Formal Languages, Automata and Codes

Oleg Gutik

Lecture 3

1.3 SOME APPLICATIONS

Although we stress the abstract and mathematical nature of formal languages and automata, it turns out that these concepts have widespread applications in computer science and are, in fact, a common theme that connects many specialty areas. In this lecture, we present some simple examples to give the student some assurance that what we study here is not just a collection of abstractions, but is something that helps us understand many important, real problems.

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The grammars that describe a typical language like Pascal or C are very extensive. For an example, let us take a smaller language that is part of a larger one.

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Example 1.15

The rules for variable identifiers in C are

1. An identifier is a sequence of letters, digits, and underscores.
2. An identifier must start with a letter or an underscore.
3. Identifiers allow upper- and lowercase letters.

Formally, these rules can be described by a grammar.

$$\langle \text{id} \rangle \to \langle \text{letter} \rangle \langle \text{rest} \rangle \mid \langle \text{undrscr} \rangle \langle \text{rest} \rangle$$

$$\langle \text{rest} \rangle \to \langle \text{letter} \rangle \langle \text{rest} \rangle \mid \langle \text{digit} \rangle \langle \text{rest} \rangle \mid \langle \text{undrscr} \rangle \langle \text{rest} \rangle \mid \lambda$$

$$\langle \text{letter} \rangle \to a \mid b \mid \ldots \mid z \mid A \mid B \mid \ldots \mid Z$$

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In this grammar, the variables are $\langle \text{id} \rangle$, $\langle \text{letter} \rangle$, $\langle \text{digit} \rangle$, $\langle \text{undrscr} \rangle$, and $\langle \text{rest} \rangle$. The letters, digits, and the underscore are terminals. A derivation of $a0$ is

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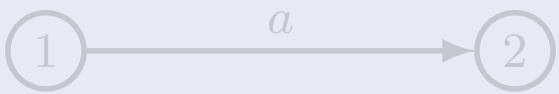

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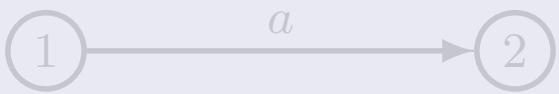

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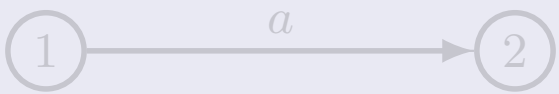

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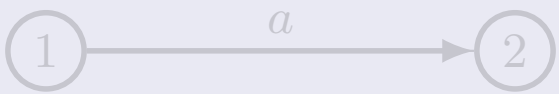

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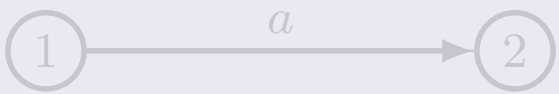

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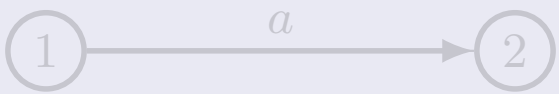

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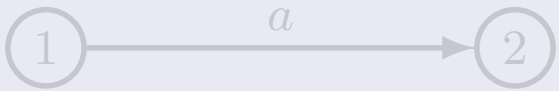

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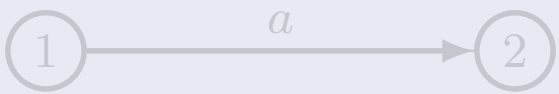

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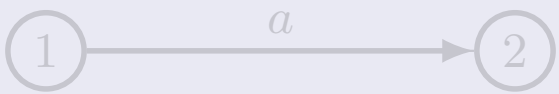

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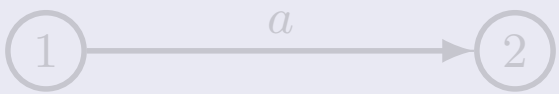

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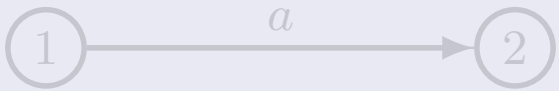

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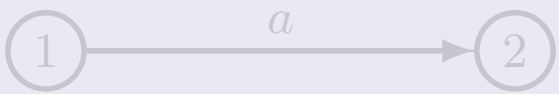

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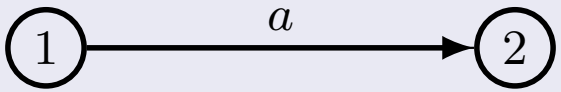

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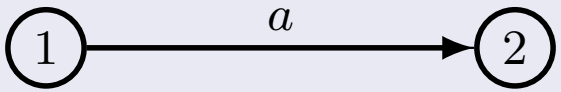

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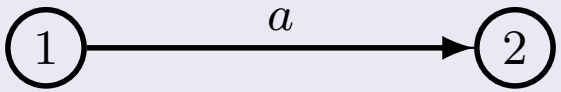

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The following Figure is an automaton that accepts all legal C identifiers.

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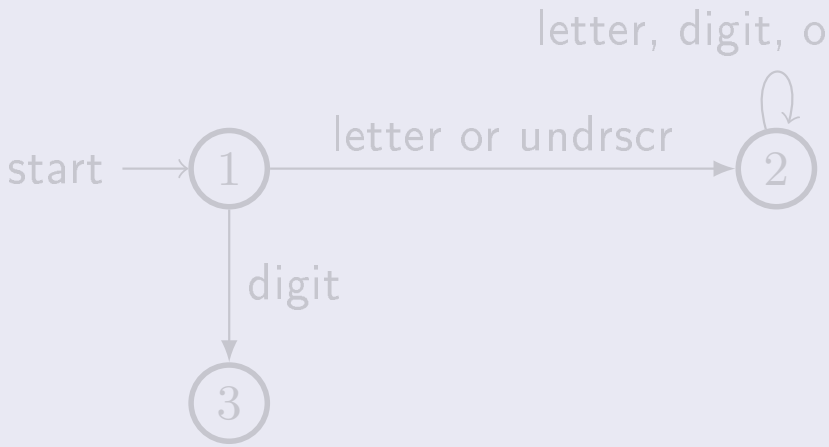

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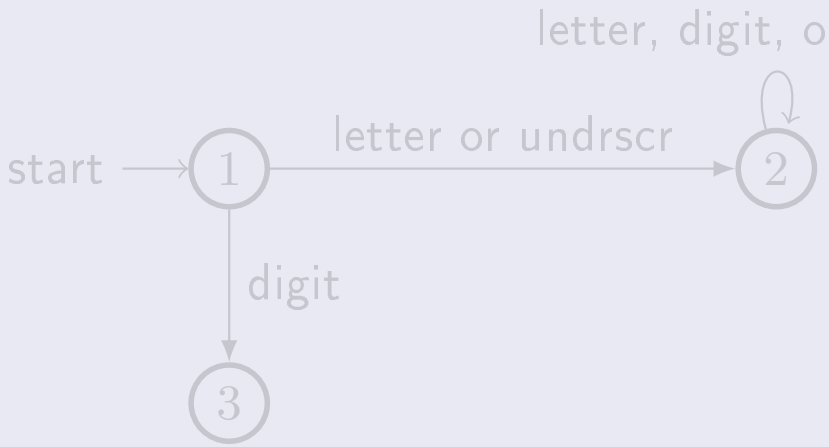

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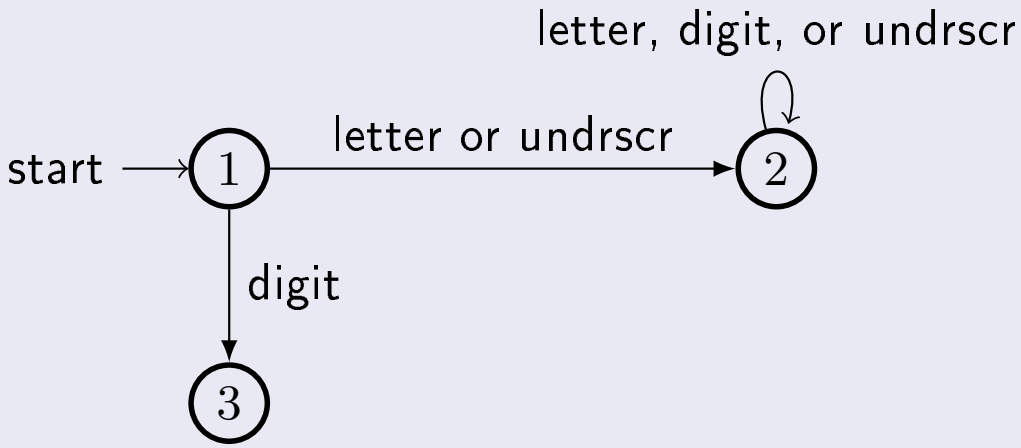

