Formal Languages, Automata and Codes

Oleg Gutik

Lecture 2

2 THREE BASIC CONCEPTS

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The *length* of a string w, denoted by $|w|$, is the number of symbols in the string. We shall frequently need to refer to the *empty string*, which is a string with no symbols at all. It will be denoted by λ. The following simple relations

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$$|\lambda| = 0,$$

$$\lambda w = w\lambda = w$$

hold for all string w.

The concatenation of two strings w and v is the string obtained by appending the symbols of v to the right end of w, that is, if

$$w = a_1 a_2 \cdots a_n$$

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$$w = vu,$$

then the substrings v and u are said to be a *prefix* and a *suffix* of w, respectively. For example, if $w = abbab$, then

$$\{\lambda, a, ab, abb, abba, abbab\}$$

is the set of all prefixes of w, while bab, ab, b are some of its suffixes. Simple properties of strings, such as their length, are very intuitive and probably need little elaboration. For example, if u and v are strings, then the length of their concatenation is the sum of the individual lengths, that is,

$$|uv| = |u| + |v|. \quad (1)$$

But although this relationship is obvious, it is useful to be able to make it precise and prove it. The techniques for doing so are important in more complicated situations.

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Example 1.8

Show that formula

$$|uv| = |u| + |v|. \quad (1)$$

holds for any strings u and v. To prove this, we first need a definition of the length of a string. We make such a definition in a recursive fashion by

$$|a| = 1,$$
$$|wa| = |w| + 1,$$

for all $a \in \Sigma$ and w any string on Σ. This definition is a formal statement of our intuitive understanding of the length of a string: The length of a single symbol is one, and the length of any string is increased by one if we add another symbol to it. With this formal definition, we are ready to prove equality (1) by induction characters.

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Example 1.8 (continuation)

By definition, the equality

$$|uv| = |u| + |v|. \tag{1}$$

holds for all strings u of any length and all strings v of length 1, so we have a basis. As an inductive assumption, we take that equality (1) holds for all strings u of any length and all strings v of length $1, 2, \ldots, n$. Now take any string v of length $n+1$ and write it as $v = wa$. Then,

$$|v| = |w| + 1,$$
$$|uv| = |uwa| = |uw| + 1.$$

By the inductive hypothesis (which is applicable since w is of length n),

$$|uw| = |u| + |w|$$

so that

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Therefore, equality (1) holds for all u and all v of length up to $n+1$, completing the inductive step and the argument. ∎

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Therefore, equality (1) holds for all u and all v of length up to $n+1$, completing the inductive step and the argument. ∎

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If w is a string, then w^n stands for the string obtained by repeating w n times. As a special case, we define

$$w^0 = \lambda,$$

for all strings w.

If Σ is an alphabet, then we use Σ^* to denote the set of strings obtained by concatenating zero or more symbols from Σ. The set Σ^* always contains the empty string λ. To exclude the empty string, we define

$$\Sigma^+ = \Sigma^* - \{\lambda\}.$$

While Σ is finite by assumption, Σ^* and Σ^+ are always infinite because there is no limit on the length of the strings in these sets. A *language* is defined very generally as a subset of Σ^*. A string in a language L will be called a *sentence* of L. This definition is quite broad; any set of strings on an alphabet Σ can be considered as a language. Later we will study methods by which specific languages can be defined and described; this will enable us to give some structure to this rather broad concept. For the moment, though, we shall just look at a few specific examples.

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Example 1.9

Let $\Sigma = \{a, b\}$. Then

$$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, \ldots\}.$$

The set

$$\{a, aa, aab\}$$

is a language on Σ. Since it has a finite number of sentences, we call it a *finite language*. The set

$$L = \{a^n b^n : n \geq 0\}$$

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Let $\Sigma = \{a, b\}$. Then

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$$L^* = L^0 \cup L_1 \cup L_2 \cup \cdots$$

and the *positive closure* of L as

$$L^+ = L^1 \cup L_2 \cup L_3 \cup \cdots .$$

Example 1.10

If

$$L = \{a^n b^n : n \geq 0\},$$

then

$$L^2 = \{a^n b^n a^m b^m : n \geq 0, m \geq 0\}.$$

Note that n and m in the above are unrelated; the string $aabbaaabbb$ is in L^2.

