Formal Languages, Automata and Codes

Oleg Gutik

Lecture 4

The first type of automaton we study in detail are finite accepters that are deterministic in their operation. We start with a precise formal definition of deterministic accepters.

The following Figures represent two simple automata, with some common features:

2.1 Deterministic Finite Accepters: Deterministic Accepters and Transition Graphs

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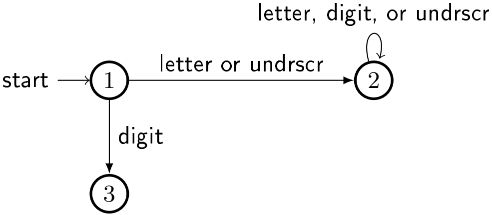

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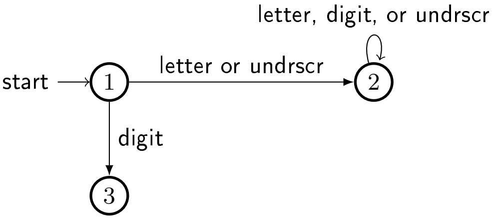

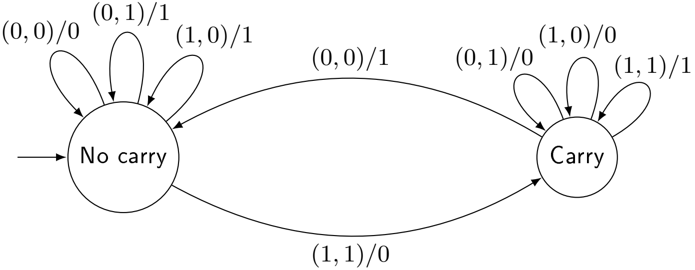

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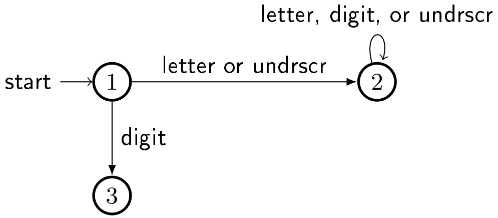

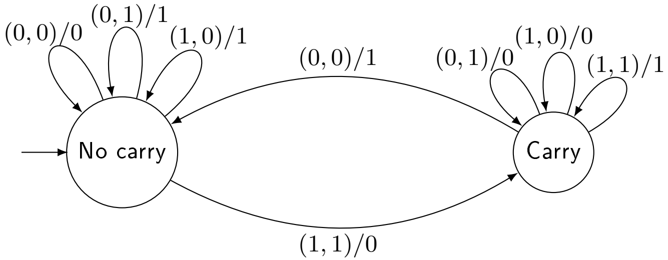

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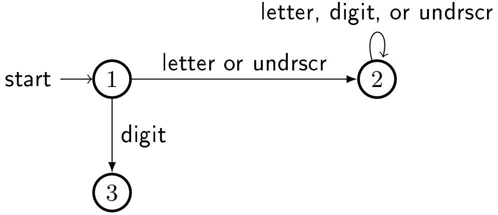

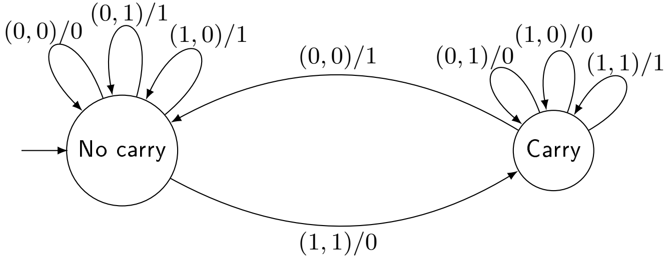

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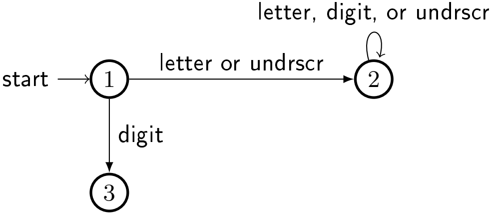

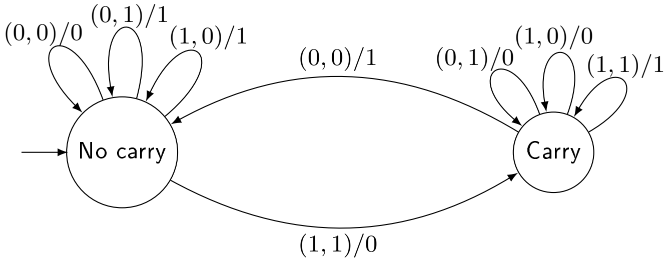

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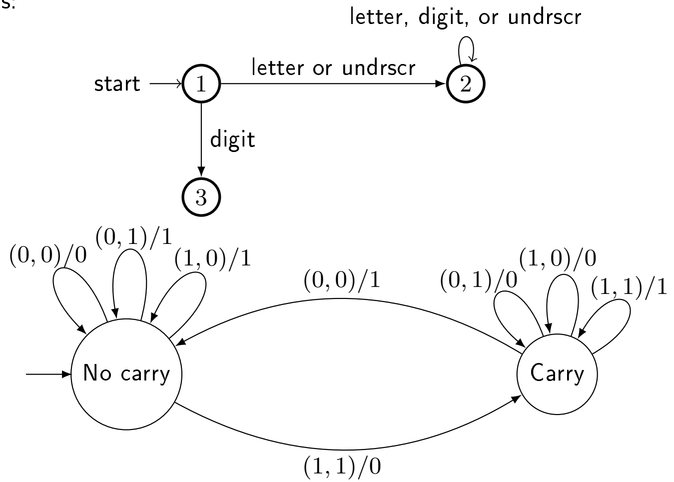

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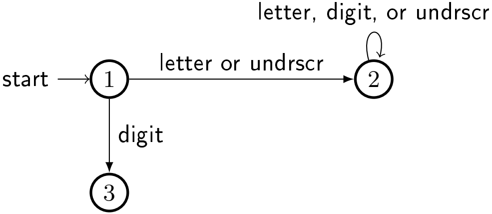

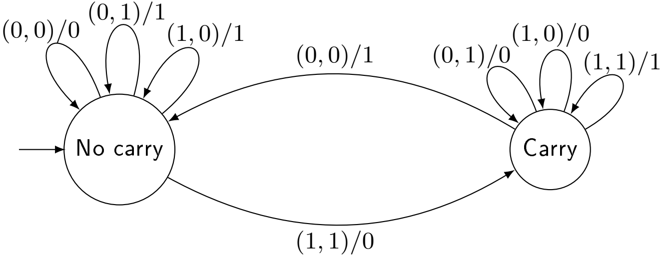

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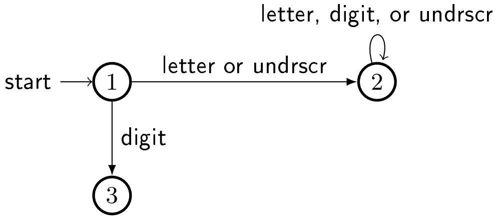

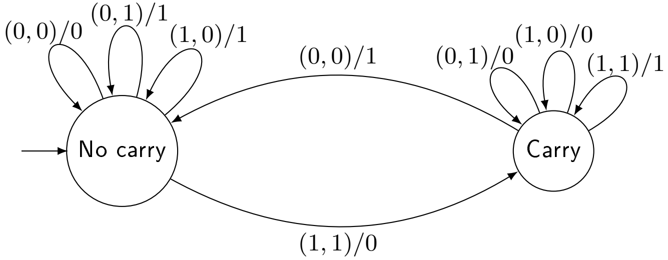

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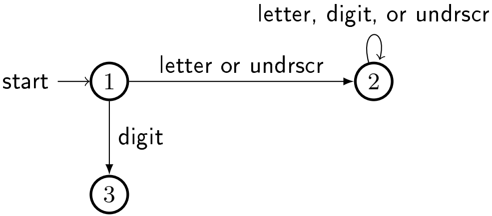

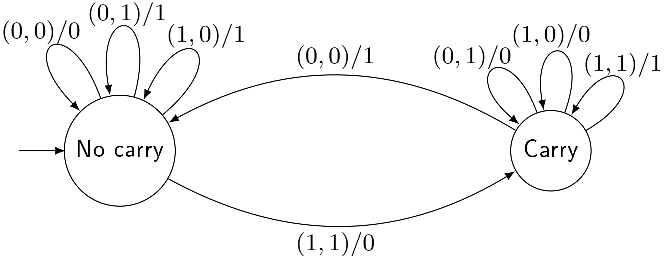

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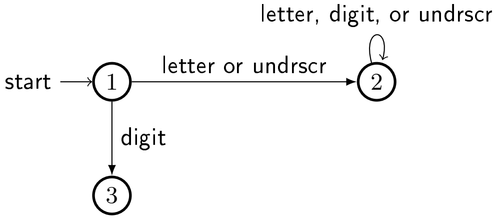

