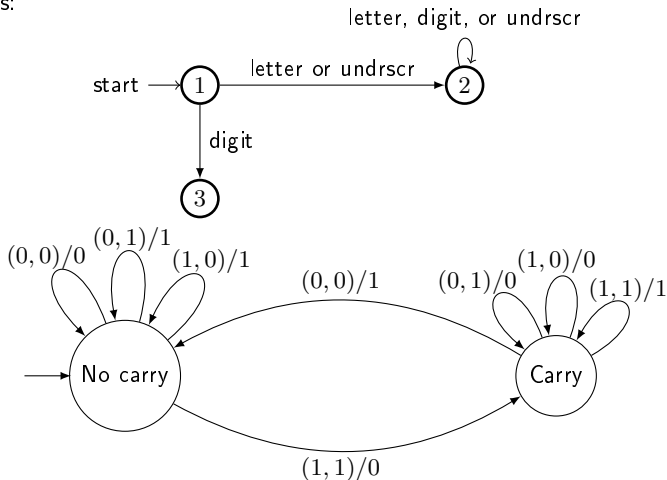
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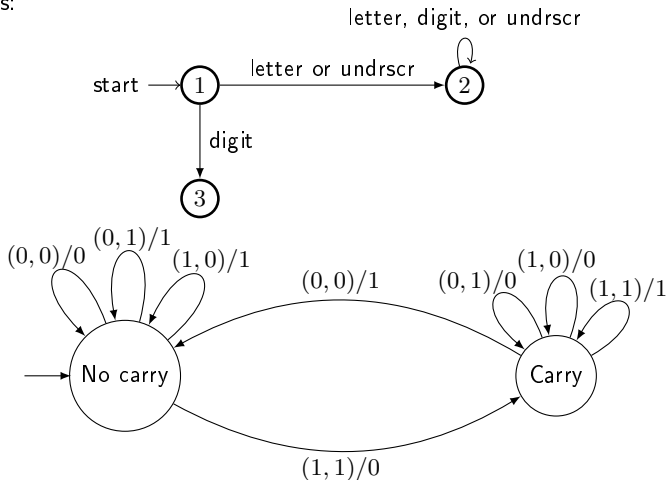
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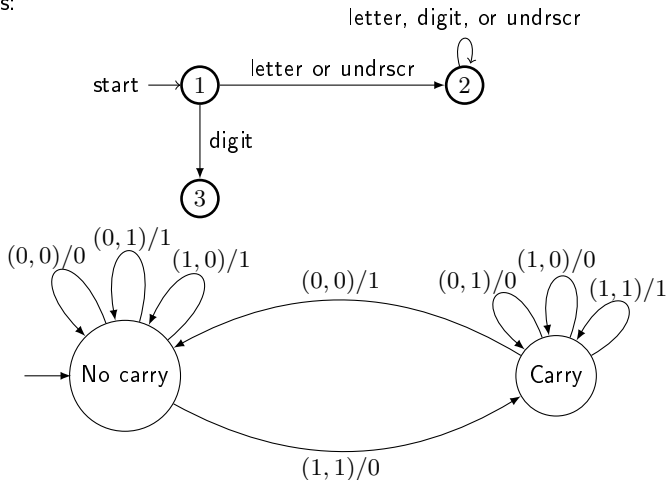
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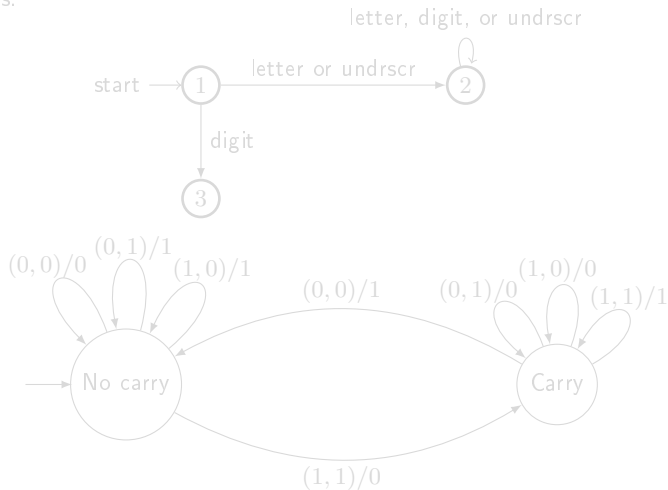
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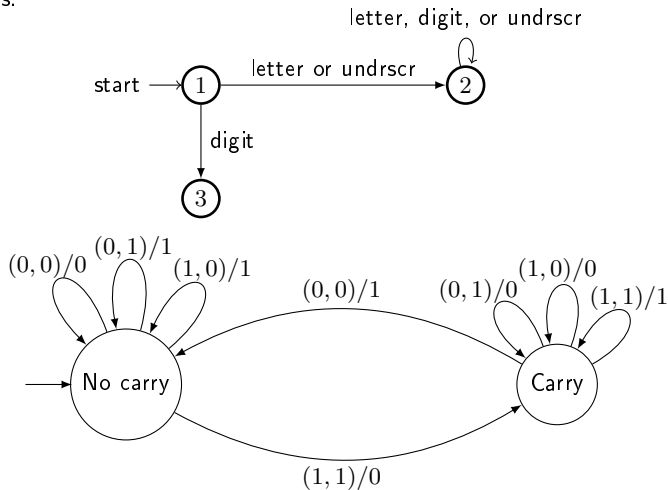
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- Both produce some output, but in a slightly different form. The automaton in the first figure only accepts or rejects the input, but also produces some output. Figure 2.10 shows an output DFA.

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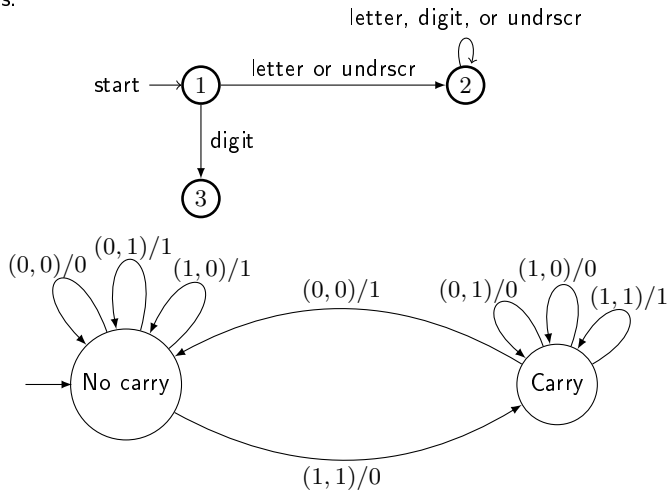
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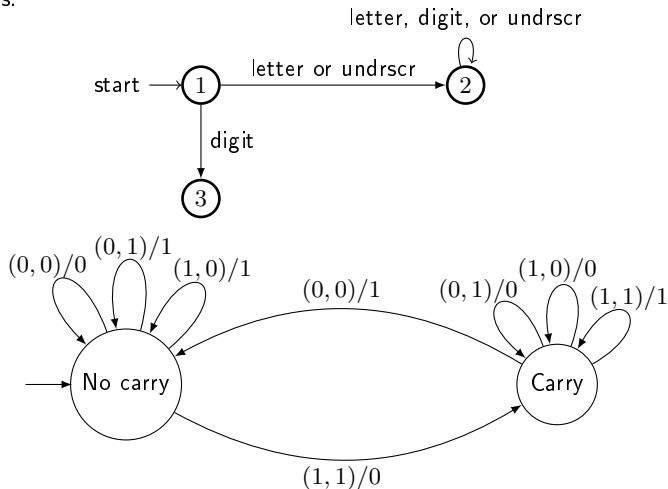


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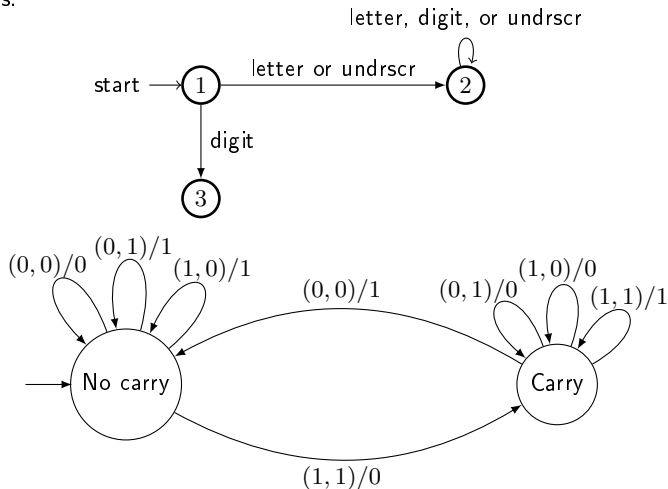
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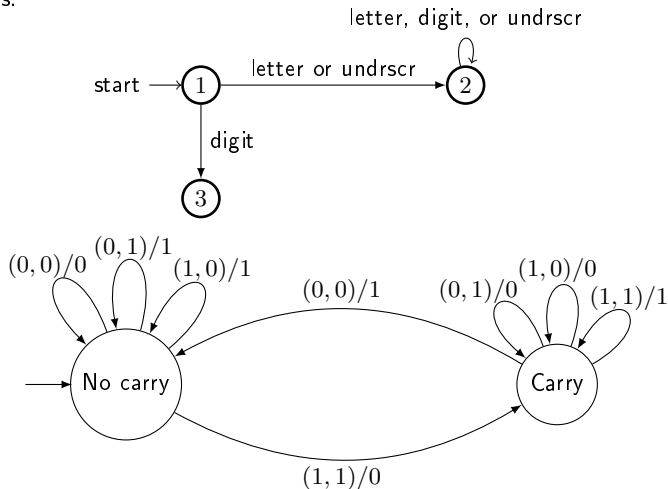
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Notice also that both automata have a single well-defined transition at each step. All of these features are incorporated in the following definition.