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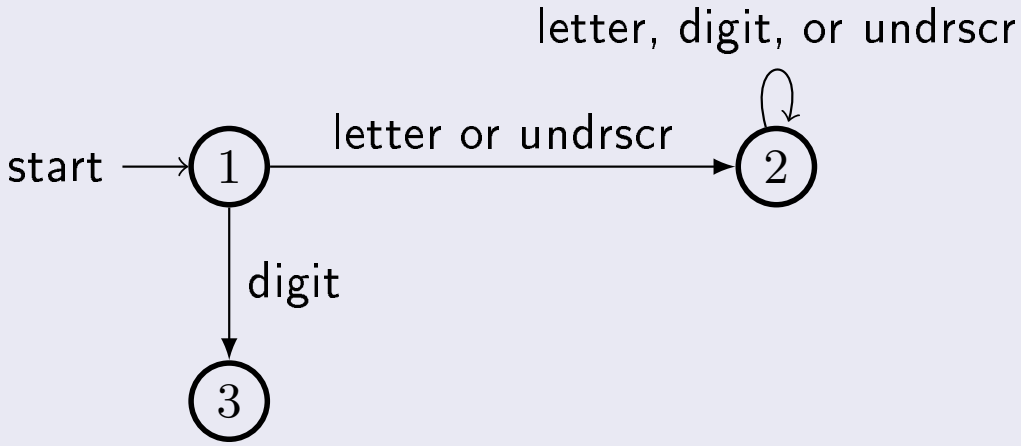

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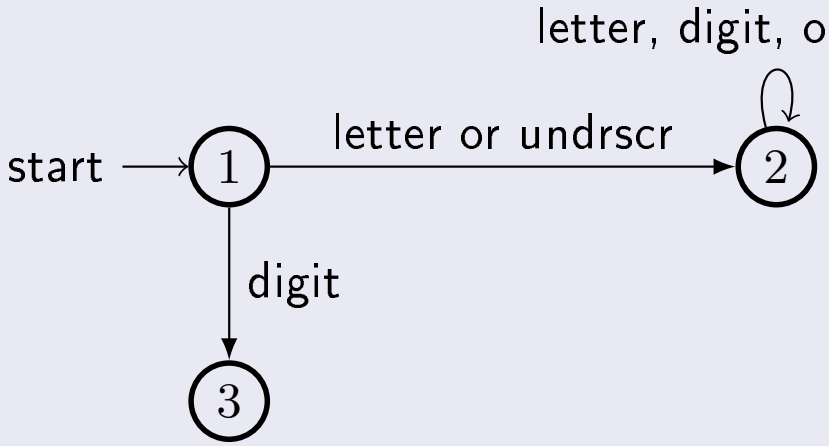

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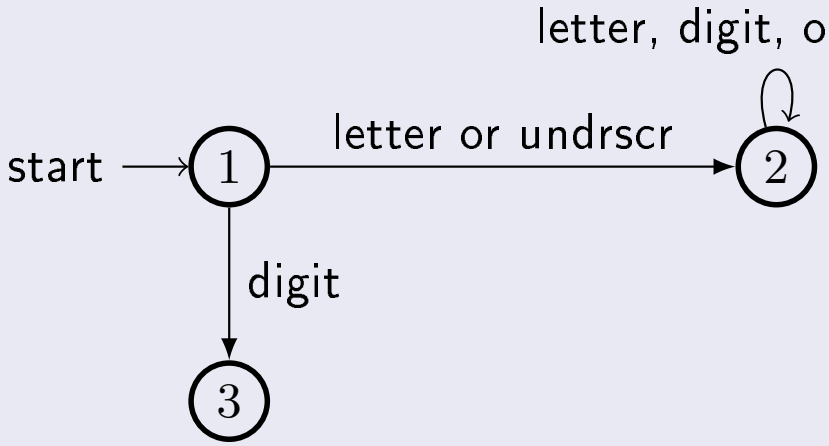

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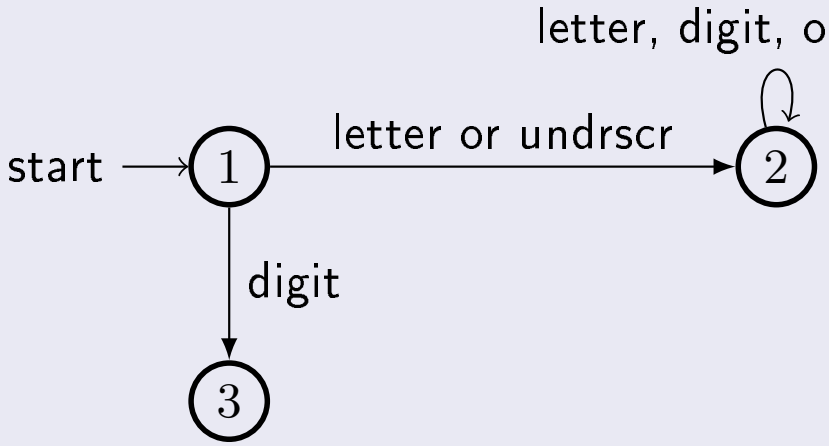

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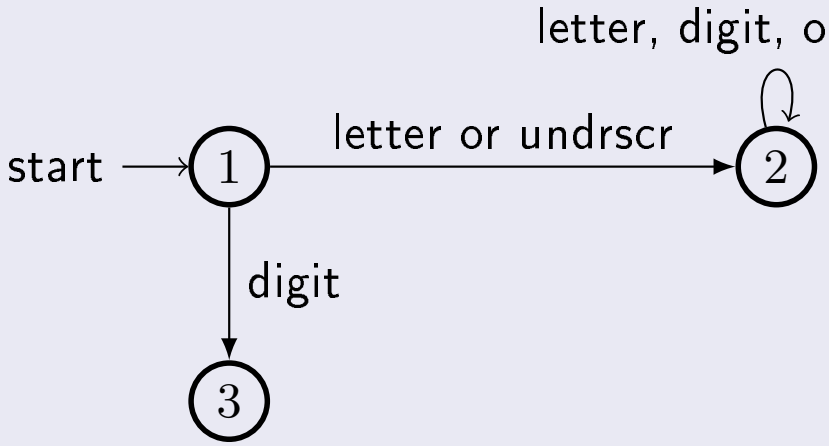

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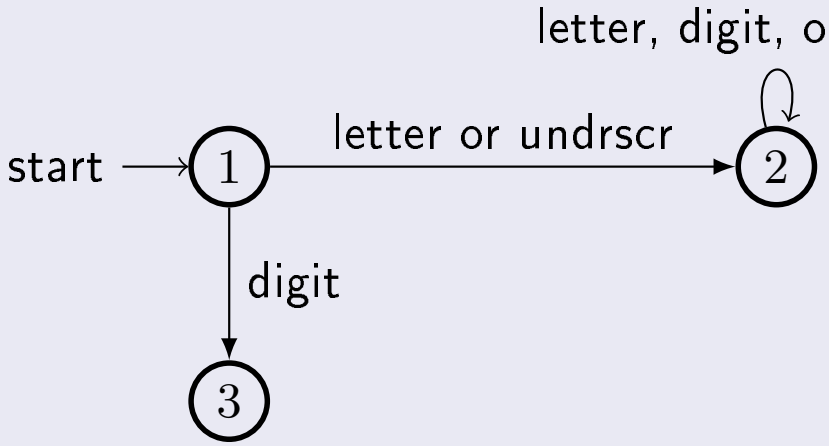

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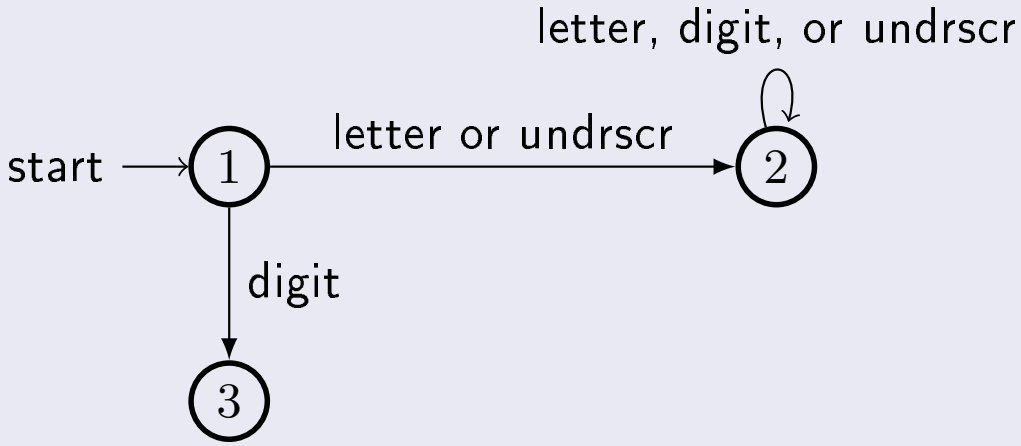