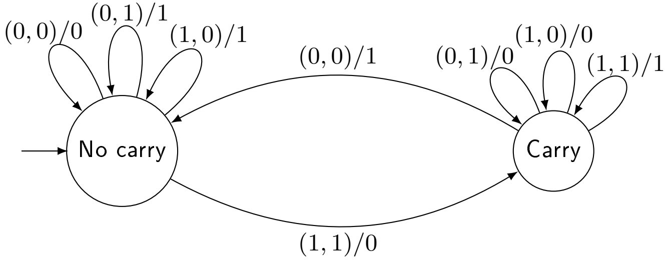

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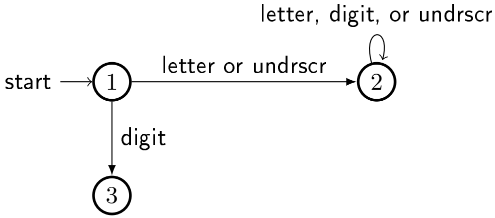

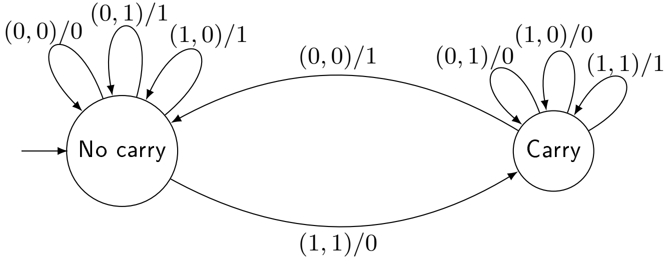

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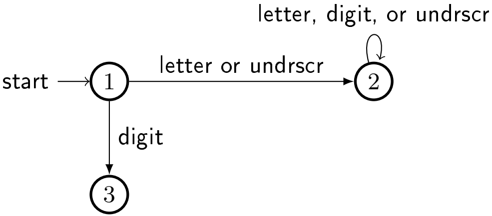

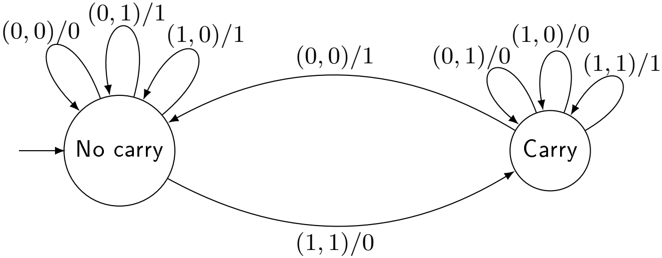

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Notice also that both automata have a single well-defined transition at each step. All of these features are incorporated in the following definition.

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A *deterministic finite accepter* or *DFA* is defined by the quintuple

$$M = (Q, \Sigma, \delta, q_0, F),$$

where

- Q is a finite set of *internal states*,
- Σ is a finite set of symbols called the *input alphabet*,
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A deterministic finite accepter operates in the following manner. At the initial time, it is assumed to be in the initial state q_0, with its input mechanism on the leftmost symbol of the input string. During each move of the automaton, the input mechanism advances one position to the right, so each move consumes one input symbol. When the end of the string is reached, the string is accepted if the automaton is in one of its final states. Otherwise the string is rejected. The input mechanism can move only from left to right and reads exactly one symbol on each step. The transitions from one internal state to another are governed by the transition function δ. For example, if

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Example 2.1

The graph in the following Figure

represents the DFA

$$M = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_1\}),$$

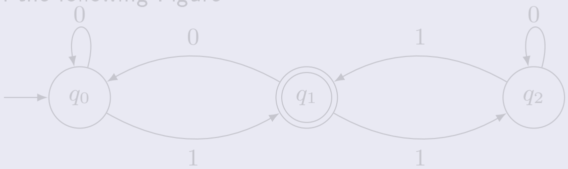
where δ is given by

$$\delta(q_0, 0) = q_0, \qquad \delta(q_0, 1) = q_1,$$
$$\delta(q_1, 0) = q_0, \qquad \delta(q_1, 1) = q_2,$$
$$\delta(q_2, 0) = q_2, \qquad \delta(q_2, 1) = q_1.$$

This DFA accepts the string 01. Starting in state q_0, the symbol 0 is read first. Looking at the edges of the graph, we see that the automaton remains in state q_0. Next, the 1 is read and the automaton goes into state q_1. We are now at the end of the string and, at the same time, in a final state q_1. Therefore, the string 01 is accepted. The DFA does not accept the string 00, because after reading two consecutive 0's, it will be in state q_0. By similar reasoning, we see that the automaton will accept the strings 101, 0111, and 11001, but not 100 or 1100.

Example 2.1

The graph in the following Figure



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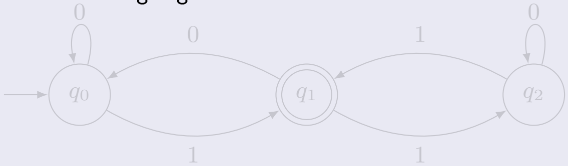

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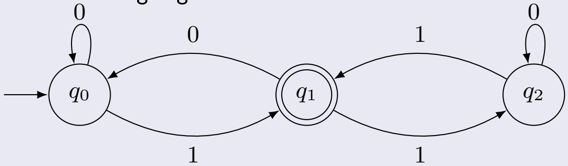

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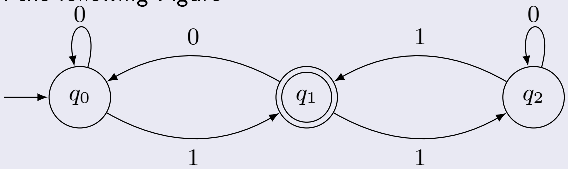

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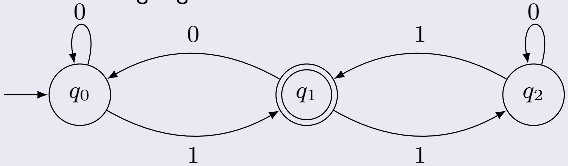

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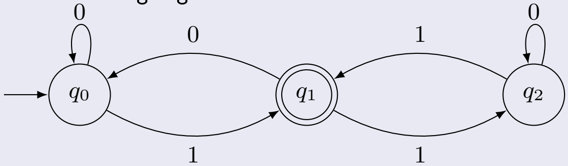

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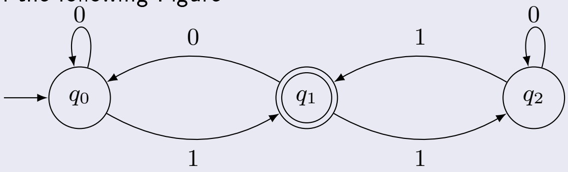

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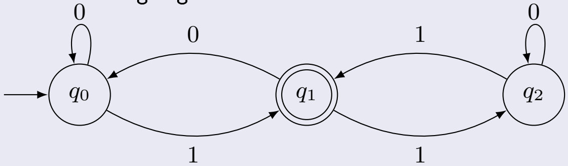

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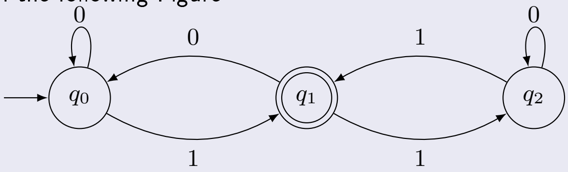

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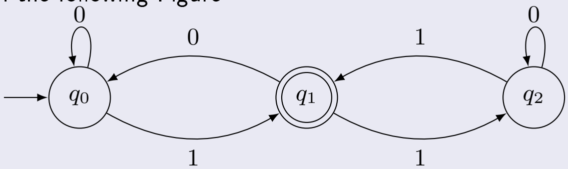

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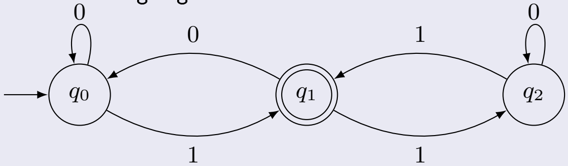

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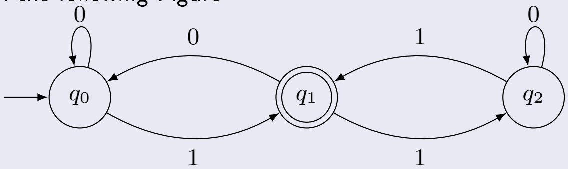

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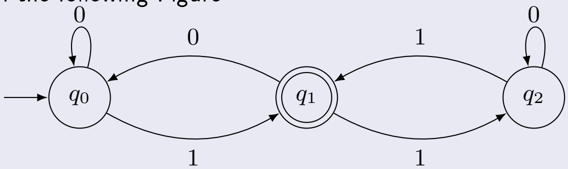