### Definition 2.1

A *deterministic finite acceptor* or *DFA* is defined by the quintuple

$$M = (Q, \Sigma, \delta, q_0, F),$$

where

- $Q$  is a finite set of internal states,
- $\Sigma$  is a finite set of symbols called the input alphabet,
- $\delta: Q \times \Sigma \rightarrow Q$  is a total function called the transition function,
- $q_0 \in Q$  is the start state,
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$$\delta(q_0, a) = q_1,$$

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## 2.1 Deterministic Finite Accepters: Deterministic Accepters and Transition Graphs

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## 2.1 Deterministic Finite Accepters: Deterministic Accepters and Transition Graphs

### Example 2.1

The graph in the following Figure



represents the DFA

$$M = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_1\}),$$

where  $\delta$  is given by

$$\delta(q_0, 0) = q_0,$$

$$\delta(q_0, 1) = q_1,$$

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$$\delta(q_1, 1) = q_2,$$

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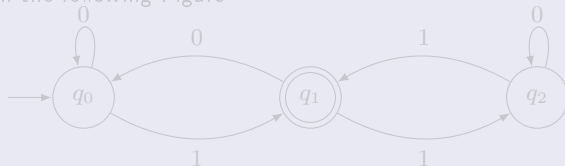
$$\delta(q_2, 1) = q_2.$$

This DFA accepts the string 01. Starting in state  $q_0$ , the symbol 0 is read first. Looking at the edges of the graph, we see that the automaton remains in state  $q_0$ . Next, the 1 is read and the automaton goes into state  $q_1$ . We are now at the end of the string and, at the same time, in a final state  $q_1$ . Therefore, the string 01 is accepted. The DFA does not accept the string 00, because after reading two consecutive 0's, it will be in state  $q_0$ . By similar reasoning, we see that the automaton will accept the strings 101, 0111, and 11001, but not 100 or 1100.

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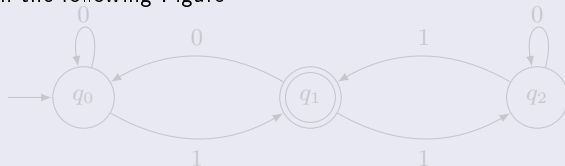
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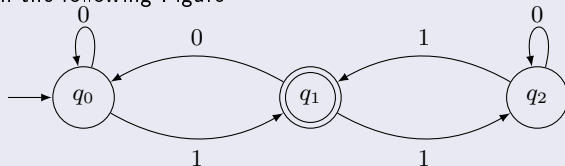
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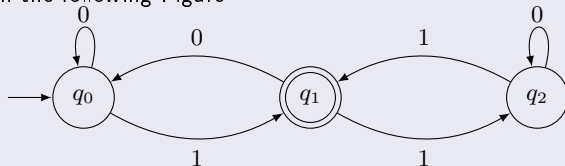
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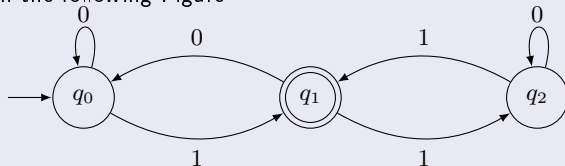
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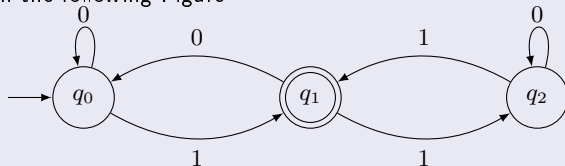
$$\delta(q_2, 1) = q_1.$$

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## 2.1 Deterministic Finite Accepters: Deterministic Accepters and Transition Graphs

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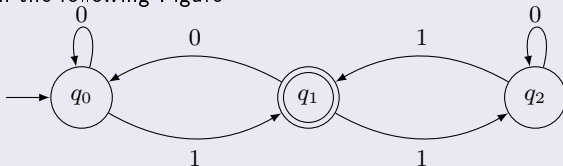
$$\delta(q_2, 1) = q_1.$$

This DFA accepts the string 01. Starting in state  $q_0$ , the symbol 0 is read first. Looking at the edges of the graph, we see that the automaton remains in state  $q_0$ . Next, the 1 is read and the automaton goes into state  $q_1$ . We are now at the end of the string and, at the same time, in a final state  $q_1$ . Therefore, the string 01 is accepted. The DFA does not accept the string 00, because after reading two consecutive 0's, it will be in state  $q_0$ . By similar reasoning, we see that the automaton will accept the strings 101, 0111, and 11001, but not 100 or 1100.

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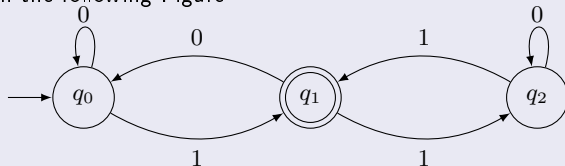
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## 2.1 Deterministic Finite Acceptors: Deterministic Acceptors and Transition Graphs

### Example 2.1

The graph in the following Figure



represents the DFA

where  $\delta$  is given by  $M = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_1\})$ ,

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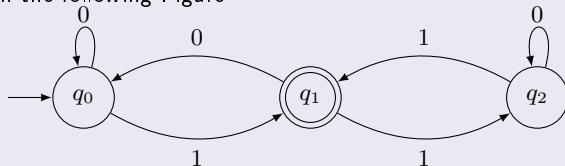
$$\delta(q_2, 1) = q_1.$$

This DFA accepts the string 01. Starting in state  $q_0$ , the symbol 0 is read first. Looking at the edges of the graph, we see that the automaton remains in state  $q_0$ . Next, the 1 is read and the automaton goes into state  $q_1$ . We are now at the end of the string and, at the same time, in a final state  $q_1$ . Therefore, the string 01 is accepted. The DFA does not accept the string 00, because after reading two consecutive 0's, it will be in state  $q_0$ . By similar reasoning, we see that the automaton will accept the strings 101, 0111, and 11001, but not 100 or 1100.

## 2.1 Deterministic Finite Accepters: Deterministic Accepters and Transition Graphs

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The graph in the following Figure



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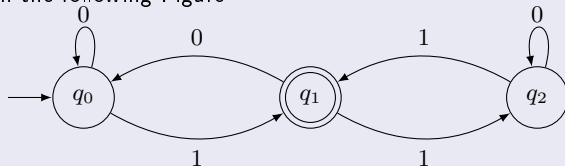
$$\delta(q_2, 1) = q_1.$$

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## 2.1 Deterministic Finite Accepters: Deterministic Accepters and Transition Graphs

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The graph in the following Figure