The reverse of L is easily described in set notation as

$$L^R = \{b^n a^n : n \geq 0\},$$

but it is considerably harder to describe $\overline{L}$ or L^* this way. A few tries will quickly convince you of the limitation of set notation for the specification of complicated languages.

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then

$$L^2 = \{a^n b^n a^m b^m : n \geq 0, m \geq 0\}.$$

Note that n and m in the above are unrelated; the string $aabbaaabbb$ is in L^2.

The reverse of L is easily described in set notation as

$$L^R = \{b^n a^n : n \geq 0\},$$

but it is considerably harder to describe $\overline{L}$ or L^* this way. A few tries will quickly convince you of the limitation of set notation for the specification of complicated languages.

Finally, we define the *star-closure of a language* L as

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This example illustrates the definition of a general concept in terms of simple ones. We start with the top-level concept, here $\langle \text{sentence} \rangle$, and successively reduce it to the irreducible building blocks of the language. The generalization of these ideas leads us to formal grammars.

Definition 1.1

A *grammar* G is defined as a quadruple

$$G = (V, T, S, P),$$

where

- V is a finite set of objects called *variables*,
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It will be assumed without further mention that the sets V and T are nonempty and disjoint.

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The production rules are the heart of a grammar; they specify how the grammar transforms one string into another, and through this they define a language associated with the grammar. In our discussion we will assume that all production rules are of the form

$$x \to y,$$

where x is an element of $(V \cup T)^+$ and y is in $(V \cup T)^*$. The productions are applied in the following manner: Given a string w of the form

$$w = uxv,$$

we say the production $x \to y$ is applicable to this string, and we may use it to replace x with y, thereby obtaining a new string

$$z = uyv.$$

This is written as

$$w \Rightarrow z.$$

We say that w *derives* z or that z is *derived* from w. Successive strings are derived by applying the productions of the grammar in arbitrary order. A production can be used whenever it is applicable, and it can be applied as often as desired. If

$$w_1 \Rightarrow w_2 \Rightarrow \cdots \Rightarrow w_n,$$

we say that w_1 *derives* w_n and write

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The star $*$ indicates that an unspecified number of steps (including zero) can be taken to derive w_n from w_1.

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By applying the production rules in a different order, a given grammar can normally generate many strings. The set of all such terminal strings is the language defined or generated by the grammar.

Definition 1.2

Let $G = (V, T, S, P)$ be a grammar. Then the set

$$L(G) = \left\{ w \in T^* : S \stackrel{*}{\Rightarrow} w \right\}$$

is the *language generated* by G.

If $w \in L(G)$ then the sequence

$$S \Rightarrow w_1 \Rightarrow w_2 \Rightarrow \cdots \Rightarrow w_n \Rightarrow w$$

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$$L(G) = \left\{ w \in T^* : S \overset{*}{\Rightarrow} w \right\}$$

is the *language generated* by G.

If $w \in L(G)$ then the sequence

$$S \Rightarrow w_1 \Rightarrow w_2 \Rightarrow \cdots \Rightarrow w_n \Rightarrow w$$

is a *derivation* of the sentence w. The strings $S, w_1, w_2, \ldots, w_n$, which contain variables as well as terminals, are called *sentential forms* of the derivation.

Example 1.11

Consider the grammar

$$G = (\{S\}, \{a, b\}, S, P),$$

with P given by

$$S \to aSb,$$
$$S \to \lambda.$$

Then

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb,$$

so we can write

$$S \overset{*}{\Rightarrow} aabb.$$

The string $aabb$ is a sentence in the language generated by G, while the string $aaSbb$ is a sentential form.