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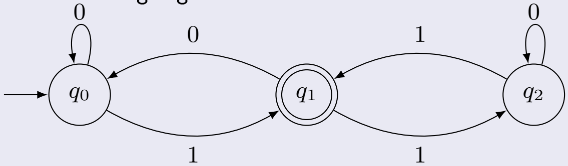

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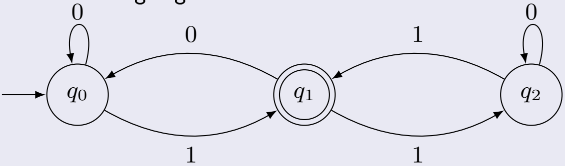

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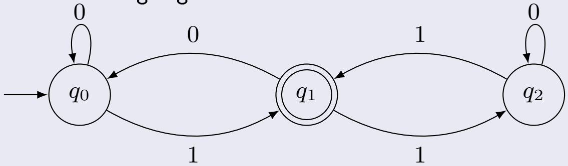

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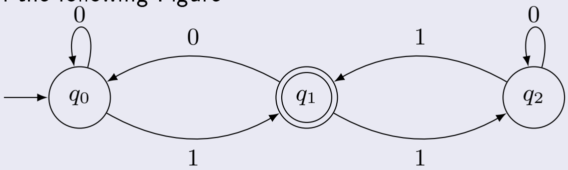

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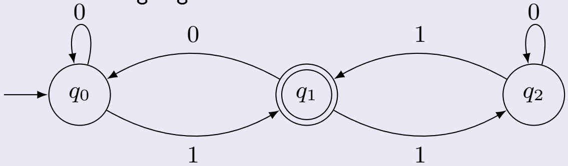

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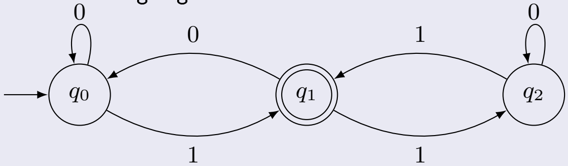

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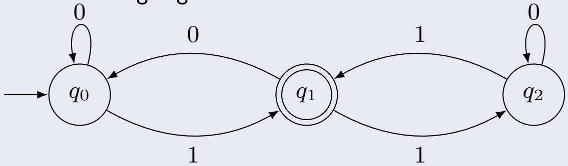

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It is convenient to introduce the extended transition function $\delta^*\colon Q \times \Sigma^* \to Q$. The second argument of δ^* is a string, rather than a single symbol, and its value gives the state the automaton will be in after reading that string. For example, if

$$\delta(q_0, a) = q_1$$

and

$$\delta(q_1, b) = q_2,$$

then

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Having made a precise definition of an accepter, we are now ready to define formally what we mean by an associated language. The association is obvious: The language is the set of all the strings accepted by the automaton.

Definition 2.2

The language accepted by a DFA $M = (Q, \Sigma, \delta, q_0, F)$ is the set of all strings on Σ accepted by M. In formal notation,

$$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}.$$

Note that we require that δ, and consequently δ^*, be total functions. At each step, a unique move is defined, so that we are justified in calling such an automaton deterministic. A DFA will process every string in Σ^* and either accept it or not accept it. Nonacceptance means that the DFA stops in a nonfinal state, so that

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Consider the DFA in the following Figure.

In the drawing we allowed the use of two labels on a single edge. Such multiply labeled edges are shorthand for two or more distinct transitions: The transition is taken whenever the input symbol matches any of the edge labels.

The automaton in the Figure remains in its initial state q_0 until the first b is encountered. If this is also the last symbol of the input, then the string is accepted. If not, the DFA goes into state q_2, from which it can never escape. The state q_2 is a trap state. We see clearly from the graph that the automaton accepts all strings consisting of an arbitrary number of a's, followed by a single b. All other input strings are rejected. In set notation, the language accepted by the automaton is

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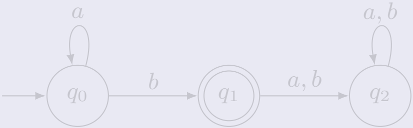

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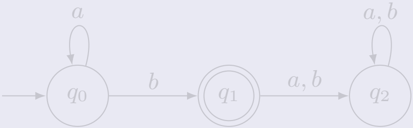

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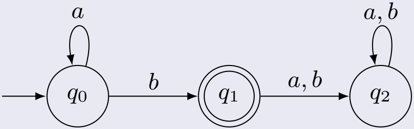

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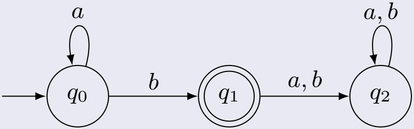

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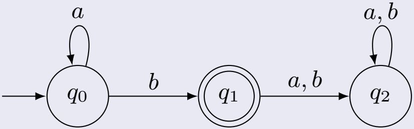

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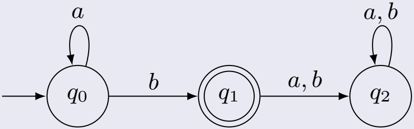

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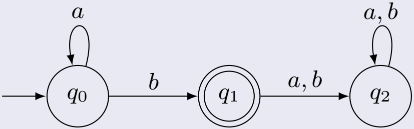

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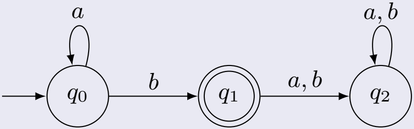

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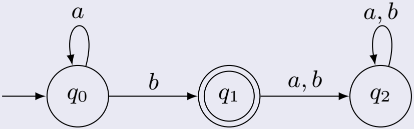

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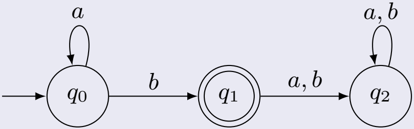

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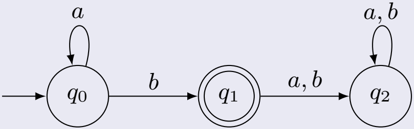

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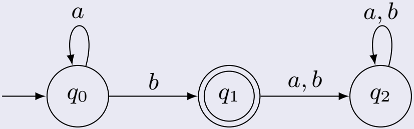

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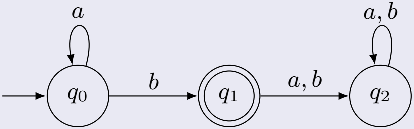

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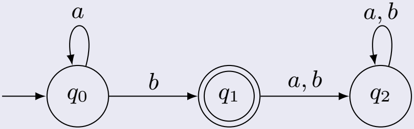

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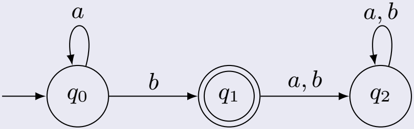

In the drawing we allowed the use of two labels on a single edge. Such multiply labeled edges are shorthand for two or more distinct transitions: The transition is taken whenever the input symbol matches any of the edge labels.

The automaton in the Figure remains in its initial state q_0 until the first b is encountered. If this is also the last symbol of the input, then the string is accepted. If not, the DFA goes into state q_2, from which it can never escape. The state q_2 is a trap state. We see clearly from the graph that the automaton accepts all strings consisting of an arbitrary number of a's, followed by a single b. All other input strings are rejected. In set notation, the language accepted by the automaton is

$$L = \{a^n b : n \geqslant 0\}.$$

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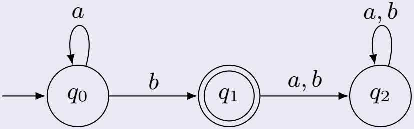

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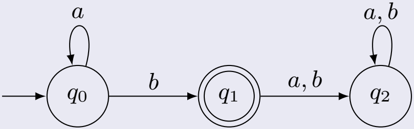

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These examples show how convenient transition graphs are for working with finite automata. While it is possible to base all arguments strictly on the properties of the transition function and its extension through (1) and (2),

$$\delta^*(q, \lambda) = q, \tag{1}$$
$$\delta^*(q, wa) = \delta(\delta^*(q, w), a), \tag{2}$$

the results are hard to follow. In our discussion, we use graphs, which are more intuitive, as far as possible. To do so, we must, of course, have some assurance that we are not misled by the representation and that arguments based on graphs are as valid as those that use the formal properties of δ. The following preliminary result gives us this assurance.