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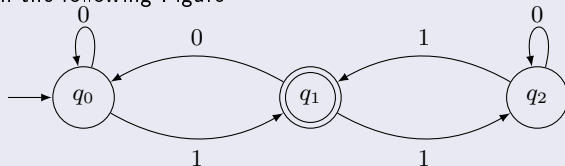
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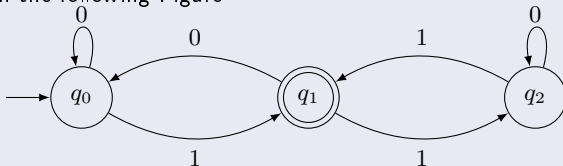
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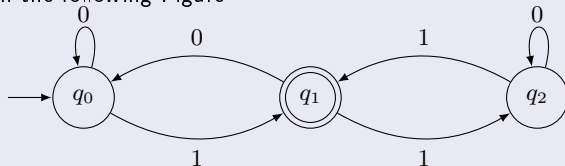
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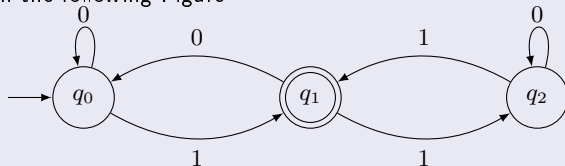
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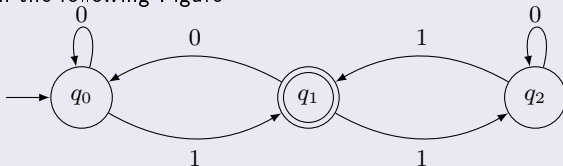
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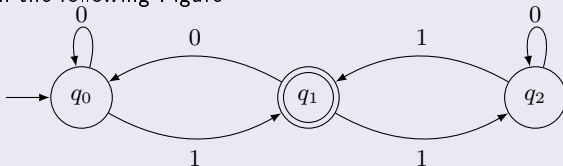
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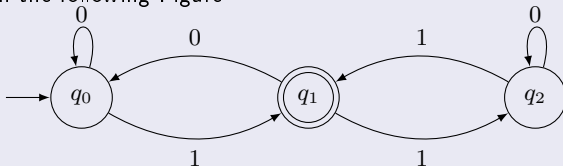
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This DFA accepts the string 01. Starting in state  $q_0$ , the symbol 0 is read first. Looking at the edges of the graph, we see that the automaton remains in state  $q_0$ . Next, the 1 is read and the automaton goes into state  $q_1$ . We are now at the end of the string and, at the same time, in a final state  $q_1$ . Therefore, the string 01 is accepted. The DFA does not accept the string 00, because after reading two consecutive 0's, it will be in state  $q_0$ . By similar reasoning, we see that the automaton will accept the strings 101, 0111, and 11001, but not 100 or 1100.

## 2.1 Deterministic Finite Accepters: Deterministic Accepters and Transition Graphs

### Example 2.1

The graph in the following Figure



represents the DFA

where  $\delta$  is given by  $M = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_1\})$ ,

$$\delta(q_0, 0) = q_0,$$

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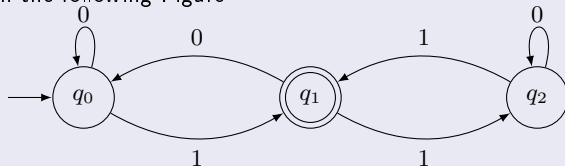
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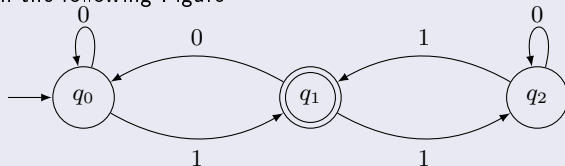
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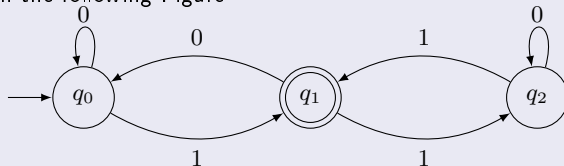
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### Example 2.2

Consider the DFA in the following Figure.



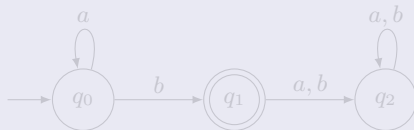
In the drawing we allowed the use of two labels on a single edge. Such multiply labeled edges are shorthand for two or more distinct transitions: The transition is taken whenever the input symbol matches any of the edge labels.

The automaton in the Figure remains in its initial state  $q_0$  until the first  $b$  is encountered. If this is also the last symbol of the input, then the string is accepted. If not, the DFA goes into state  $q_2$ , from which it can never escape. The state  $q_2$  is a trap state. We see clearly from the graph that the automaton accepts all strings consisting of an arbitrary number of  $a$ 's, followed by a single  $b$ . All other input strings are rejected. In set notation, the language accepted by the automaton is

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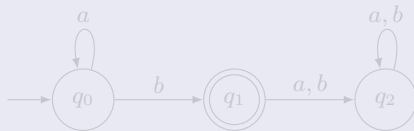
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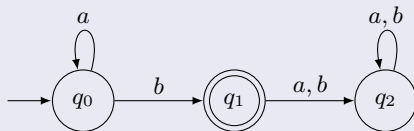
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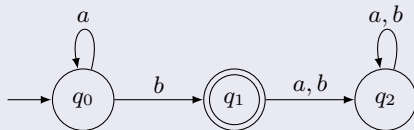
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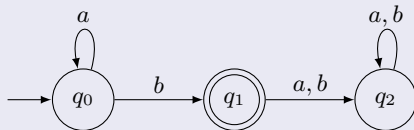
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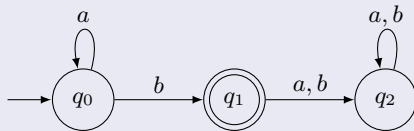
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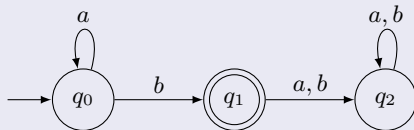
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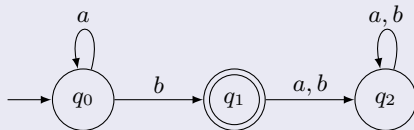
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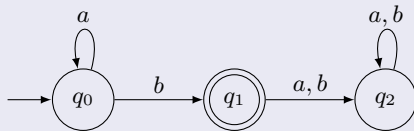
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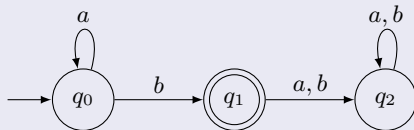
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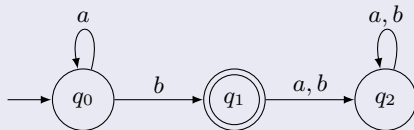
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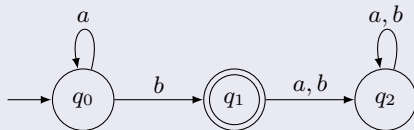
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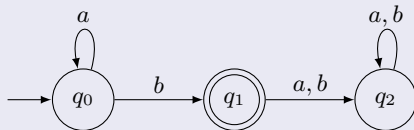
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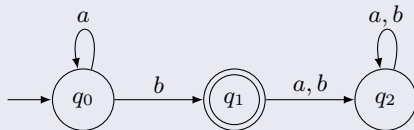
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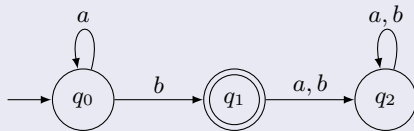
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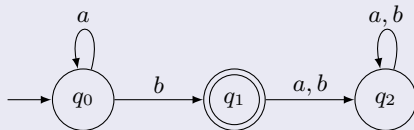
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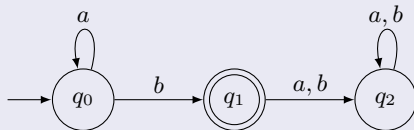
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These examples show how convenient transition graphs are for working with finite automata. While it is possible to base all arguments strictly on the properties of the transition function and its extension through (1) and (2),

$$\delta^*(q, \lambda) = q, \quad (1)$$

$$\delta^*(q, wa) = \delta(\delta^*(q, w), a), \quad (2)$$

the results are hard to follow. In our discussion, we use graphs, which are more intuitive, as far as possible. To do so, we must, of course, have some assurance that we are not misled by the representation and that arguments based on graphs are as valid as those that use the formal properties of  $\delta$ . The following preliminary result gives us this assurance.

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### Theorem 2.1

Let  $M = (Q, \Sigma, \delta, q_0, F)$  be a deterministic finite accepter, and let  $G_M$  be its associated transition graph. Then for every  $q_i, q_j \in Q$ , and  $w \in \Sigma^+$ ,  $\delta^*(q_i, w) = q_j$  if and only if there is in  $G_M$  a walk with label  $w$  from  $q_i$  to  $q_j$ .

**Proof.** This claim is fairly obvious from an examination of such simple cases as **Example 2.1**. It can be proved rigorously using an induction on the length of  $w$ . Assume that the claim is true for all strings  $v$  with  $|v| \leq n$ . Consider then any  $w$  of length  $n + 1$  and write it as

$$w = va.$$

Suppose now that  $\delta^*(q_i, v) = q_k$ . Since  $|v| = n$ , there must be a walk in  $G_M$  labeled  $v$  from  $q_i$  to  $q_k$ . But if  $\delta^*(q_i, w) = q_j$ , then  $M$  must have a transition  $\delta(q_k, a) = q_j$ , so that by construction  $G_M$  has an edge  $(q_k, q_j)$  with label  $a$ . Thus, there is a walk in  $G_M$  labeled  $va = w$  between  $q_i$  and  $q_j$ . Since the result is obviously true for  $n = 1$ , we can claim by induction that, for every  $w \in \Sigma^+$ ,

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The argument can be turned around in a straightforward way to show that the existence of such a path implies (4), thus completing the proof. ■

### Theorem 2.1

Let  $M = (Q, \Sigma, \delta, q_0, F)$  be a deterministic finite acceptor, and let  $G_M$  be its associated transition graph. Then for every  $q_i, q_j \in Q$ , and  $w \in \Sigma^+$ ,  $\delta^*(q_i, w) = q_j$  if and only if there is in  $G_M$  a walk with label  $w$  from  $q_i$  to  $q_j$ .