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A grammar G completely defines $L(G)$, but it may not be easy to get a very explicit description of the language from the grammar. Here, however, the answer is fairly clear. It is not hard to conjecture that

$$L(G) = \{a^n b^n : n \geq 0\},$$

and it is easy to prove it. If we notice that the rule $S \to aSb$ is recursive, a proof by induction readily suggests itself. We first show that all sentential forms must have the form

$$w_i = a^i S b^i. \tag{2}$$

Suppose that condition (2) holds for all sentential forms w_i of length $2i+1$ or less. To get another sentential form (which is not a sentence), we can only apply the production $S \to aSb$. This gets us

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The idea behind the previous example can be extended to this case. All we need to do is generate an extra b. This can be done with a production $S \to Ab$, with other productions chosen so that A can derive the language in the previous example. Reasoning in this fashion, we get the grammar $G = (\{S, A\}, \{a, b\}, S, P)$, with productions

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Derive a few specific sentences to convince yourself that this works. ■

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$$L = \left\{a^n b^{n+1} : n \geq 0\right\}.$$

The idea behind the previous example can be extended to this case. All we need to do is generate an extra b. This can be done with a production $S \to Ab$, with other productions chosen so that A can derive the language in the previous example. Reasoning in this fashion, we get the grammar $G = (\{S, A\}, \{a, b\}, S, P)$, with productions

$$S \to Ab,$$
$$A \to aAb,$$
$$A \to \lambda.$$

Derive a few specific sentences to convince yourself that this works. ■

The previous examples are fairly easy ones, so rigorous arguments may seem superfluous. But often it is not so easy to find a grammar for a language described in an informal way or to give an intuitive characterization of the language defined by a grammar. To show that a given language is indeed generated by a certain grammar G, we must be able to show (a) that every $w \in L$ can be derived from S using G and (b) that every string so derived is in L.

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Example 1.13

Take $\Sigma = \{a, b\}$, and let $n_a(w)$ and $n_b(w)$ denote the number of a's and b's in the string w, respectively. Then the grammar G with productions

$$S \to SS,$$
$$S \to \lambda,$$
$$S \to aSb,$$
$$S \to bSa$$

generates the language

$$L = \{w : n_a(w) = n_b(w)\}.$$

This claim is not so obvious, and we need to provide convincing arguments.

First, it is clear that every sentential form of G has an equal number of a's and b's, because the only productions that generate the string a, namely $S \to aSb$ and $S \to bSa$, simultaneously generate the string b. Therefore, every element of $L(G)$ is in L. It is a little harder to see that every string in L can be derived with G.

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Example 1.13 (continuation)

Let us begin by looking at the problem in outline, considering the various forms $w \in L$ can have. Suppose w starts with a and ends with b. Then it has the form

$$w = aw_1b,$$

where w_1 is also in L. We can think of this case as being derived starting with

$$S \Rightarrow aSb$$

if S does indeed derive any string in L. A similar argument can be made if w starts with b and ends with a. But this does not take care of all cases, because a string in L can begin and end with the same symbol. If we write down a string of this type, say $aabbba$, we see that it can be considered as the concatenation of two shorter strings $aabb$ and ba, both of which are in L. Is this true in general? To show that this is indeed so, we can use the following argument: Suppose that, starting at the left end of the string, we count $+1$ for the string a and -1 for the string b. If a string w starts and ends with a, then the count will be $+1$ after the leftmost symbol and -1 immediately before the rightmost one. Therefore, the count has to go through zero somewhere in the middle of the string, indicating that such a string must have the form

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As we will see later, it is not always easy to see if two grammars are equivalent.

Example 1.14

Consider the grammar $G_1 = (\{A, S\}, \{a, b\}, S, P_1)$, with P_1 consisting of the productions

$$S \to aAb|\lambda,$$

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Here we introduce a convenient shorthand notation in which several production rules with the same left-hand sides are written on the same line, with alternative right-hand sides separated by the symbol $|$. In this notation $S \to aAb|\lambda$ stands for the two productions $S \to aAb$ and $S \to \lambda$. This grammar is equivalent to the grammar G in Example 1.11:

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2 THREE BASIC CONCEPTS: Grammars

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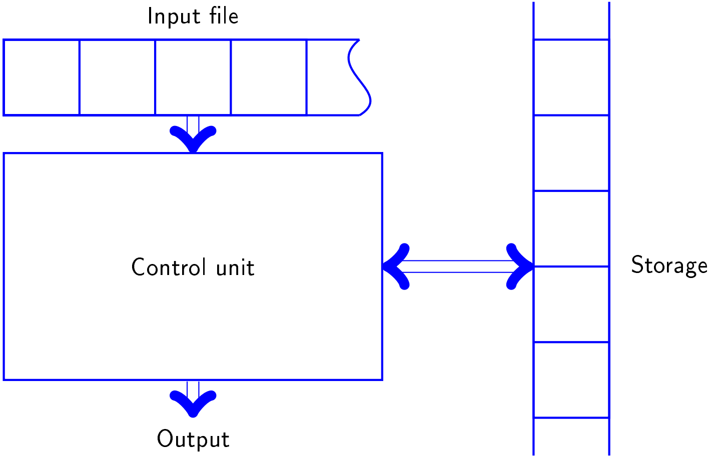