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Proof. This claim is fairly obvious from an examination of such simple cases as **Example 2.1**. It can be proved rigorously using an induction on the length of w. Assume that the claim is true for all strings v with $|v| \leqslant n$. Consider then any w of length $n+1$ and write it as

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While graphs are convenient for visualizing automata, other representations are also useful. For example, we can represent the function δ as a table. The table in the Figure

	a	b
q_0	q_0	q_1
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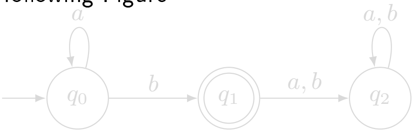

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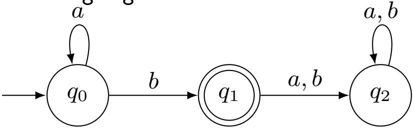

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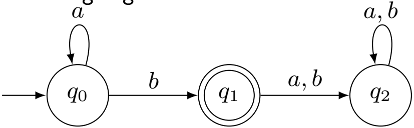

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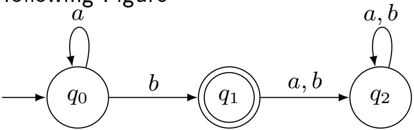

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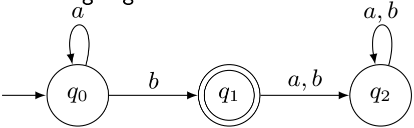

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It is apparent from this example that a DFA can easily be implemented as a computer program; for example, as a simple table-lookup or as a sequence of if statements. The best implementation or representation depends on the specific application. Transition graphs are very convenient for the kinds of arguments we want to make here, so we use them in most of our discussions.

In constructing automata for languages defined informally, we employ reasoning similar to that for programming in higher-level languages. But the programming of a DFA is tedious and sometimes conceptually complicated by the fact that such an automaton has few powerful features.

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Example 2.3

Find a deterministic finite accepter that recognizes the set of all strings on $\Sigma = \{a, b\}$ starting with the prefix ab. The only issue here is the first two symbols in the string; after they have been read, no further decisions are needed. Still, the automaton has to process the whole string before its decision is made. We can therefore solve the problem with an automaton that has four states; an initial state, two states for recognizing ab ending in a final trap state, and one nonfinal trap state. If the first symbol is the letter a and the second is the letter b, the automaton goes to the final trap state, where it will stay since the rest of the input does not matter. On the other hand, if the first symbol is not the letter a or the second one is not the letter b, the automaton enters the nonfinal trap state. The simple solution is shown in the following Figure.

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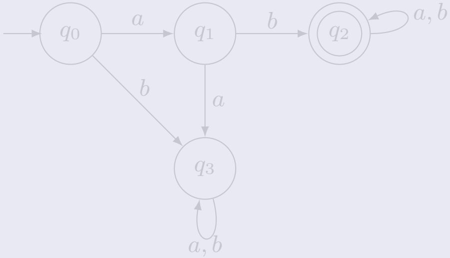

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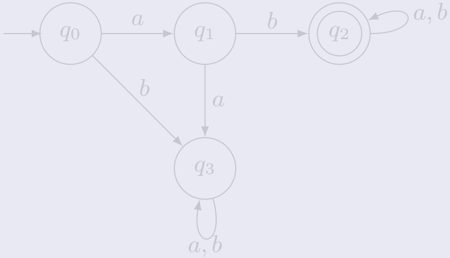

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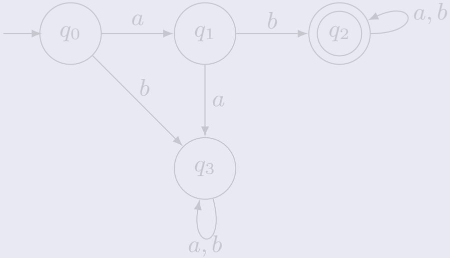

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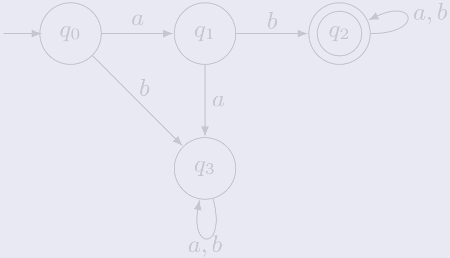

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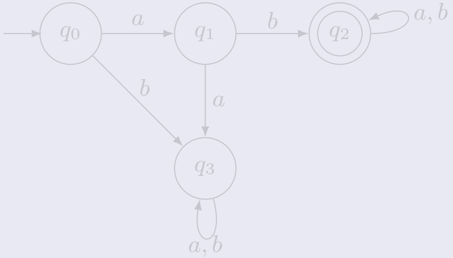

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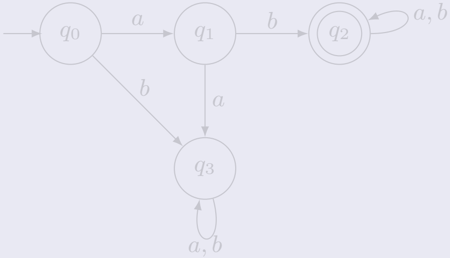

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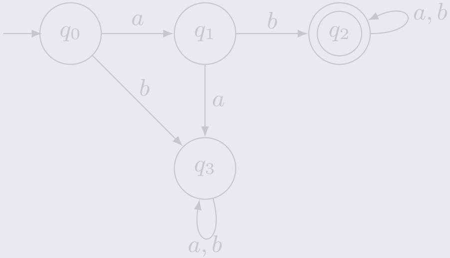

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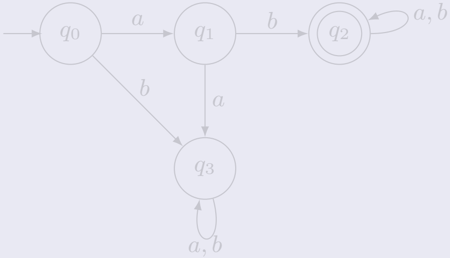

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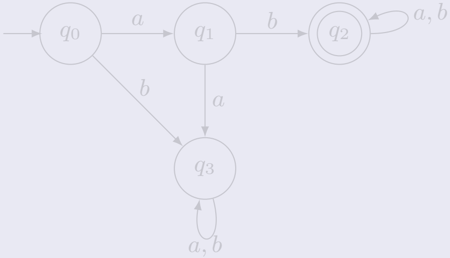

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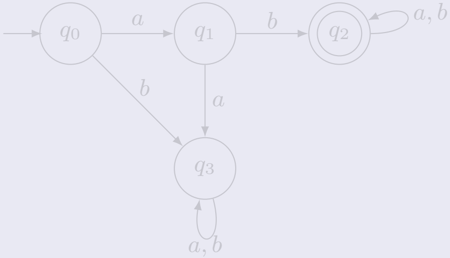

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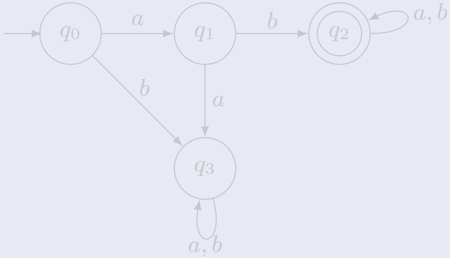

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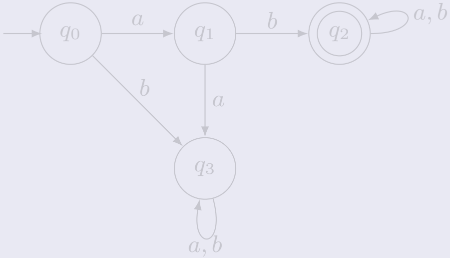

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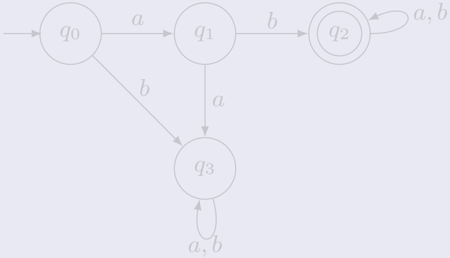

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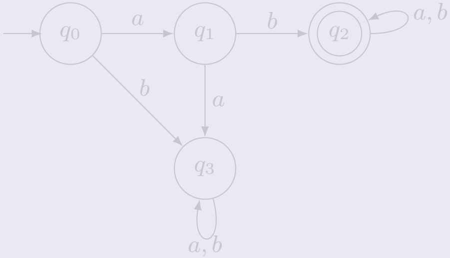

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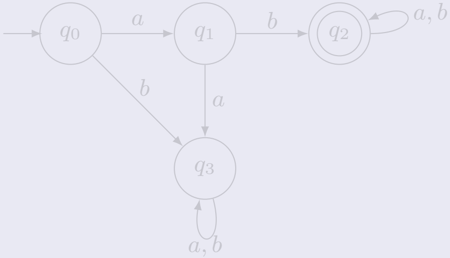

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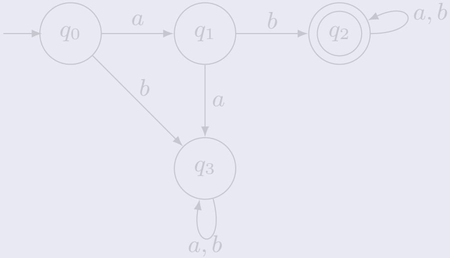

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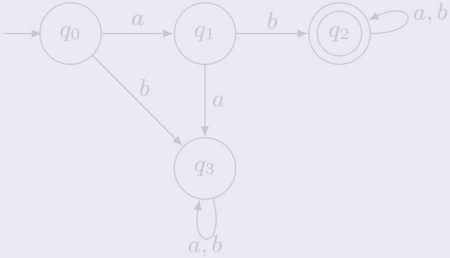

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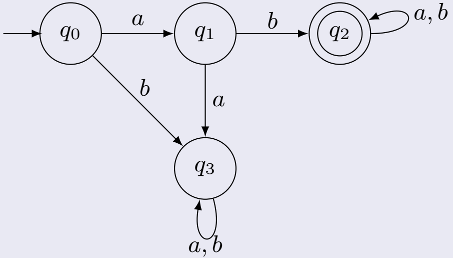

Example 2.4

Find a DFA that accepts all the strings on $\{0, 1\}$, except those containing the substring 001.