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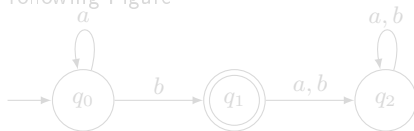
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## 2.1 Deterministic Finite Accepters: Languages and DFA's

While graphs are convenient for visualizing automata, other representations are also useful. For example, we can represent the function  $\delta$  as a table. The table in the Figure

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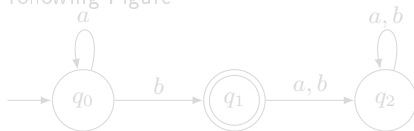
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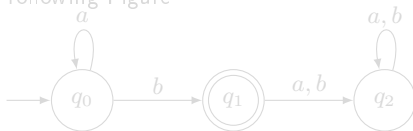
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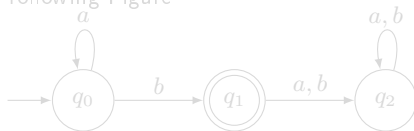
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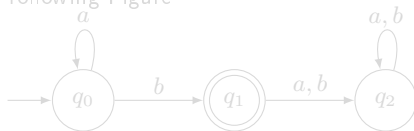


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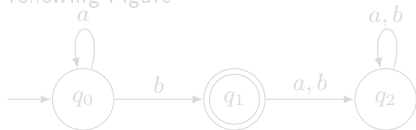
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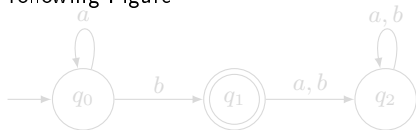
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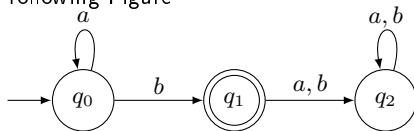
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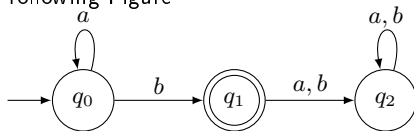
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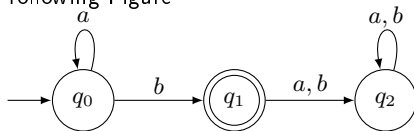
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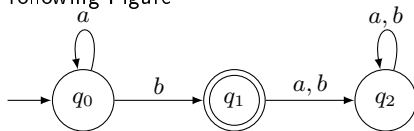
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## 2.1 Deterministic Finite Accepters: Languages and DFA's

### Example 2.3

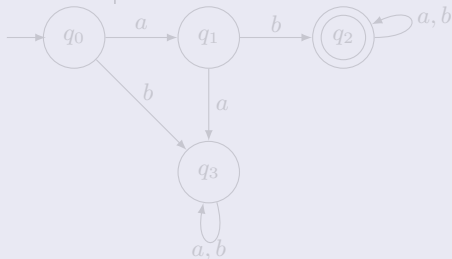
Find a deterministic finite accepter that recognizes the set of all strings on  $\Sigma = \{a, b\}$  starting with the prefix  $ab$ . The only issue here is the first two symbols in the string; after they have been read, no further decisions are needed. Still, the automaton has to process the whole string before its decision is made. We can therefore solve the problem with an automaton that has four states: an initial state, two states for recognizing  $ab$  ending in a final trap state, and one nonfinal trap state. If the first symbol is the letter  $a$  and the second is the letter  $b$ , the automaton goes to the final trap state, where it will stay since the rest of the input does not matter. On the other hand, if the first symbol is not the letter  $a$  or the second one is not the letter  $b$ , the automaton enters the nonfinal trap state. The simple solution is shown in the following Figure.



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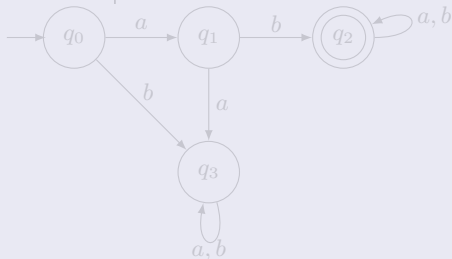
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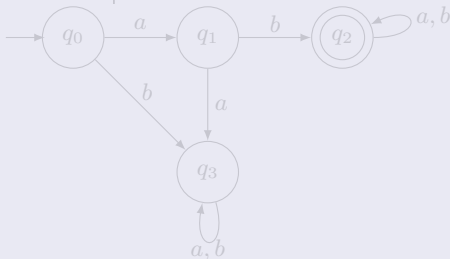
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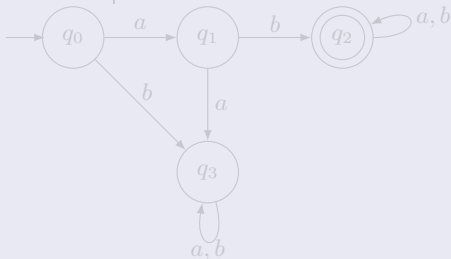
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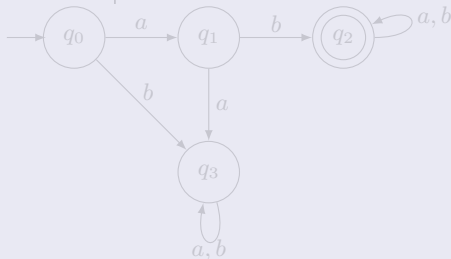
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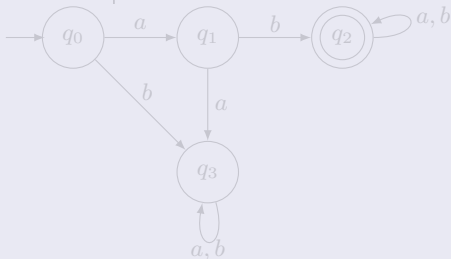
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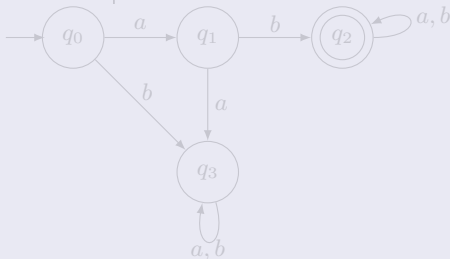
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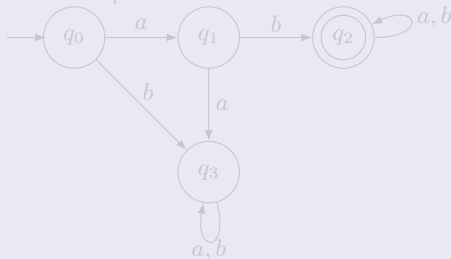




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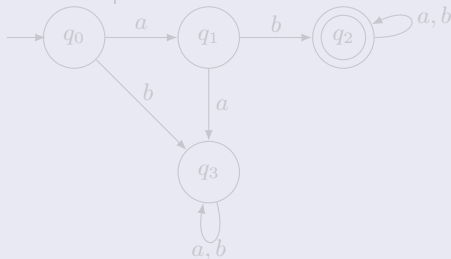
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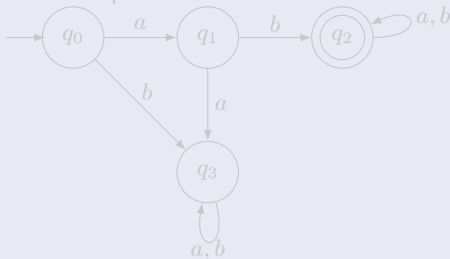
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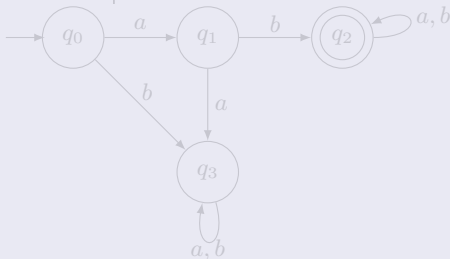
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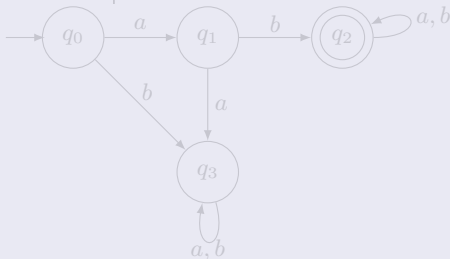
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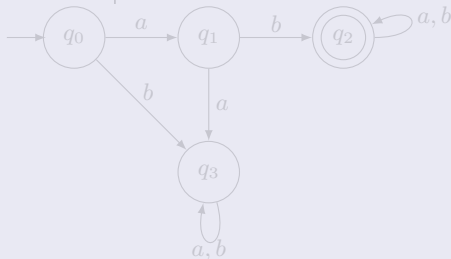
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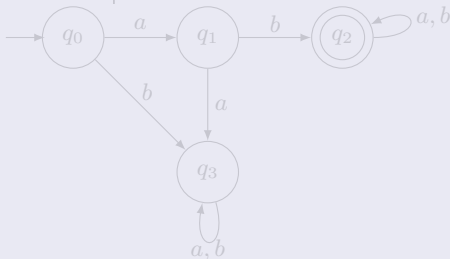
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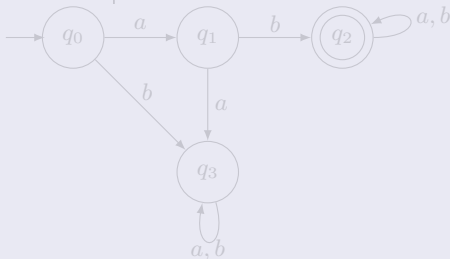
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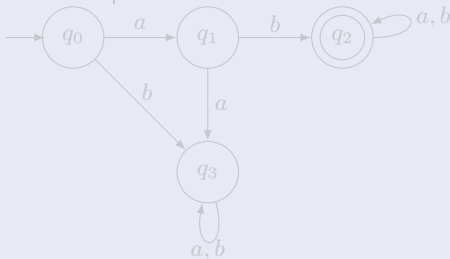