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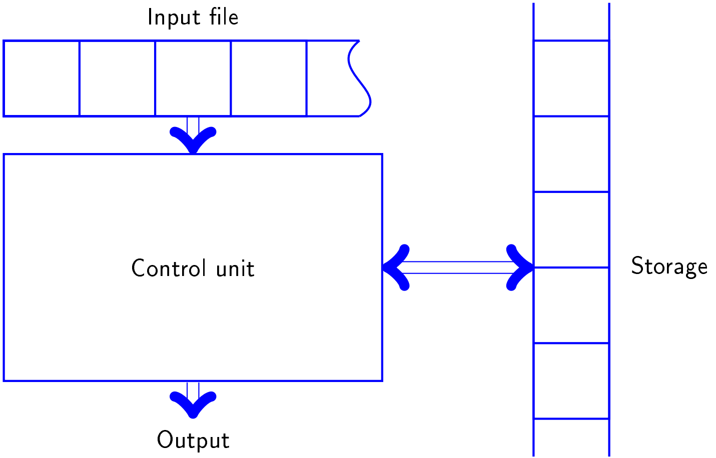

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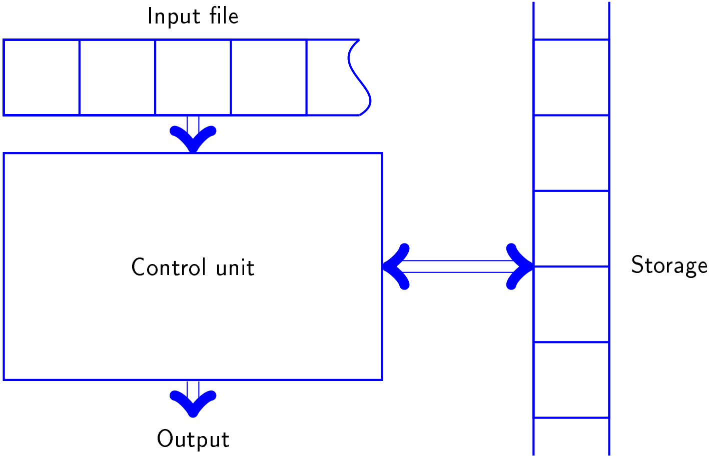

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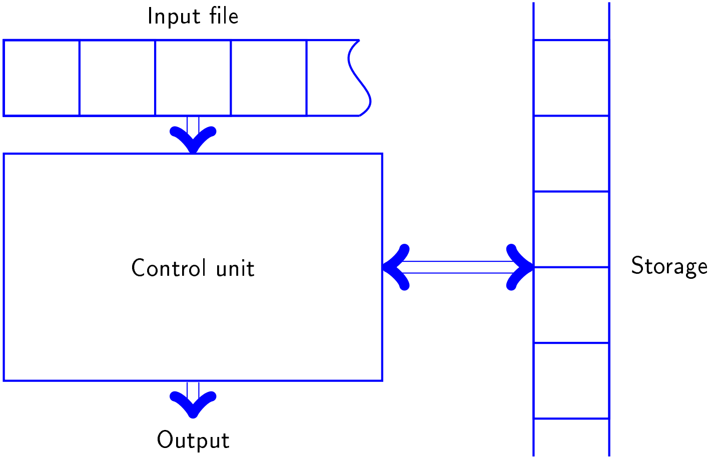