In deciding whether the substring 001 has occurred, we need to know not only the current input symbol, but we also need to remember whether or not it has been preceded by one or two 0's. We can keep track of this by putting the automaton into specific states and labeling them accordingly. Like variable names in a programming language, state names are arbitrary and can be chosen for mnemonic reasons. For example, the state in which two 0's were the immediately preceding symbols can be labeled simply 00.

If the string starts with 001, then it must be rejected. This implies that there must be a path labeled 001 from the initial state to a nonfinal state. For convenience, this nonfinal state is labeled 001. This state must be a trap state, because later symbols do not matter. All other states are accepting states.

This gives us the basic structure of the solution, but we still must add provisions for the substring 001 occurring in the middle of the input. We must define Q and δ so that whatever we need to make the correct decision is remembered by the automaton. In this case, when a symbol is read, we need to know some part of the string to the left, for example, whether or not the two previous symbols were 00.

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Example 2.4 (continuation)

If we label the states with the relevant symbols, it is very easy to see what the transitions must be. For example,

$$\delta(00, 0) = 00$$

because this situation arises only if there are three consecutive 0's. We are only interested in the last two, a fact we remember by keeping the DFA in the state 00. A complete solution is shown in the following Figure.

We see from this example how useful mnemonic labels on the states are for keeping track of things. Trace a few strings, such as 100100 and 1010100, to see that the solution is indeed correct.

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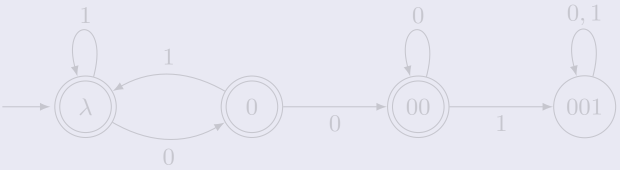

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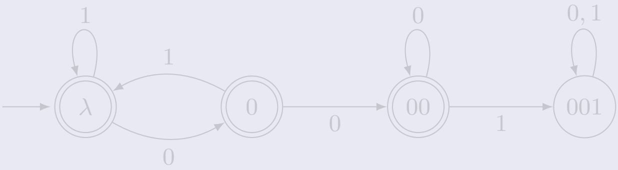

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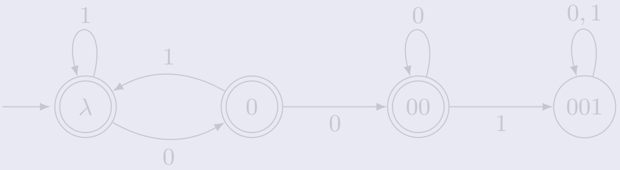

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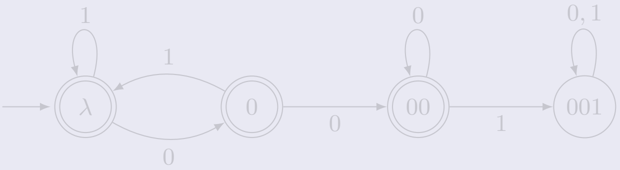

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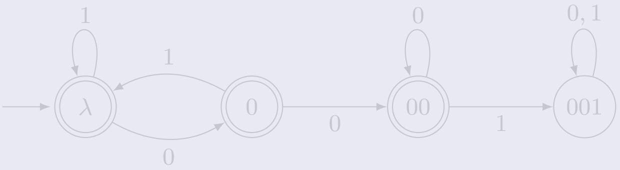

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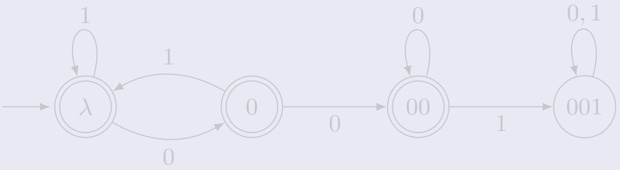

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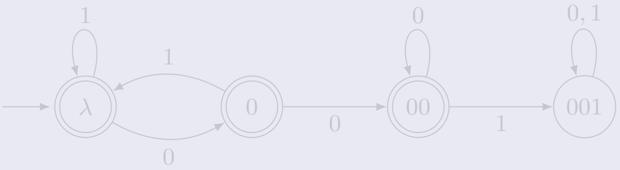

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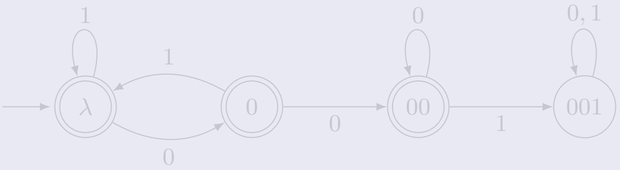

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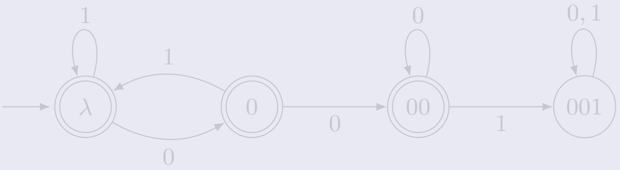

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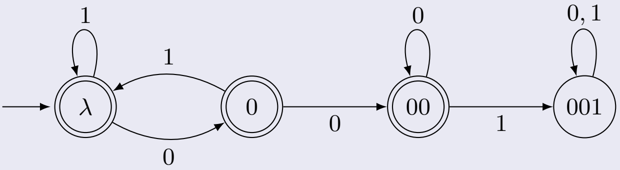

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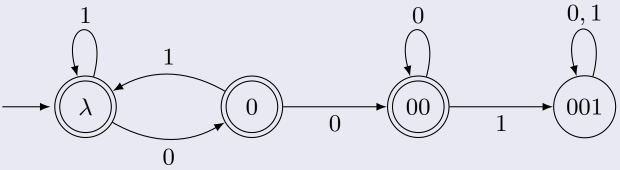

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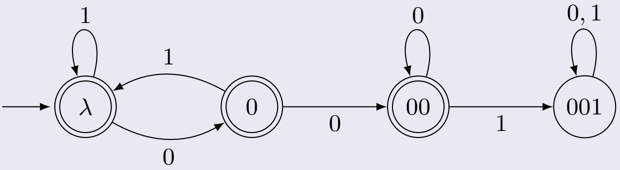

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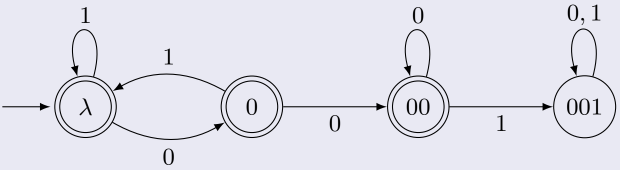

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Definition 2.2

A language L is called *regular* if and only if there exists some deterministic finite accepter M such that

$$L = L(M).$$

Example 2.5

Show that the language

$$L = \{awa : w \in \{a, b\}^*\}$$

is regular.

To show that this or any other language is regular, all we have to do is find a DFA for it. The construction of a DFA for this language is similar to Example 2.3, but a little more complicated. What this DFA must do is check whether a string begins and ends with the letter a; what is between is immaterial. The solution is complicated by the fact that there is no explicit way of testing the end of the string.

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Again, trace a few examples to see why this works. After one or two tests, it will be obvious that the DFA accepts a string if and only if it begins and ends with the letter a. Since we have constructed a DFA for the language, we can claim that, by definition, the language is regular.