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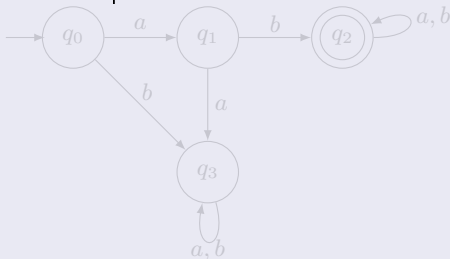
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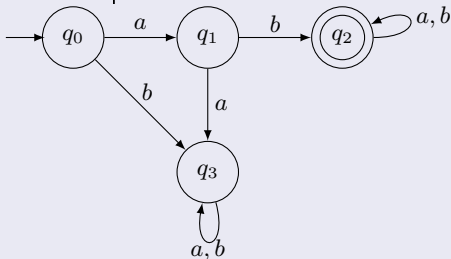
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Find a DFA that accepts all the strings on  $\{0,1\}$ , except those containing the substring 001.

In deciding whether the substring 001 has occurred, we need to know not only the current input symbol, but we also need to remember whether or not it has been preceded by one or two 0's. We can keep track of this by putting the automaton into specific states and labeling them accordingly. Like variable names in a programming language, state names are arbitrary and can be chosen for mnemonic reasons. For example, the state in which two 0's were the immediately preceding symbols can be labeled simply 00.

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If we label the states with the relevant symbols, it is very easy to see what the transitions must be. For example,

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because this situation arises only if there are three consecutive 0's. We are only interested in the last two, a fact we remember by keeping the DFA in the state 00. A complete solution is shown in the following Figure.



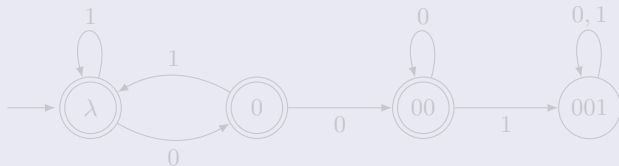
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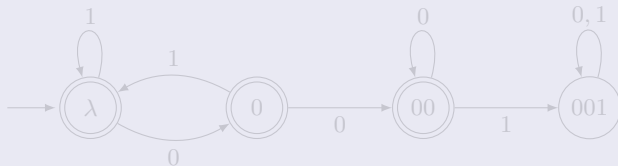
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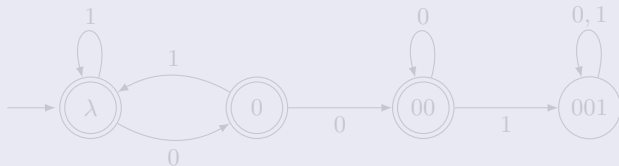
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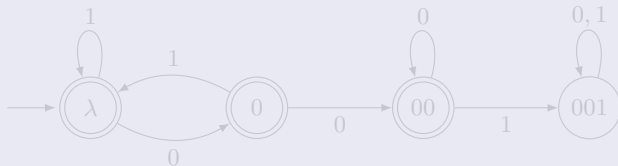
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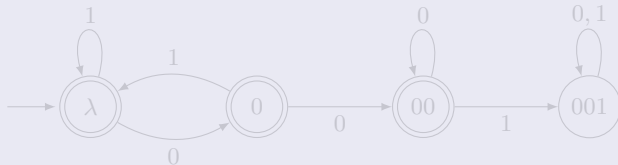
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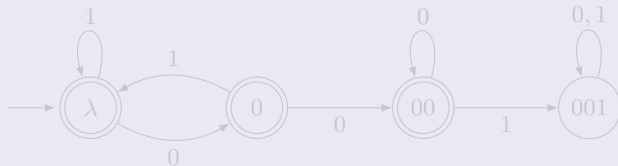
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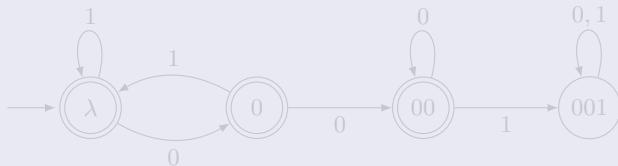


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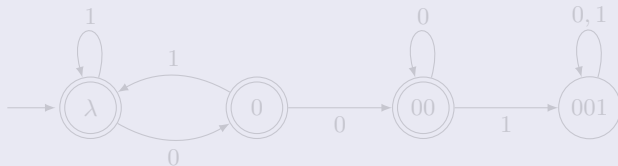
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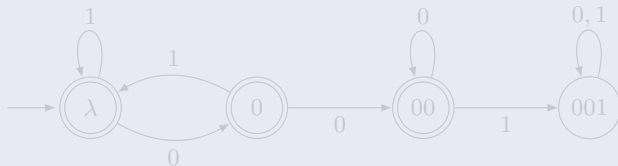
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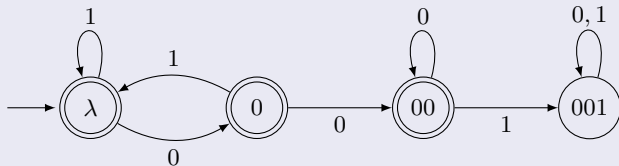
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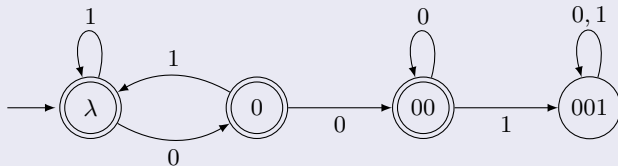
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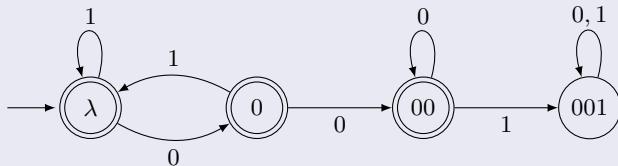
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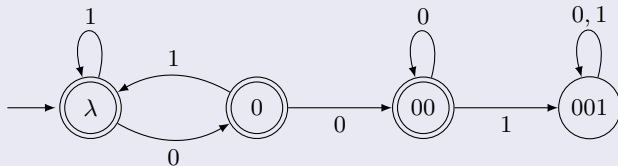
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## 2.1 Deterministic Finite Accepters: Regular Languages

Every finite automaton accepts some language. If we consider all possible finite automata, we get a set of languages associated with them. We shall call such a set of languages a *family*. The family of languages that is accepted by deterministic finite accepters is quite limited. The structure and properties of the languages in this family will become clearer as our study proceeds; for the moment we will simply attach a name to this family.

### Definition 2.2

A language  $L$  is called *regular* if and only if there exists some deterministic finite accepter  $M$  such that

$$L = L(M).$$

### Example 2.5

Show that the language

$$L = \{awa : w \in \{a,b\}^*\}$$

is regular.

To show that this or any other language is regular, all we have to do is find a DFA for it. The construction of a DFA for this language is similar to Example 2.3, but a little more complicated. What this DFA must do is check whether a string begins and ends with the letter  $a$ ; what is between is immaterial. The solution is complicated by the fact that there is no explicit way of testing the end of the string.