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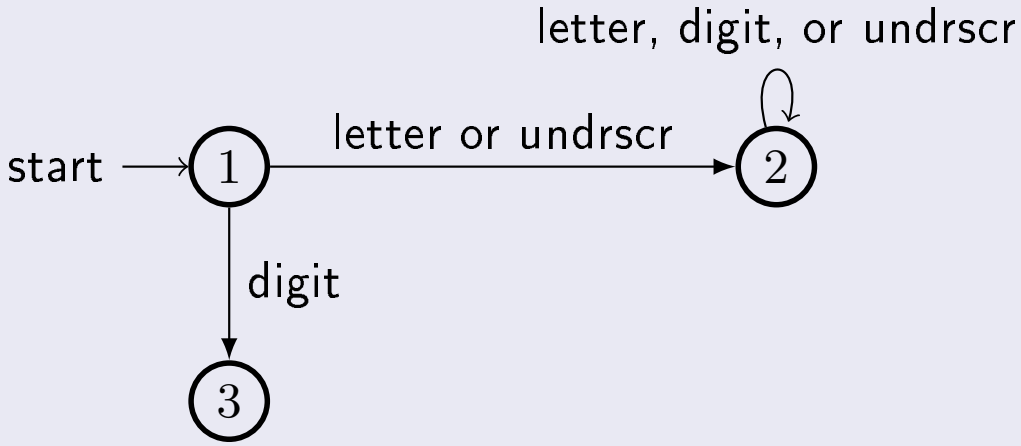

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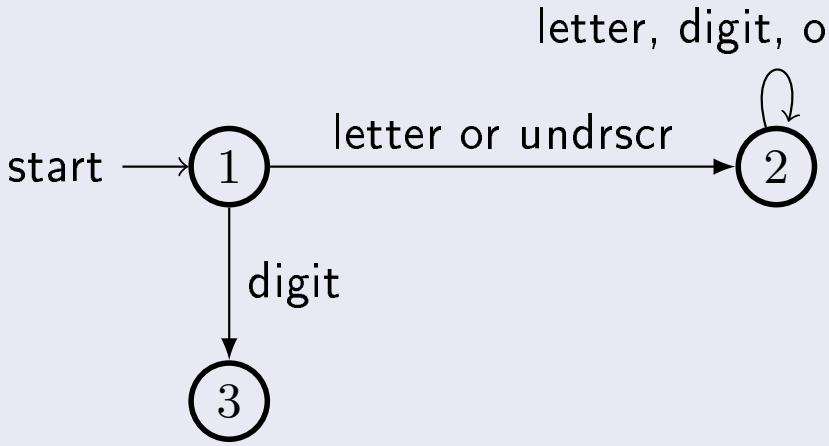

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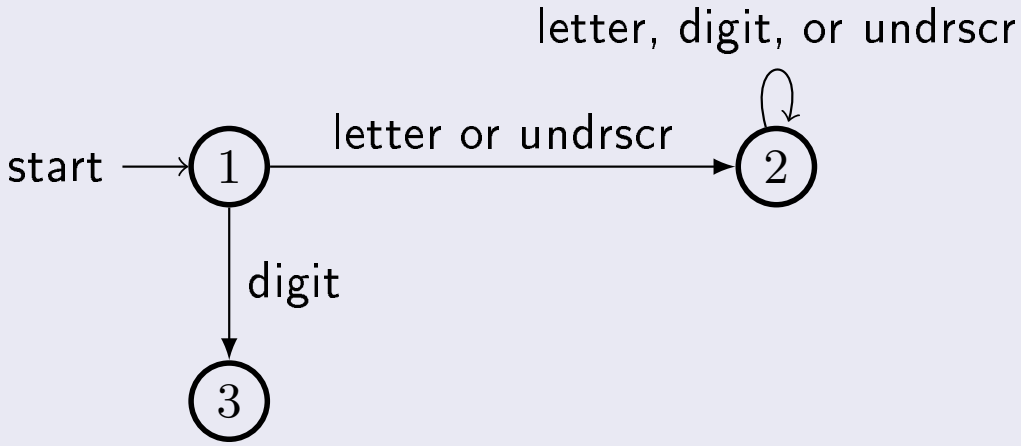

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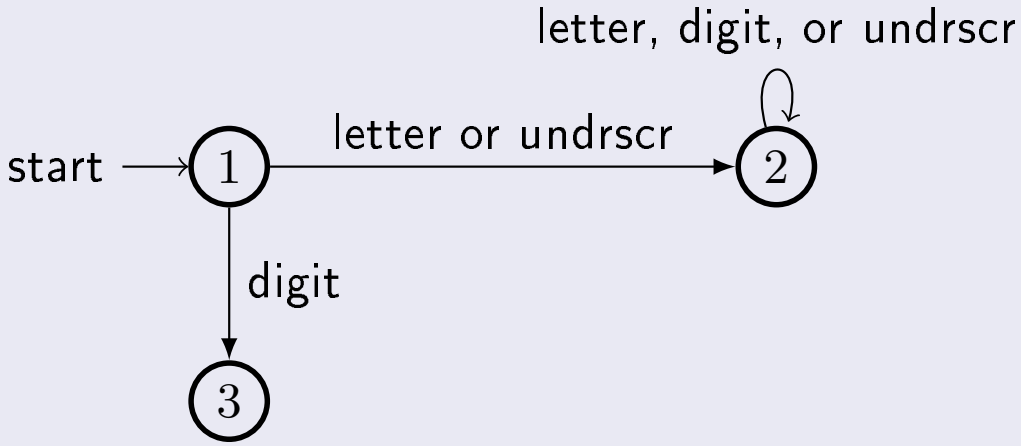

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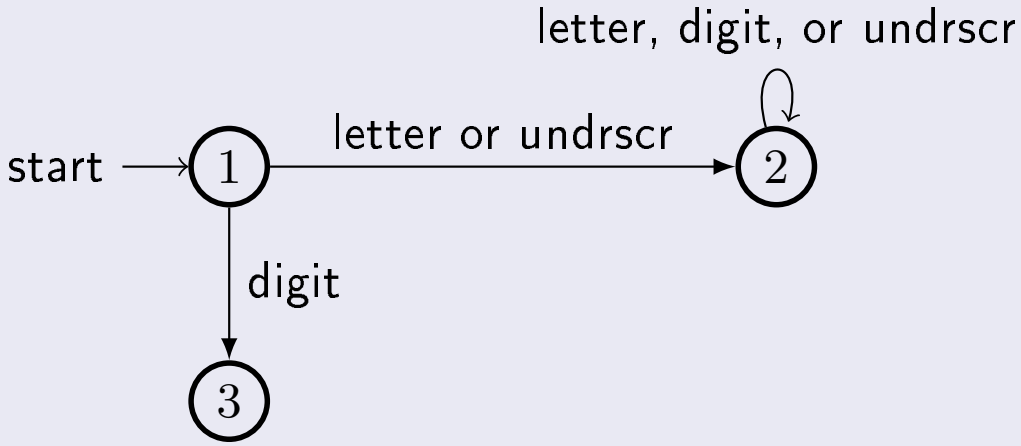

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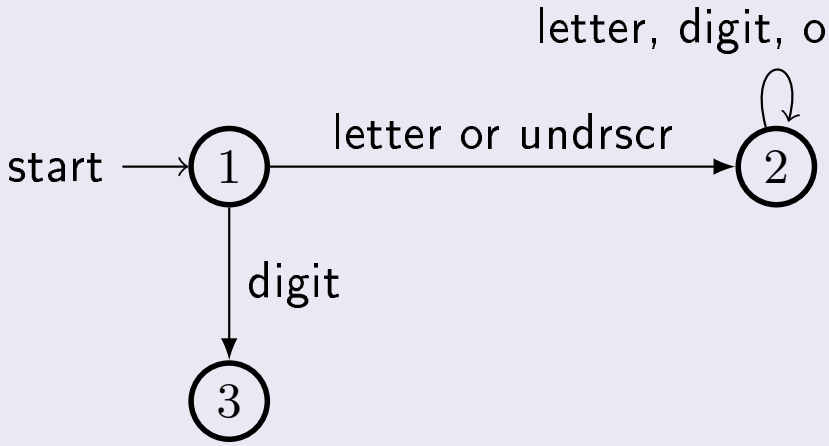

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The following example previews transducers.

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A binary adder is an integral part of any general-purpose computer. Such an adder takes two bit strings representing numbers and produces their sum as output. For simplicity, let us assume that we are dealing only with positive integers and that we use a representation in which

$$x = a_0 a_1 \cdots a_n$$

stands for the integer

$$v(x) = \sum_{i=0}^{n} a_i 2^i.$$

This is the usual binary representation in reverse.