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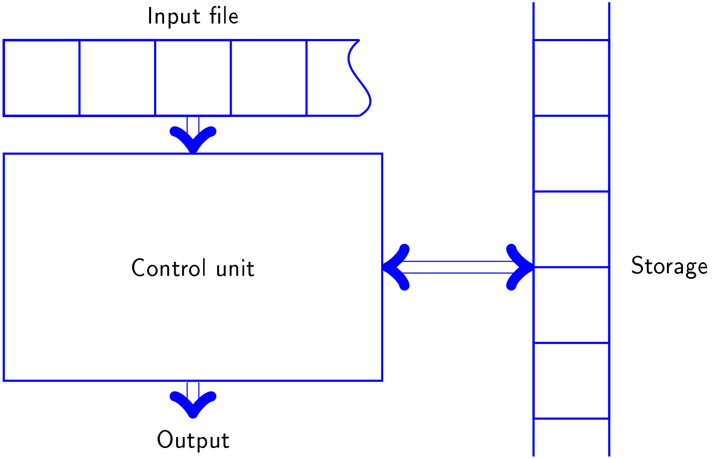

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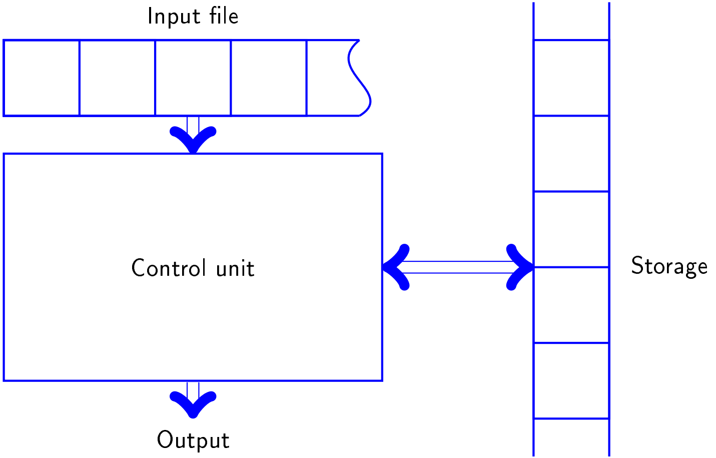

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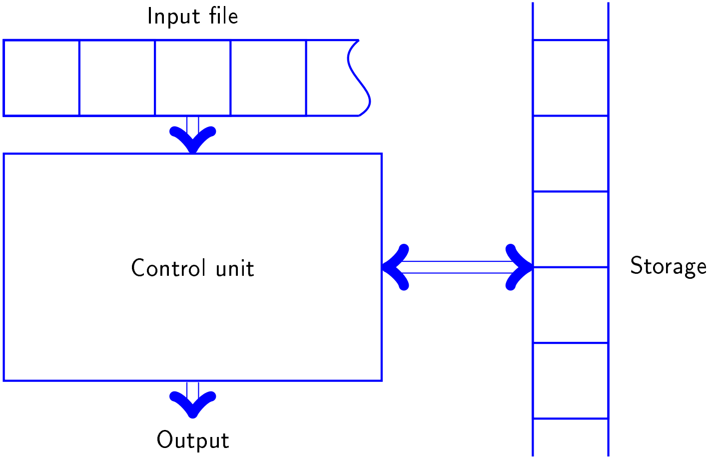

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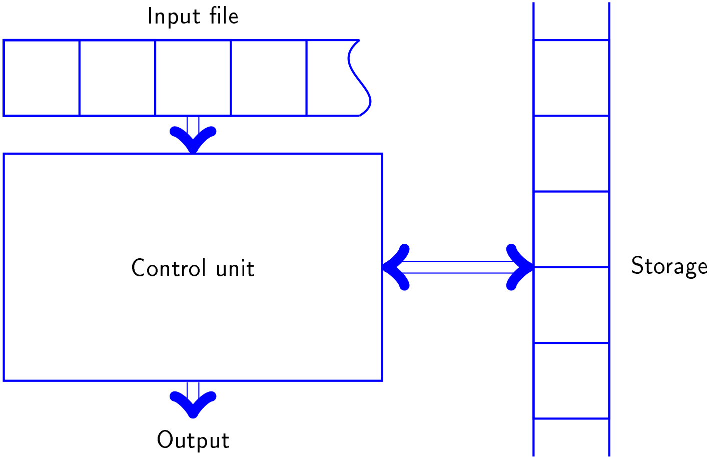