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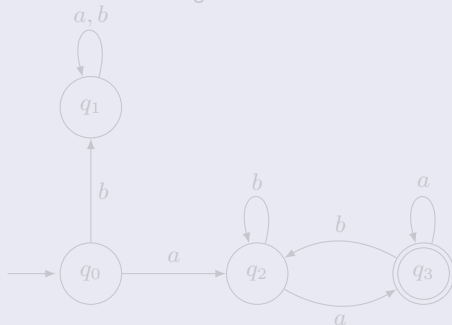


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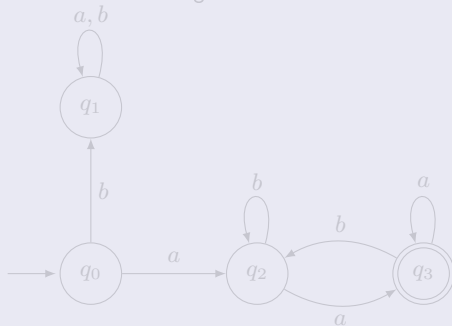


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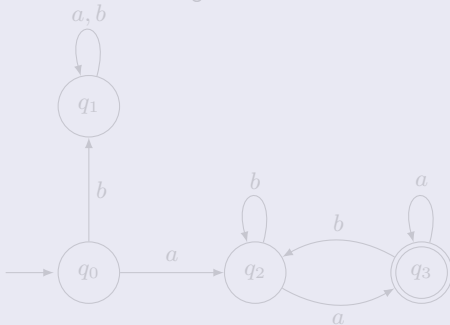
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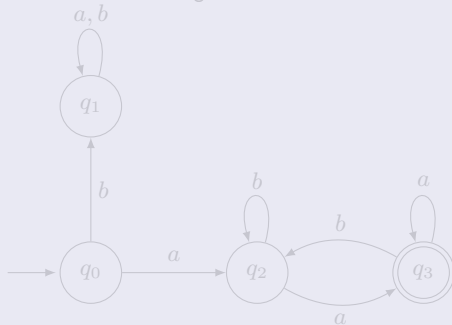


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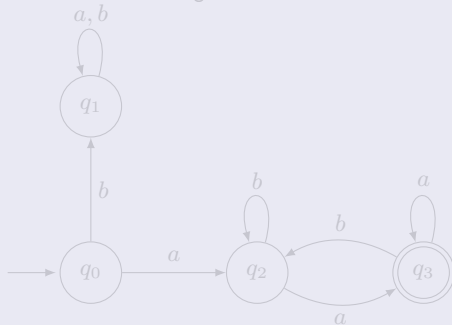


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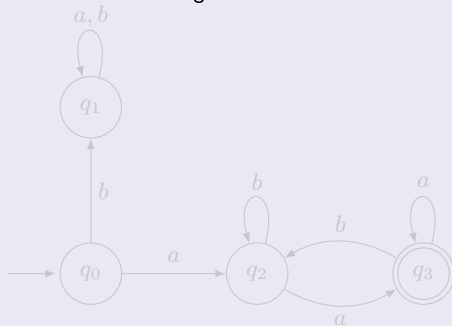


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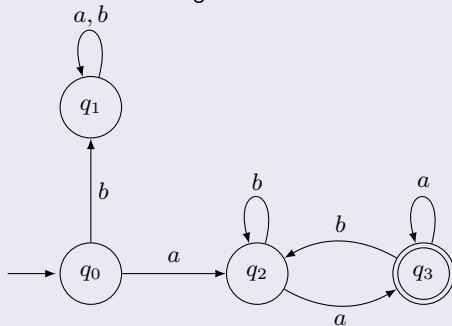


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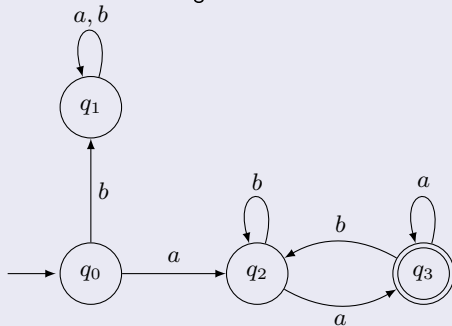


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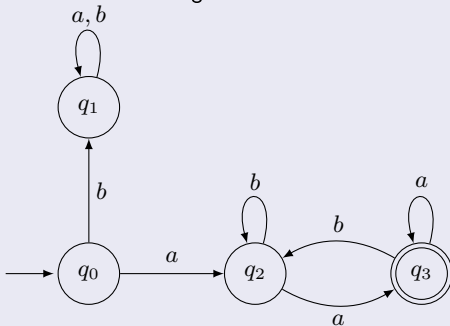


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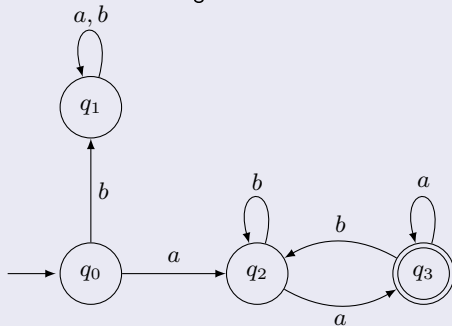


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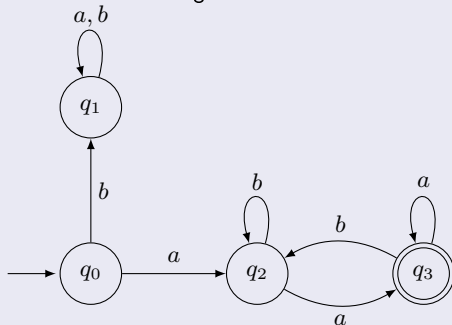
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Again, trace a few examples to see why this works. After one or two tests, it will be obvious that the DFA accepts a string if and only if it begins and ends with the letter  $a$ . Since we have constructed a DFA for the language, we can claim that, by definition, the language is regular.

### Example 2.6

Let  $L$  be the language in Example 2.5:

$$L = \{awa : w \in \{a,b\}^*\}.$$

Show that  $L^2$  is regular. Again we show that the language is regular by constructing a DFA for it. We can write an explicit expression for  $L^2$ , namely,