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The following example previews transducers.

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A binary adder is an integral part of any general-purpose computer. Such an adder takes two bit strings representing numbers and produces their sum as output. For simplicity, let us assume that we are dealing only with positive integers and that we use a representation in which

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Example 1.17 (continuation)

A serial adder processes two such numbers $x = a_0 a_1 \cdots a_n$, and $y = b_0 b_1 \cdots b_n$, bit by bit, starting at the left end. Each bit addition creates a digit for the sum as well as a carry digit for the next higher position. A binary addition table (see Figure below) summarizes the process.

a_i \ b_i	0	1
0	0 No carry	1 No carry
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A block diagram of the kind we saw when we first studied computers is given in the following Figure.

It tells us that an adder is a box that accepts two bits and produces their sum bit and a possible carry. It describes what an adder does, but explains little about its internal workings. An automaton (now a transducer) can make this much more explicit.

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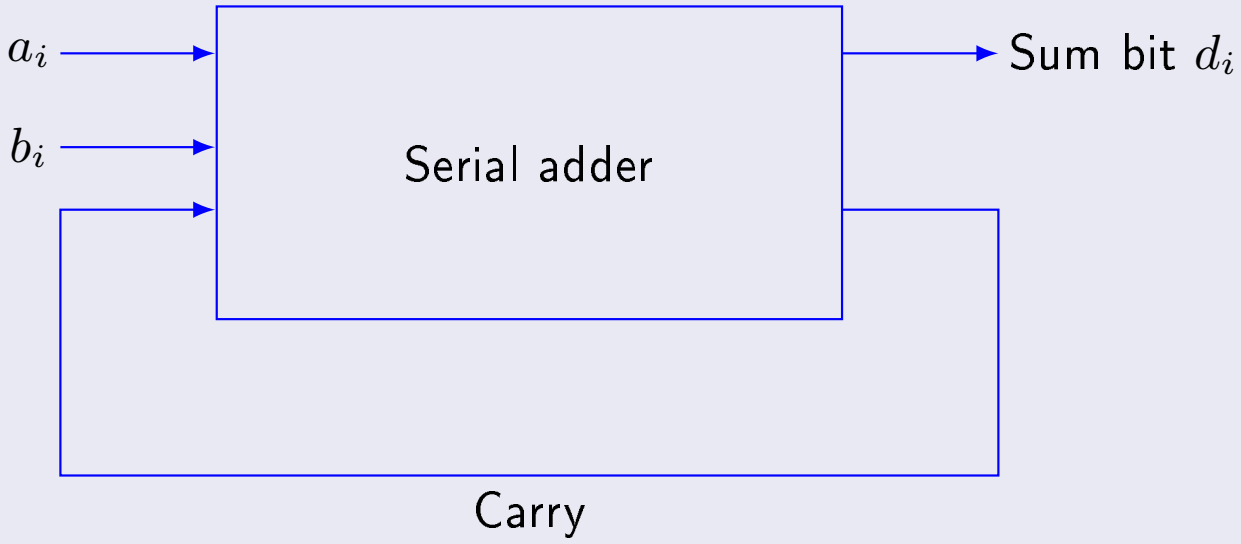

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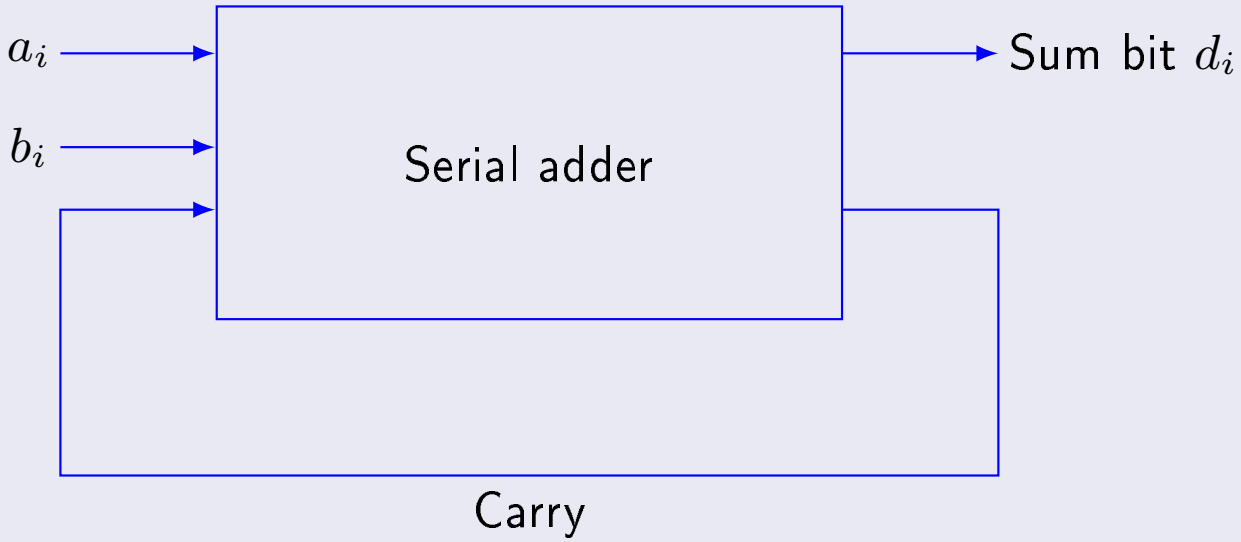

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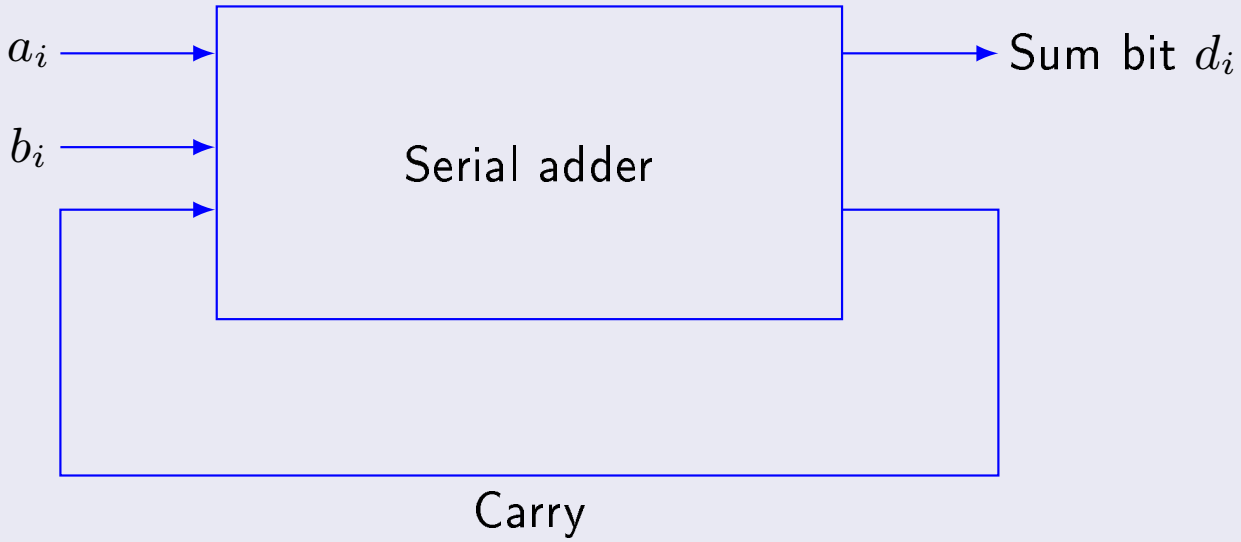