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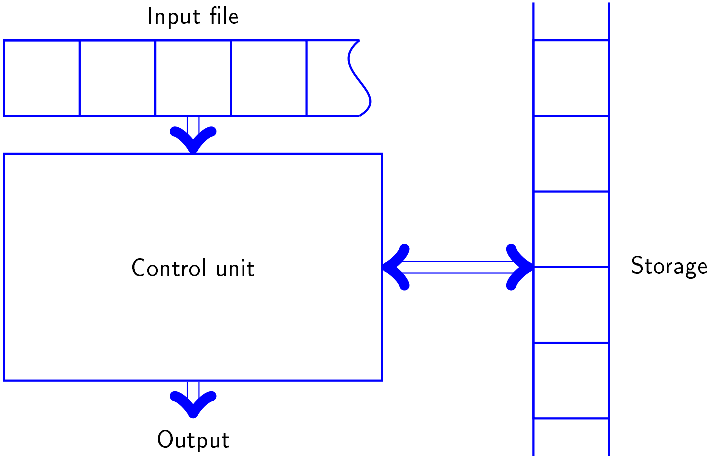

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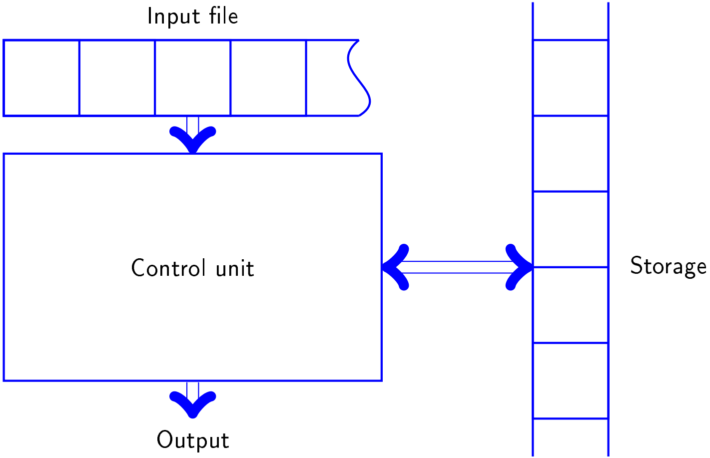

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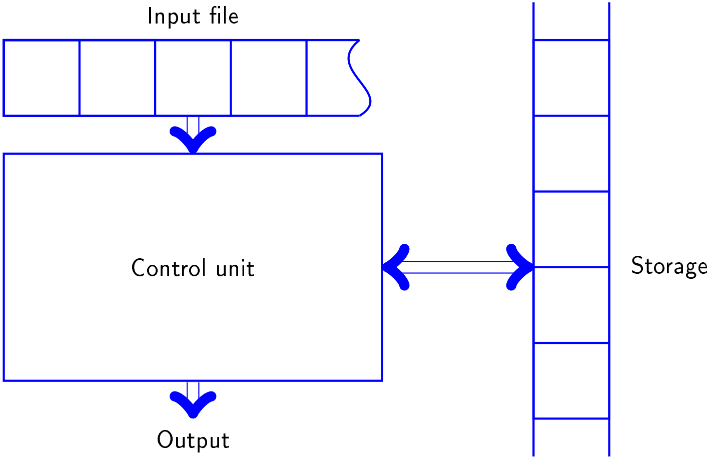

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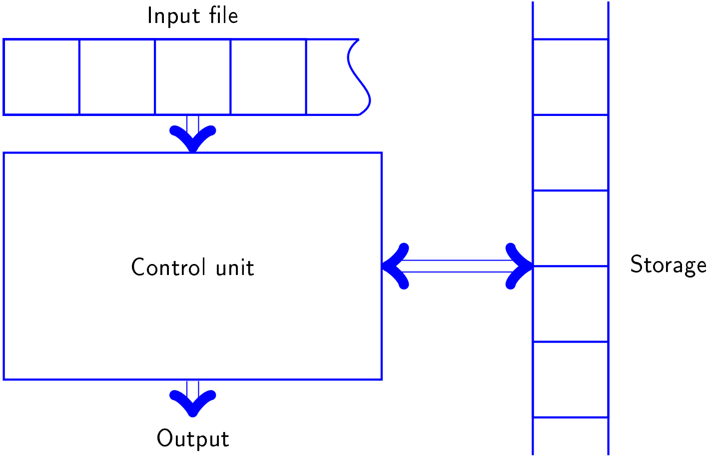

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Thank You for attention!