$$L^2 = \{aw_1aaw_2a : w_1, w_2 \in \{a,b\}^*\}.$$



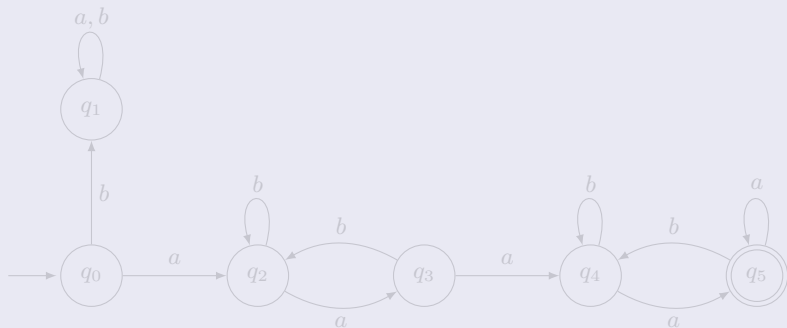
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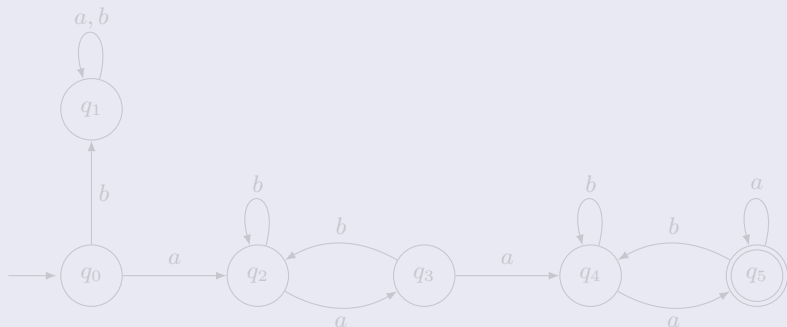
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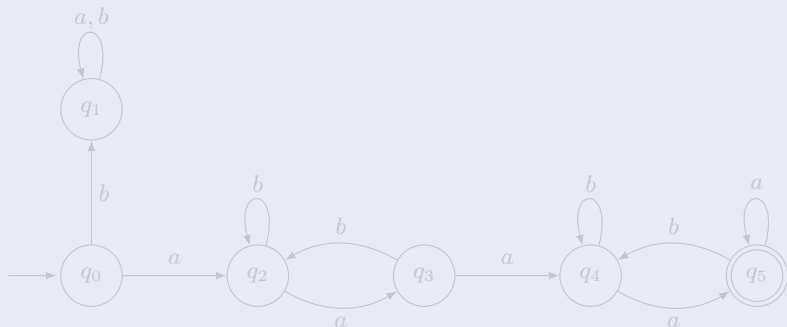
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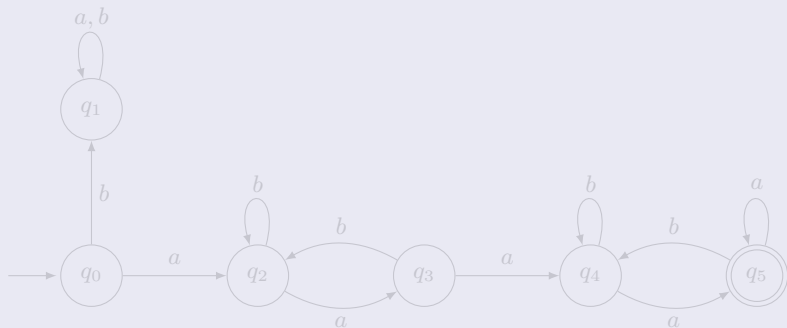
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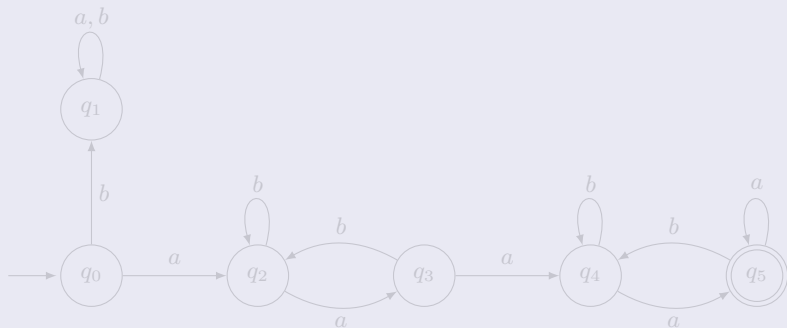
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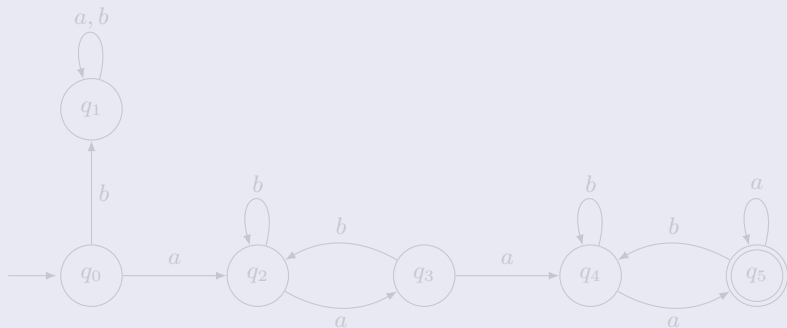
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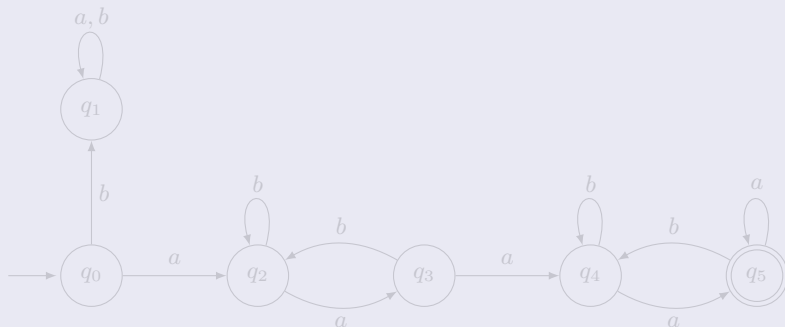
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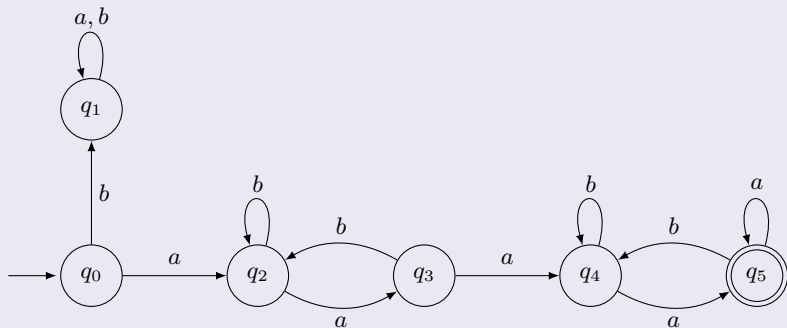
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## 2.1 Deterministic Finite Accepters: Regular Languages

### Example 2.6 (continuation)

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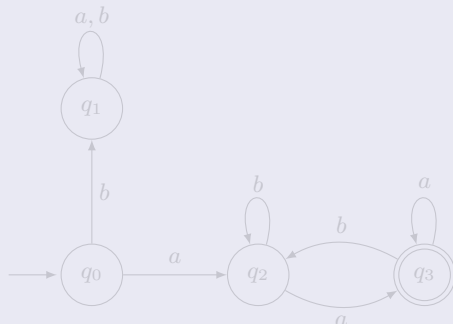


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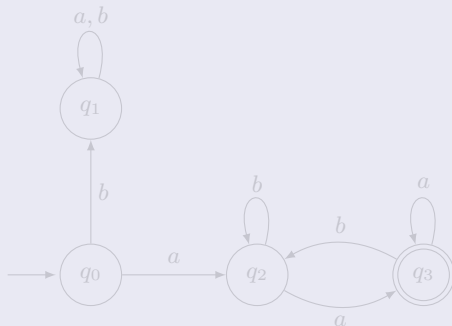


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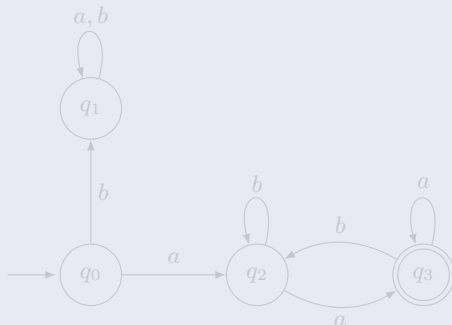


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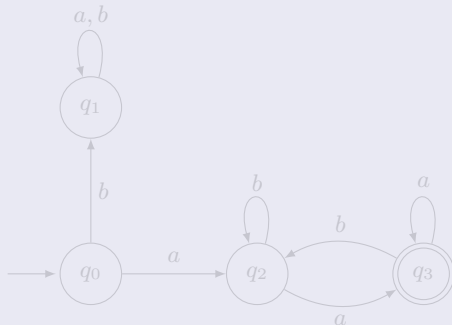


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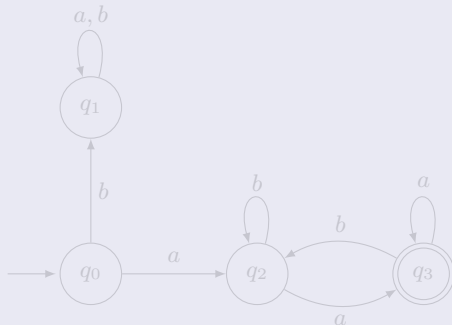


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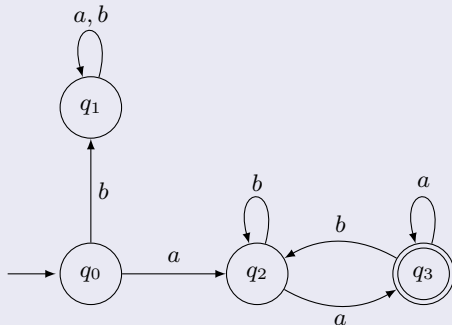
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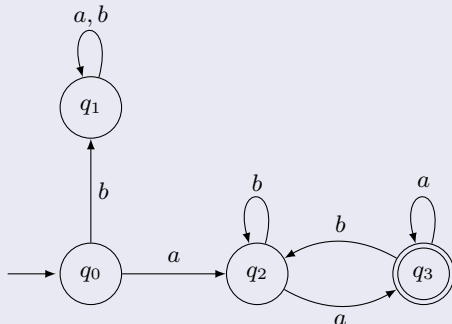


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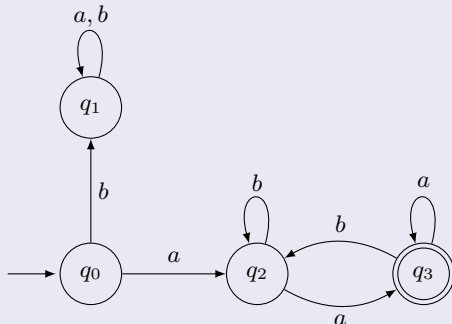


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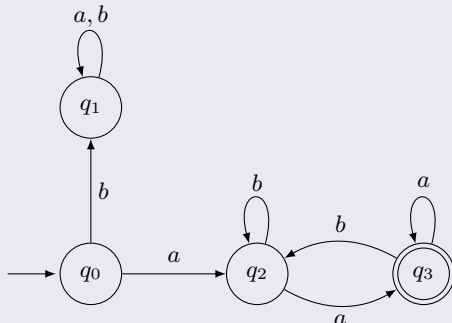