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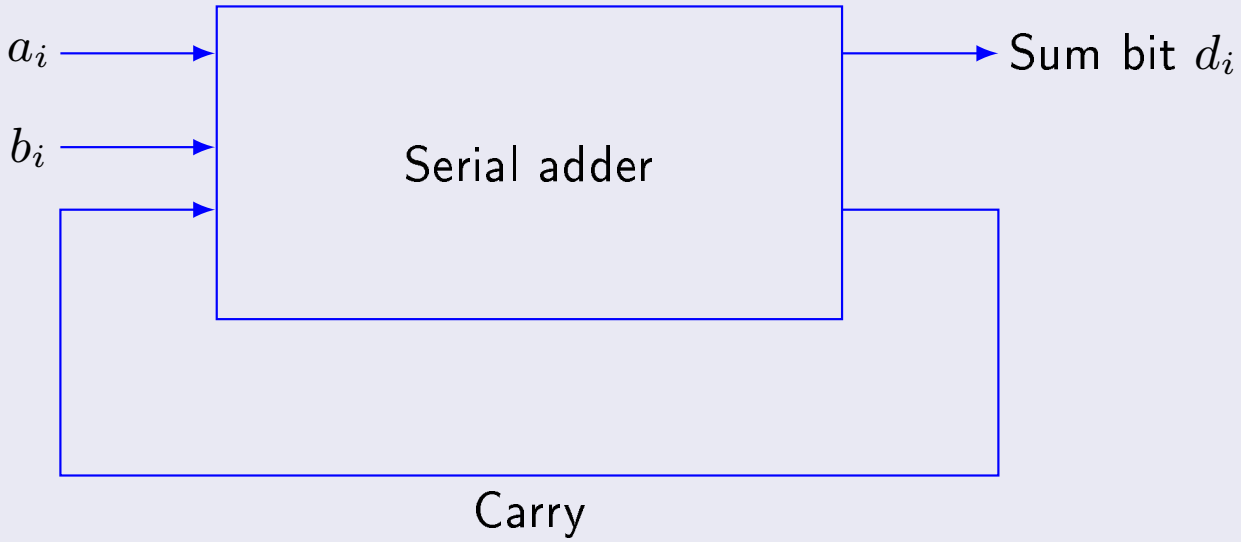

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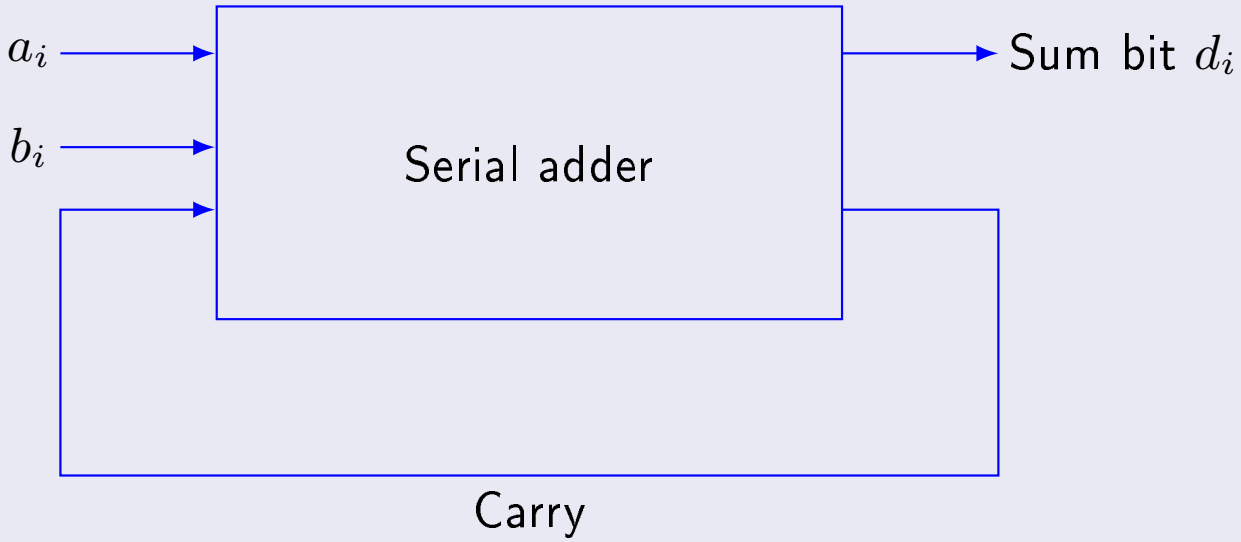

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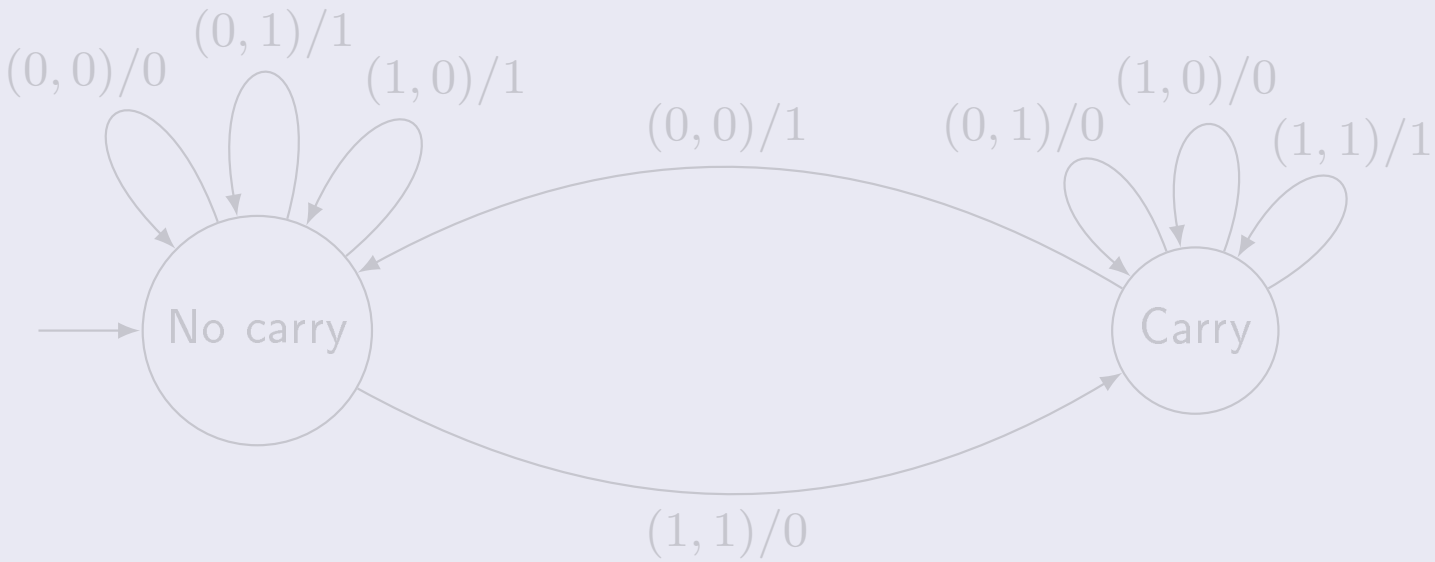

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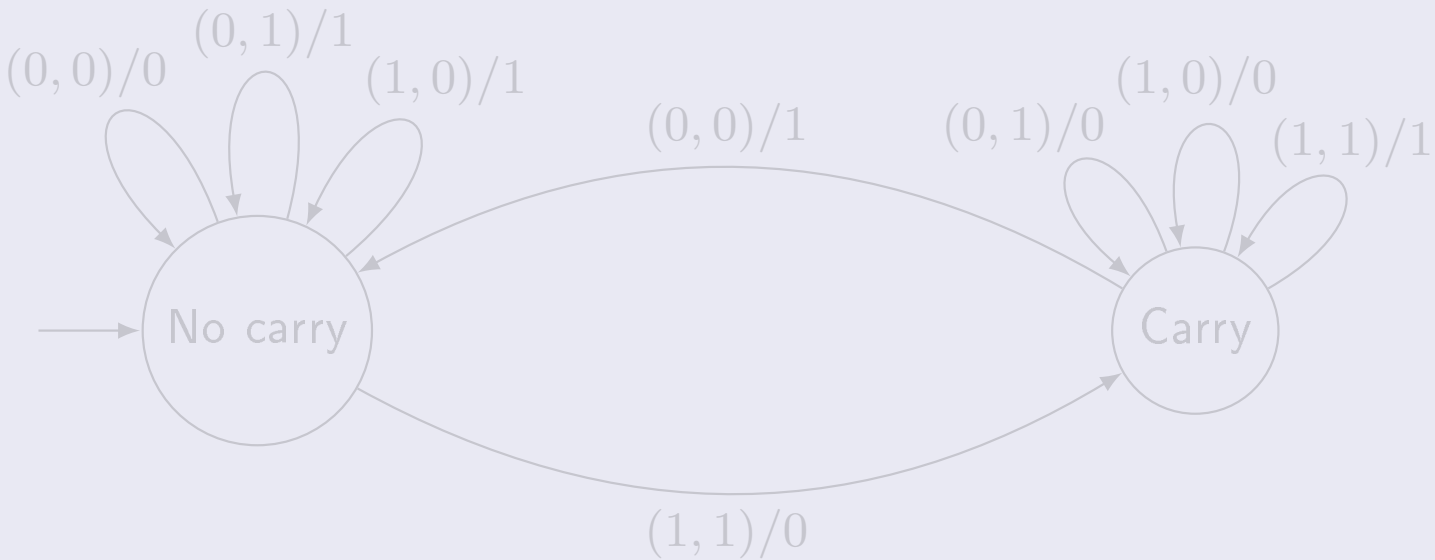