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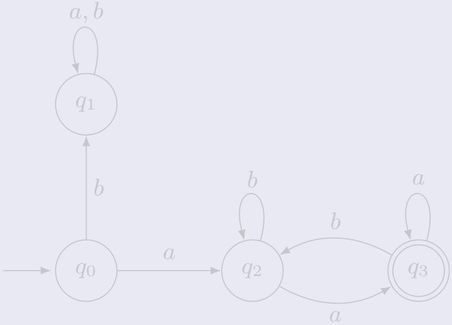

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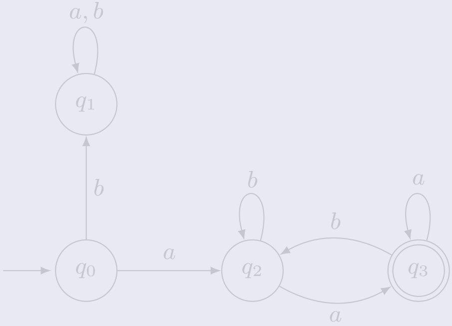

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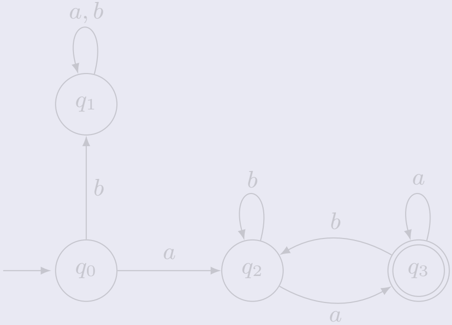

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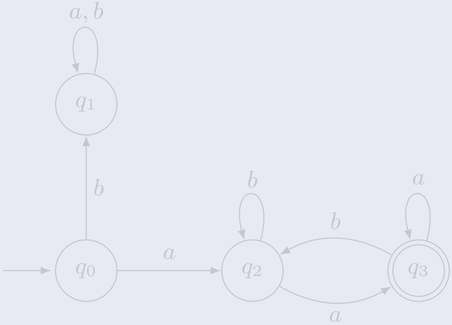

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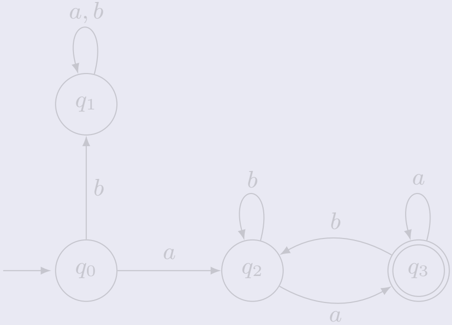

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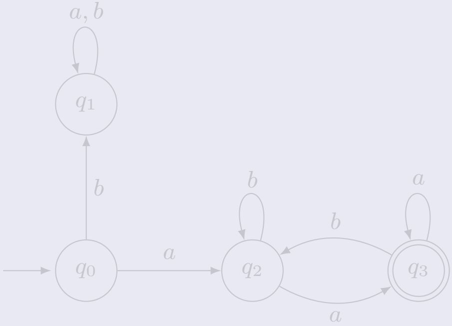

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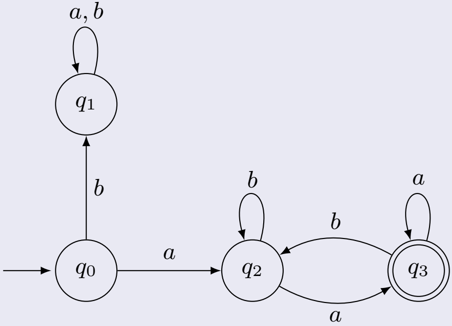

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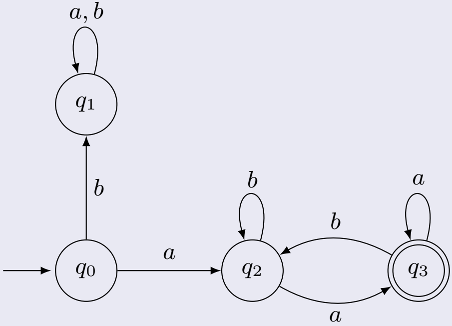

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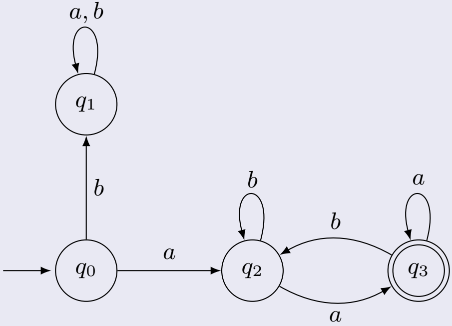

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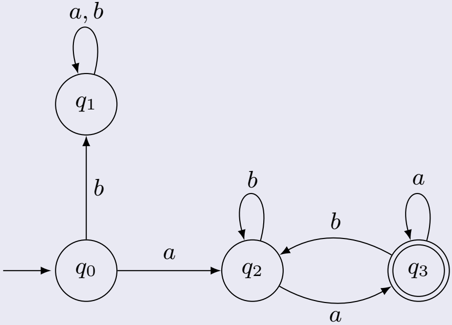

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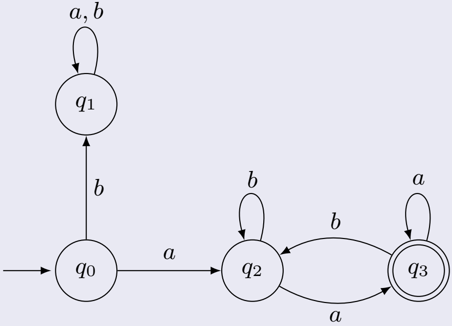

Again, trace a few examples to see why this works. After one or two tests, it will be obvious that the DFA accepts a string if and only if it begins and ends with the letter a. Since we have constructed a DFA for the language, we can claim that, by definition, the language is regular.

Example 2.6

Let L be the language in Example 2.5:

$$L = \{awa : w \in \{a, b\}^*\}.$$

Show that L^2 is regular. Again we show that the language is regular by constructing a DFA for it. We can write an explicit expression for L^2, namely,

$$L^2 = \{aw_1aaw_2a : w_1, w_2 \in \{a, b\}^*\}.$$

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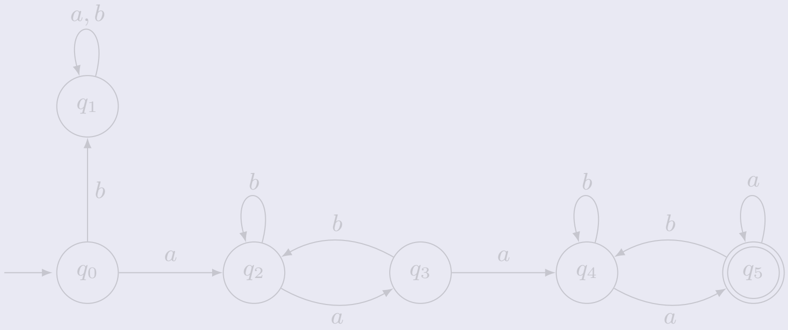

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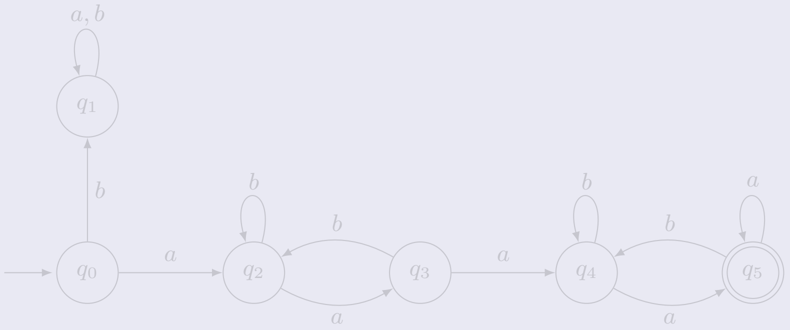

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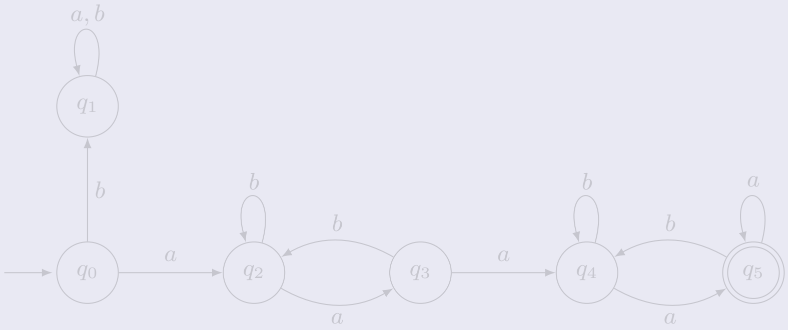

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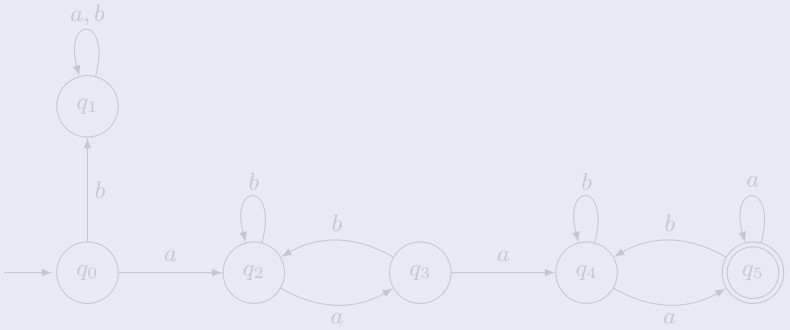