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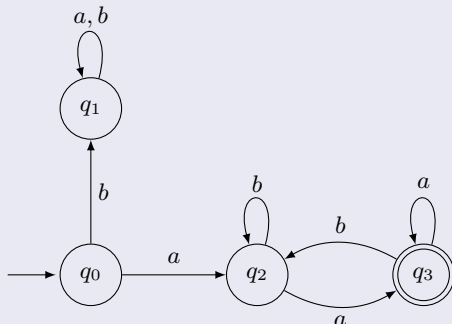


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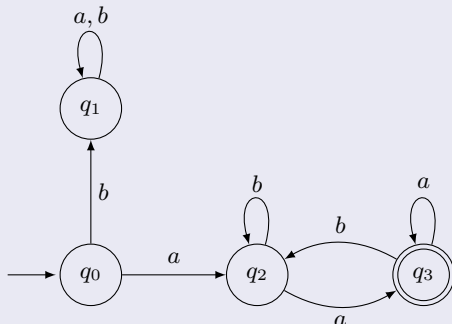


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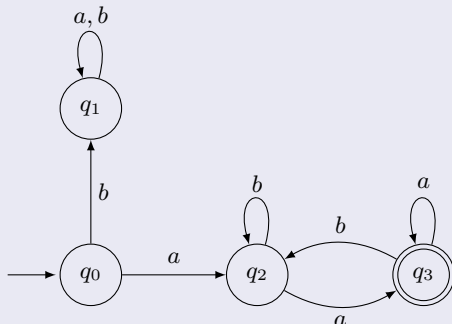


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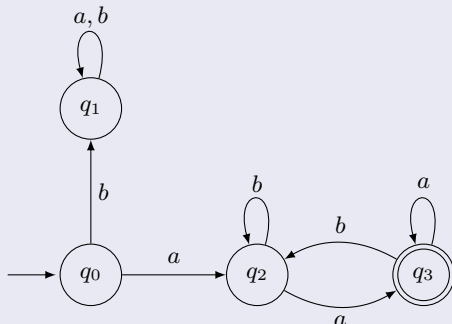


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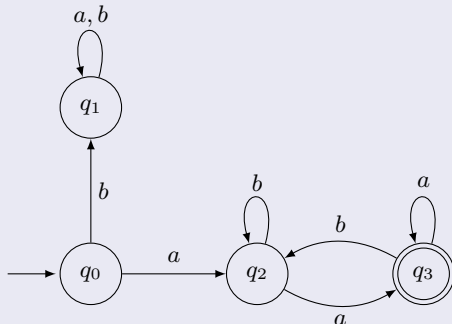
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We can do this by making  $\delta(q_3, a) = q_4$ . The complete solution is in the Figure below.

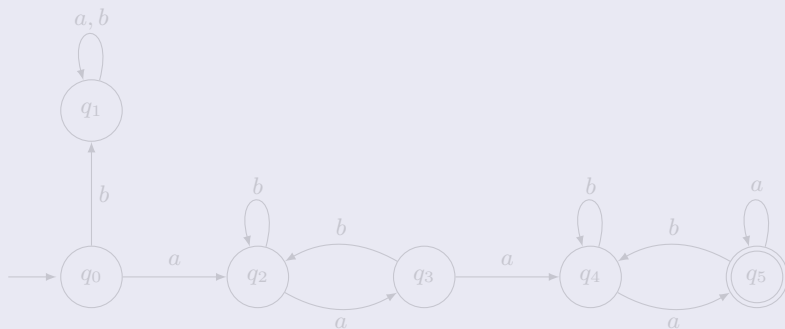


This DFA accepts  $L^2$ , which is therefore regular.

The last example suggests the conjecture that if a language  $L$  is regular, so are  $L^2, L^3, \dots$ . We shall see later that this is indeed correct.

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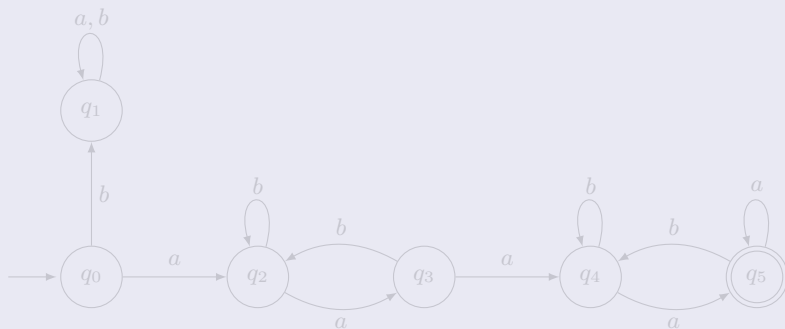


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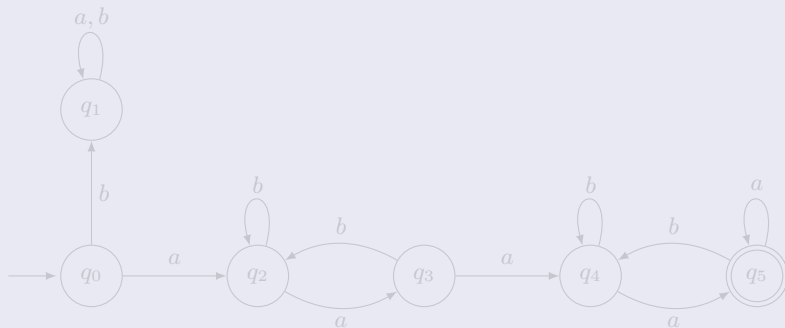


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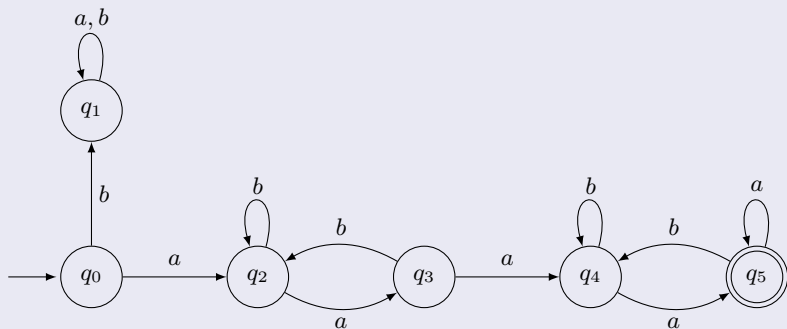


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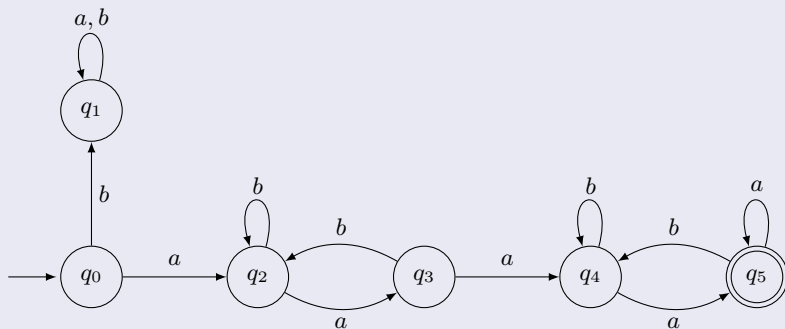


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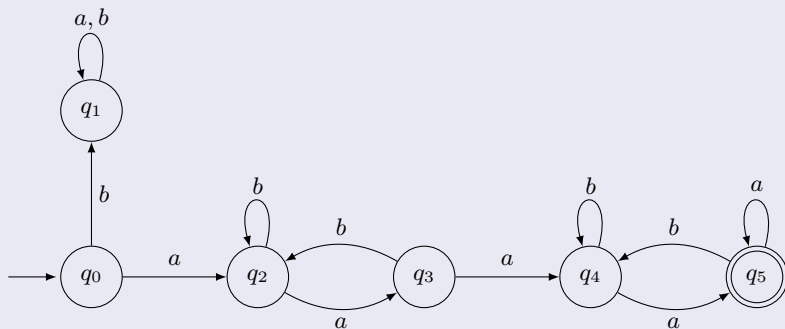


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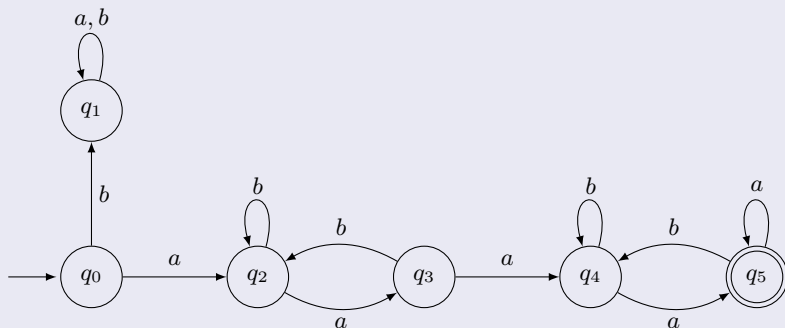
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