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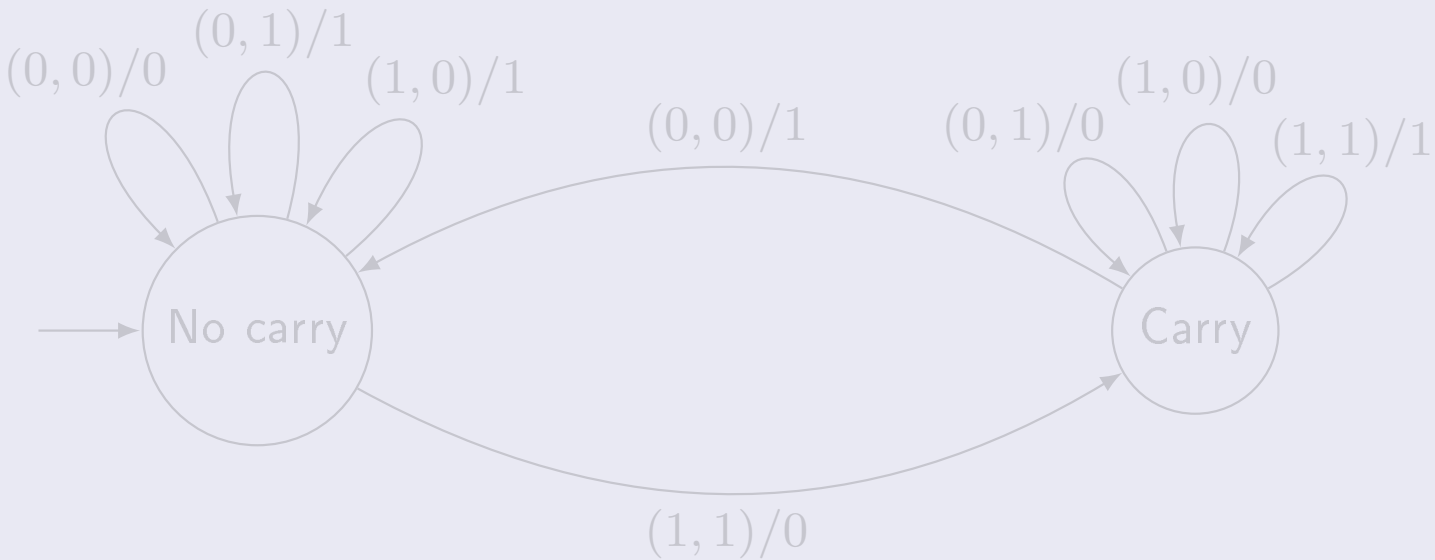

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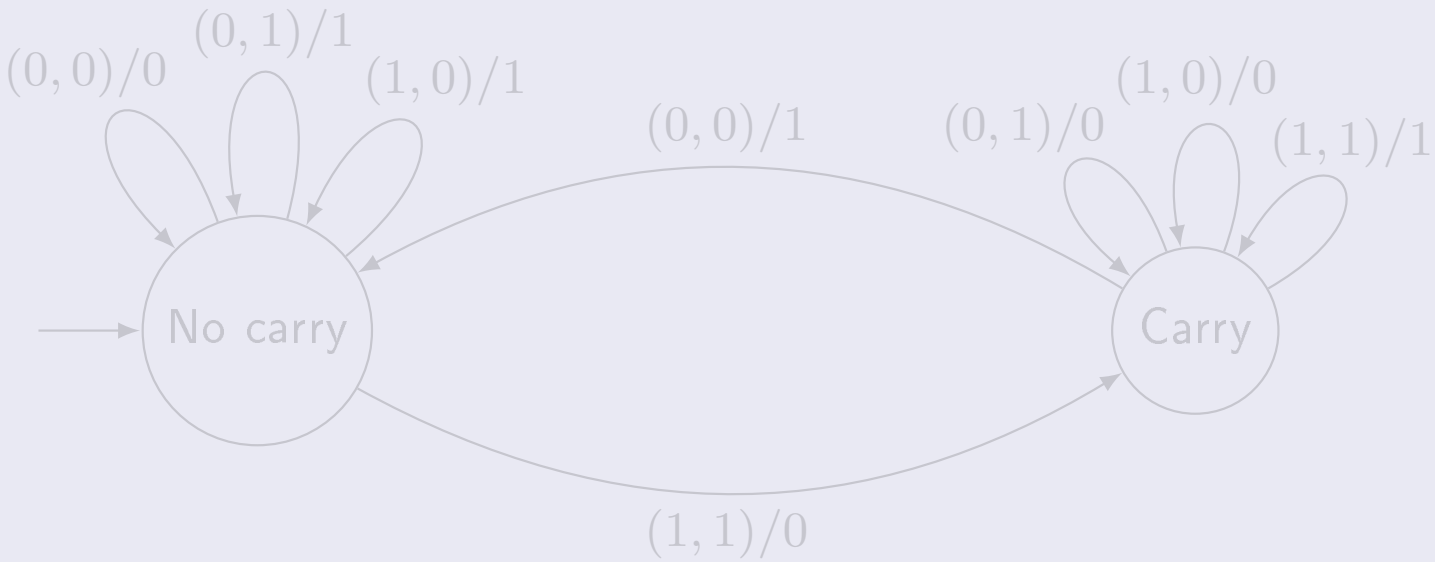

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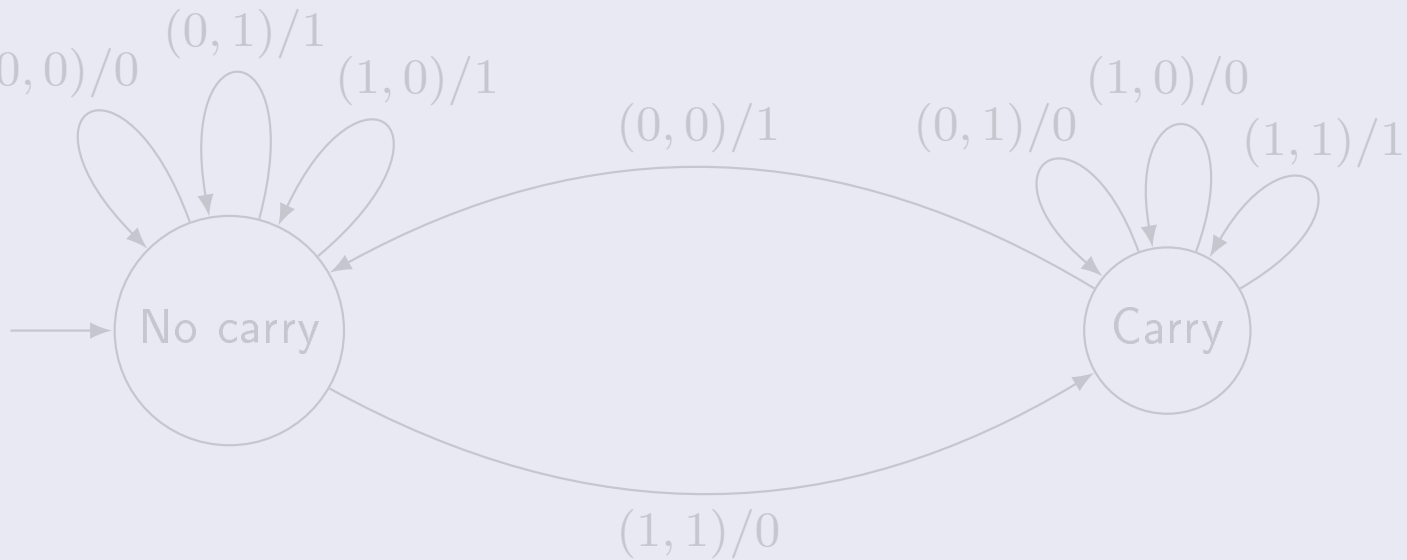