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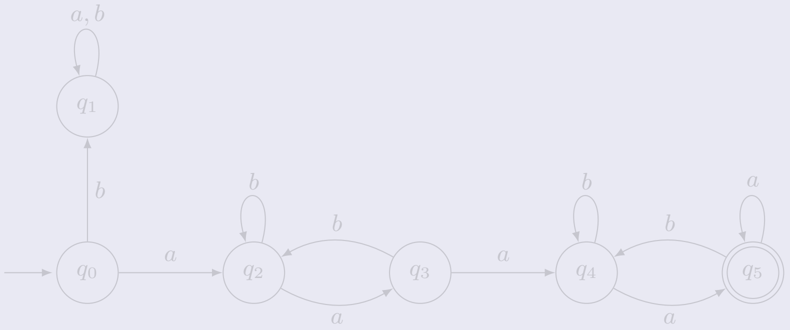

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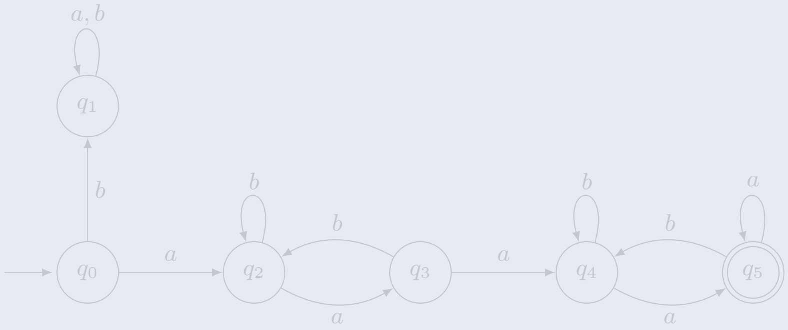

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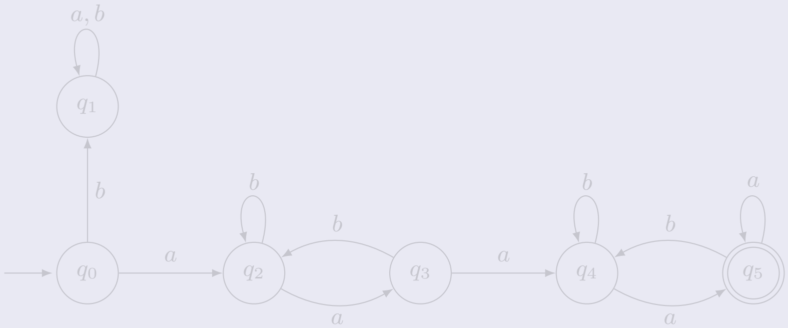

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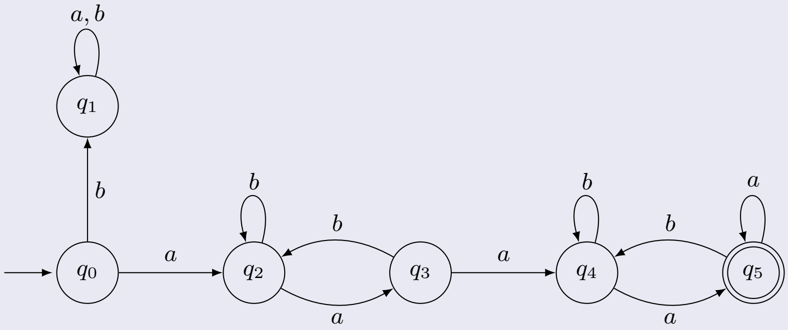

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Therefore, we need a DFA that recognizes two consecutive strings of essentially the same form (but not necessarily identical in value). The diagram in the Figure of Example 2.5 can be used as a starting point, but the vertex q_3 has to be modified.

This state can no longer be final because, at this point, we must start to look for a second substring of the form awa. To recognize the second substring, we replicate the states of the first part (with new names), with q_3 as the beginning of the second part. Since the complete string can be broken into its constituent parts wherever aa occurs, we let the first occurrence of two consecutive a's be the trigger that gets the automaton into its second part.

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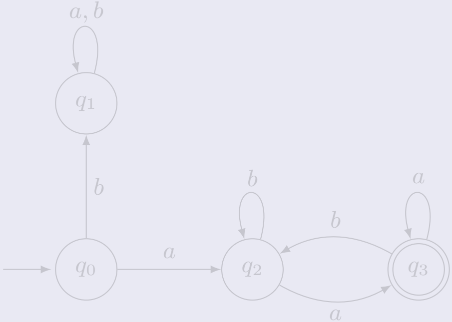

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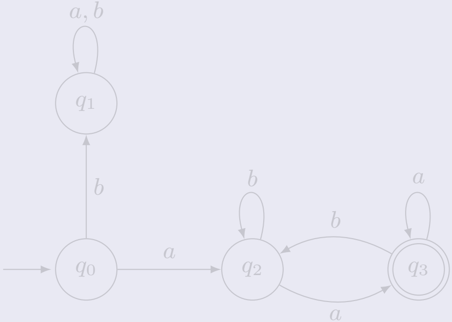

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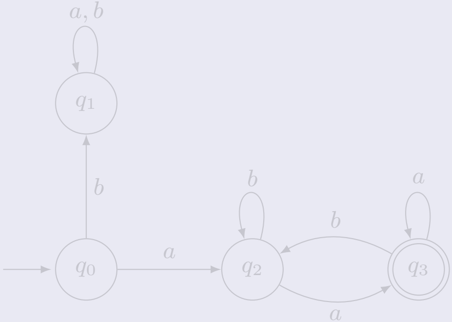

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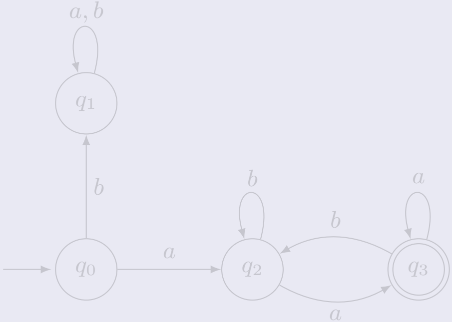

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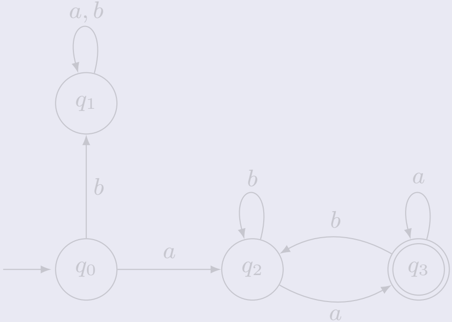

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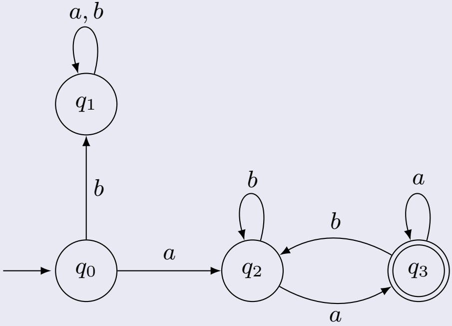

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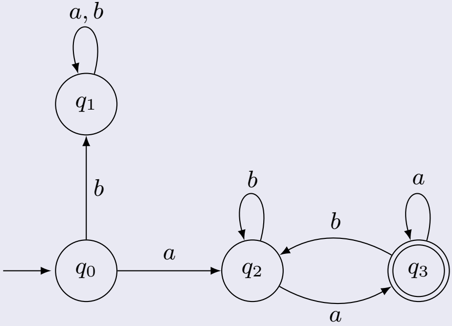

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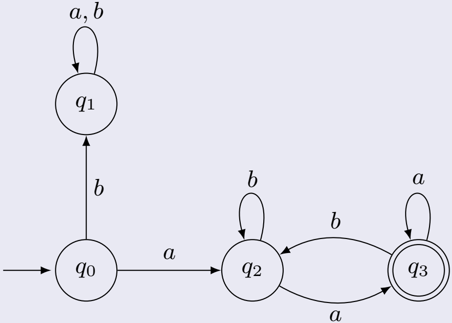

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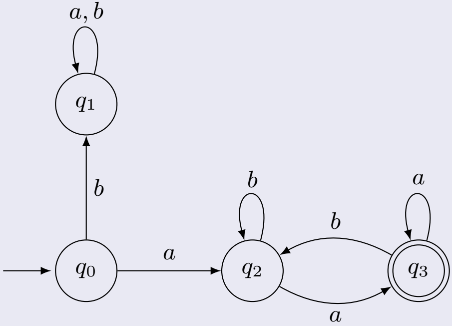