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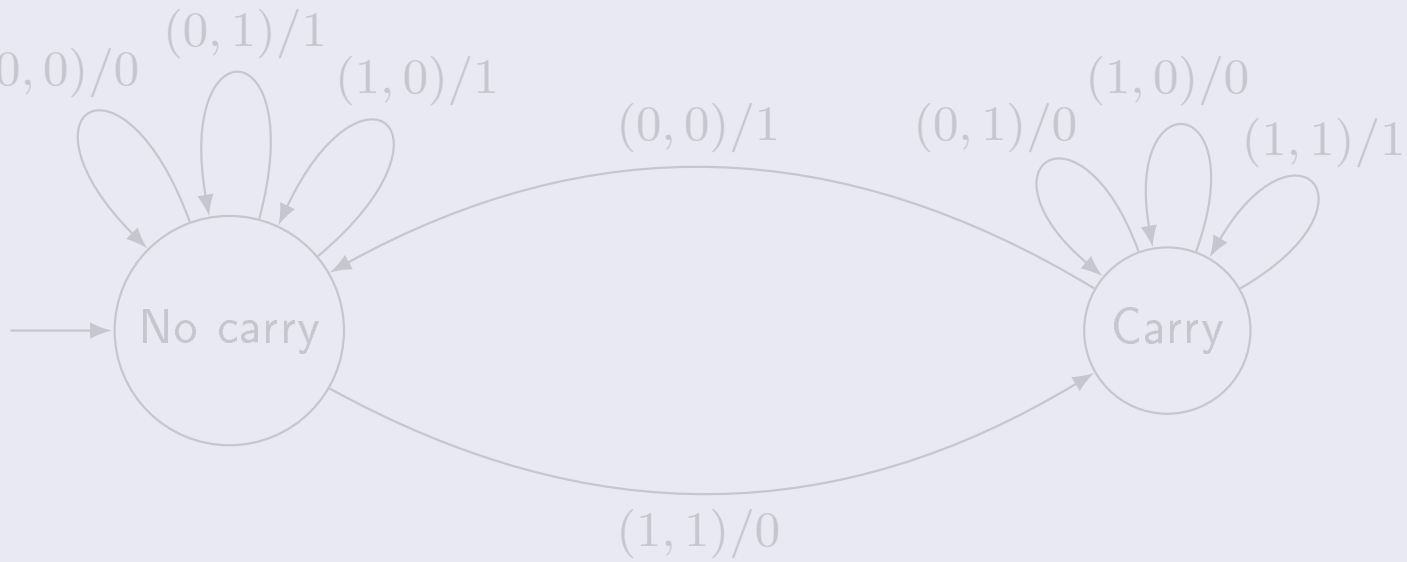

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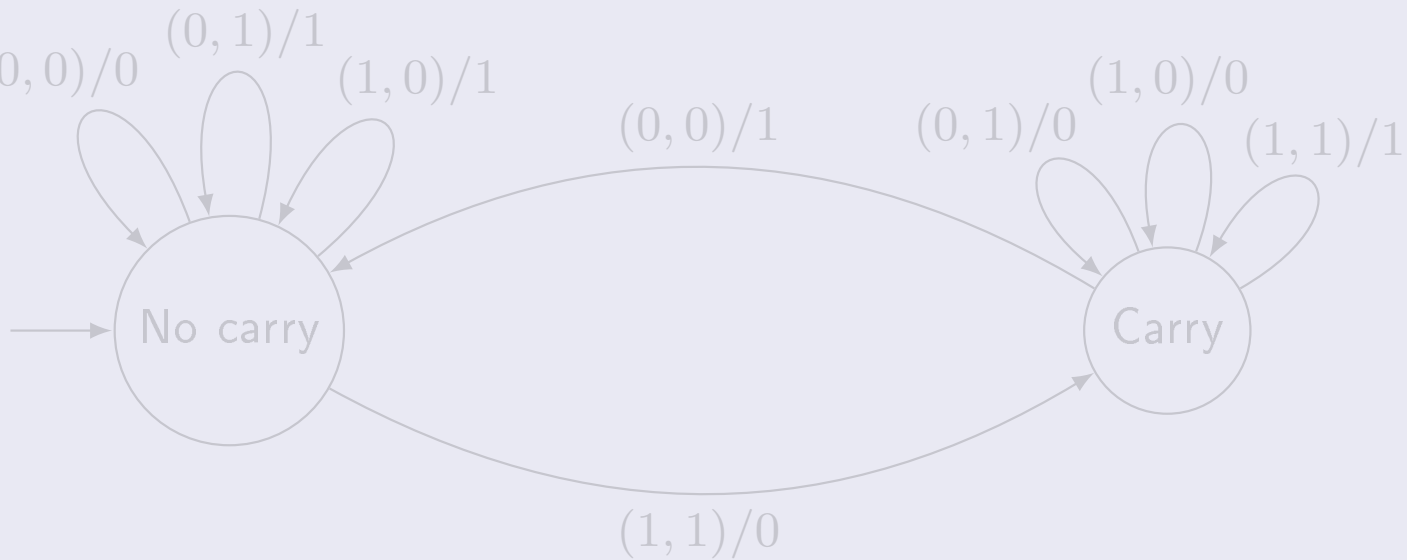

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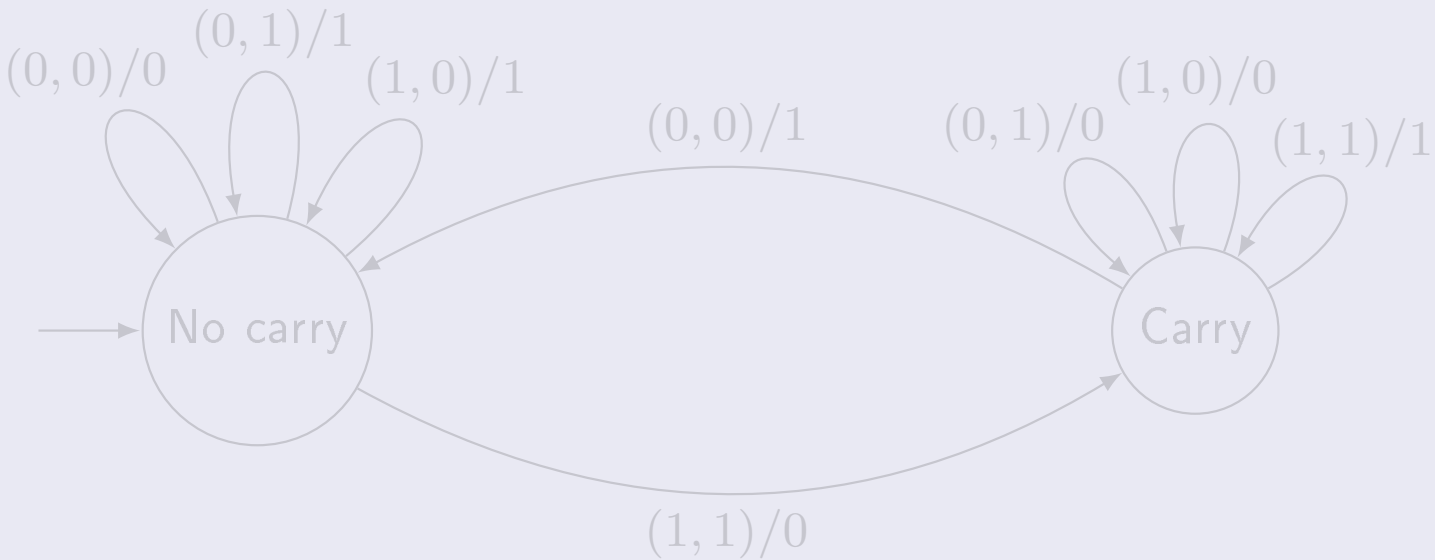

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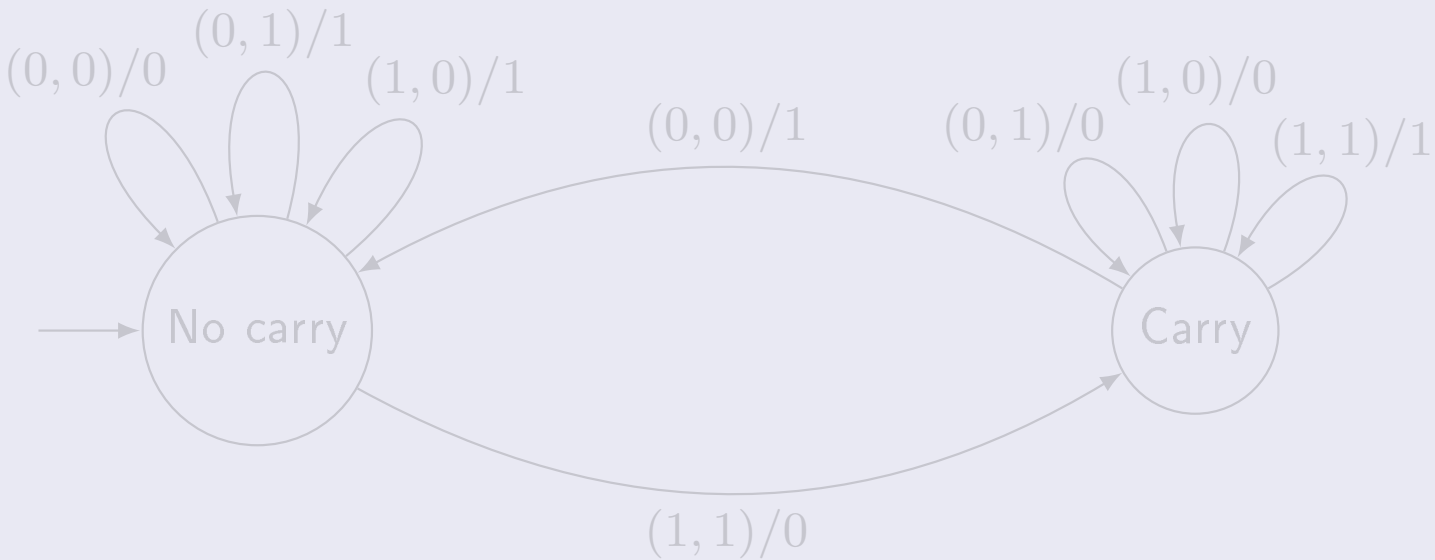

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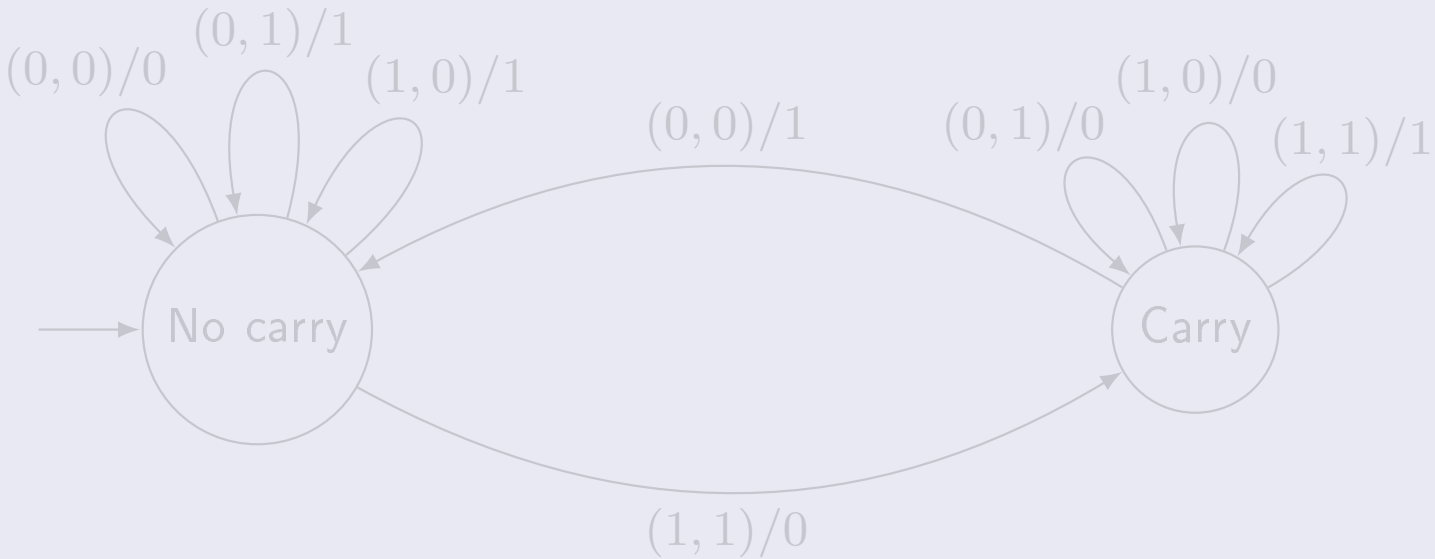

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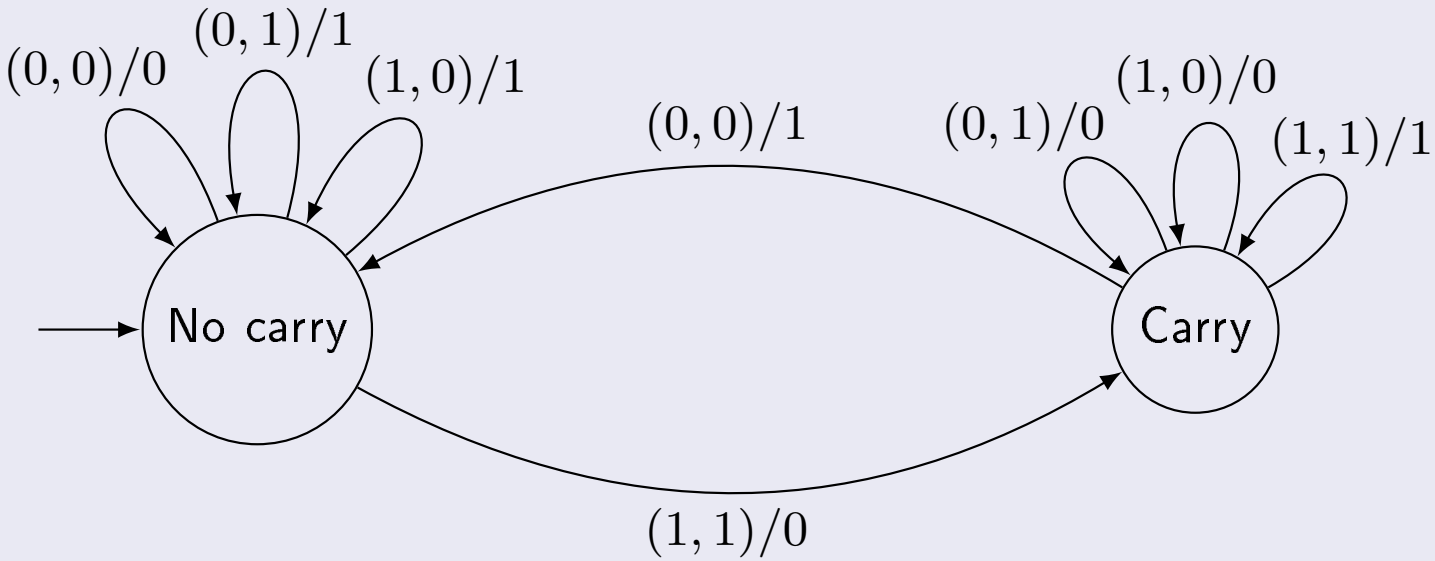

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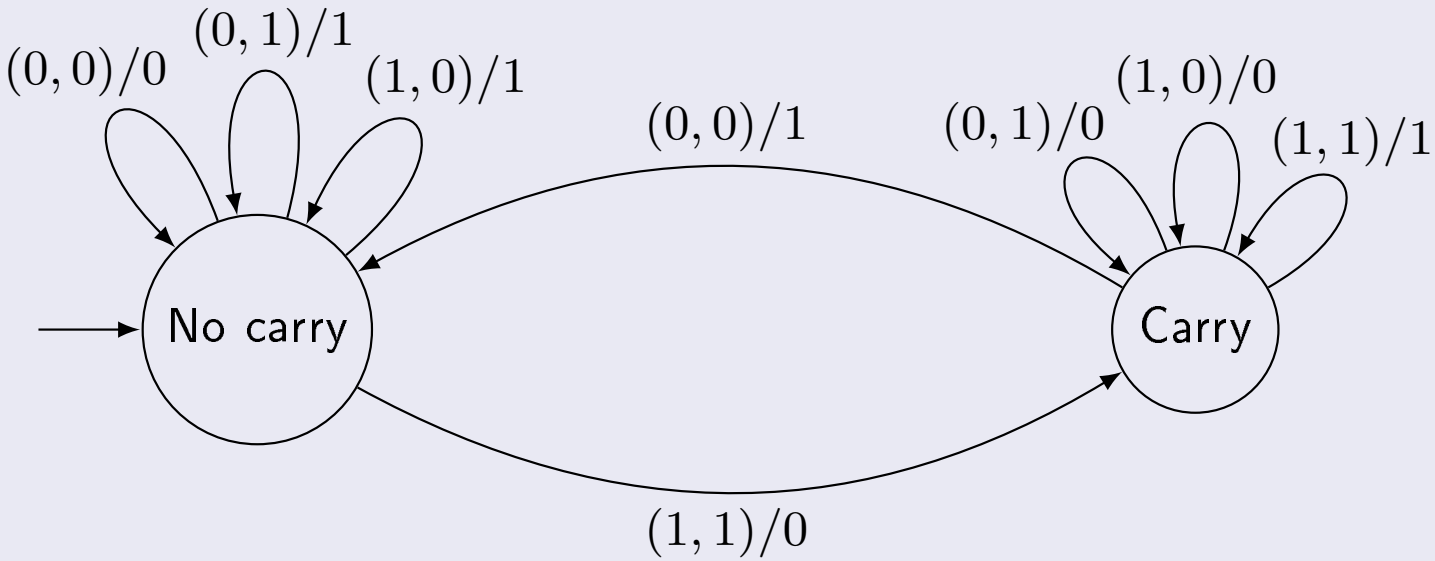

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