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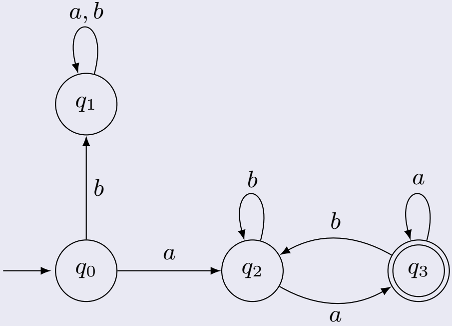

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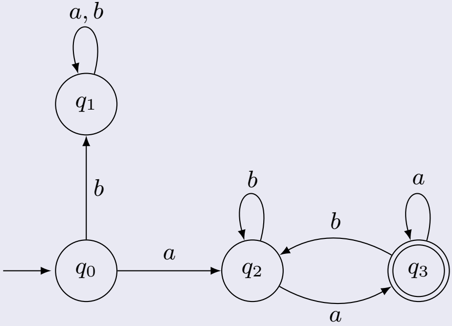

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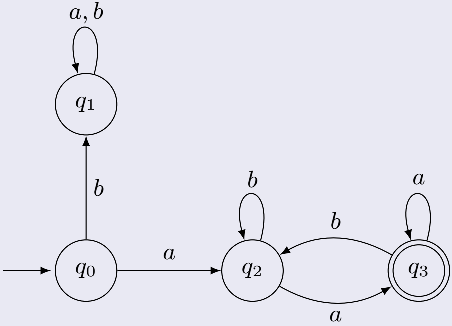

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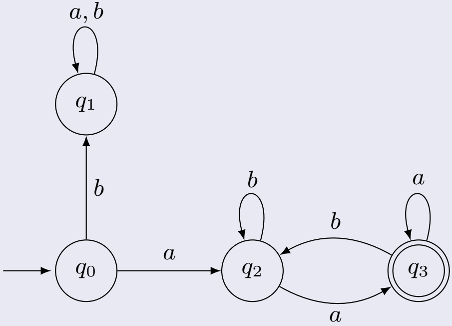

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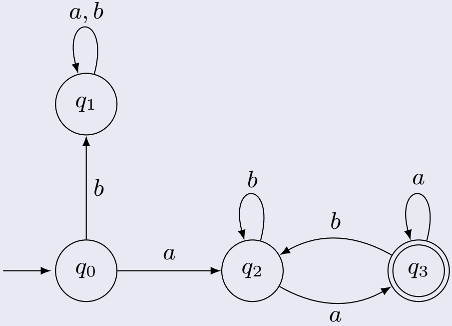

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We can do this by making $\delta(q_3, a) = q_4$. The complete solution is in the Figure below.

This DFA accepts L^2, which is therefore regular.

The last example suggests the conjecture that if a language L is regular, so are $L^2, L^3, \ldots$. We shall see later that this is indeed correct.

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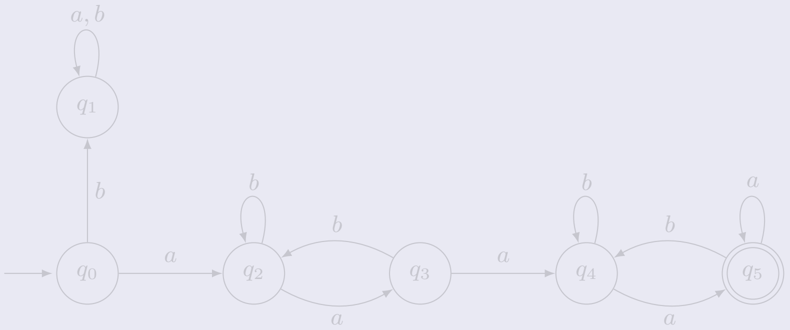

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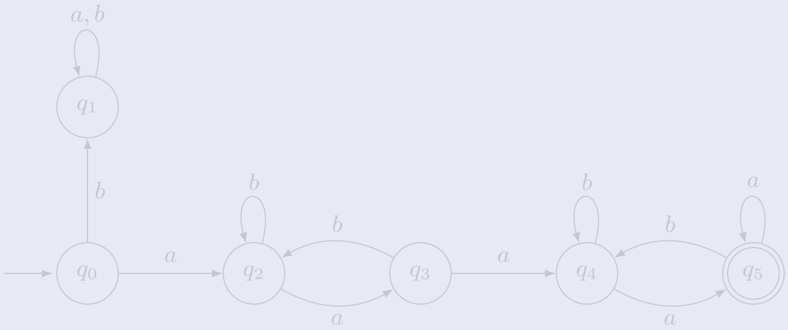

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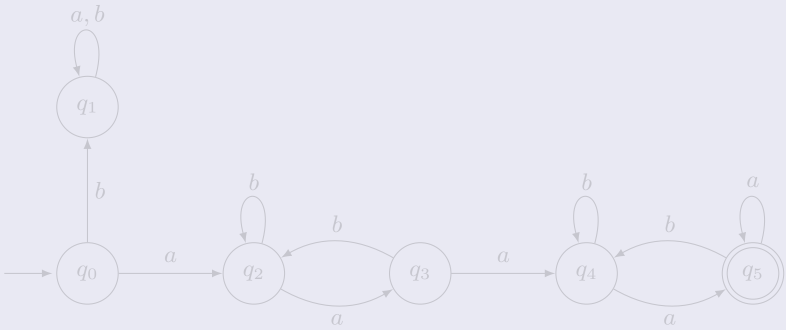

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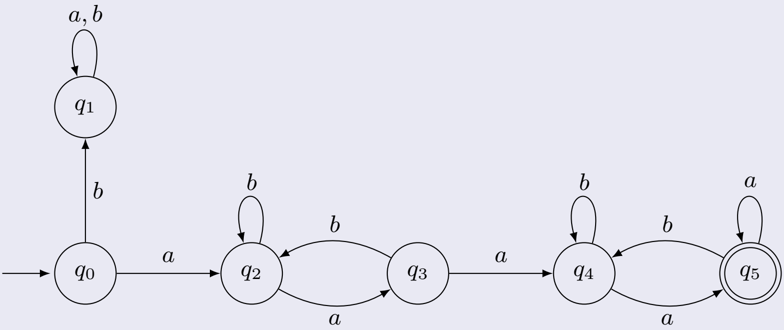

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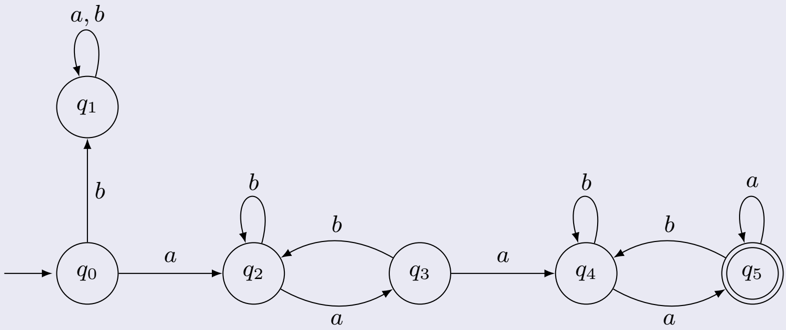

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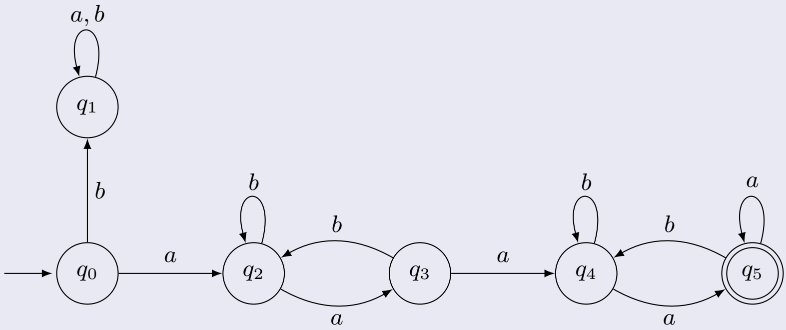

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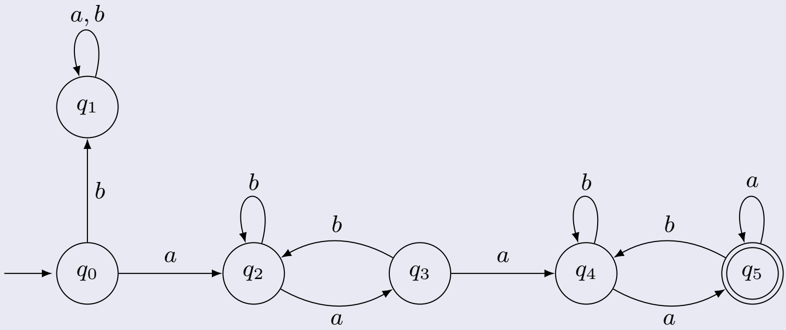

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