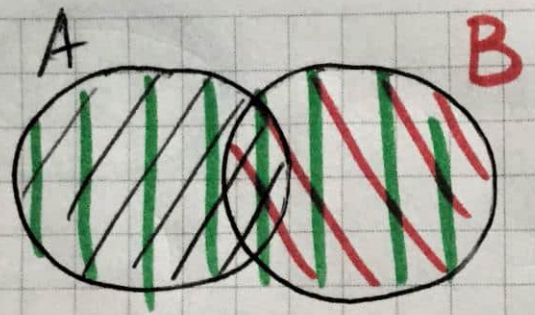
Операції над множинами. Відображення: образ і прообраз.

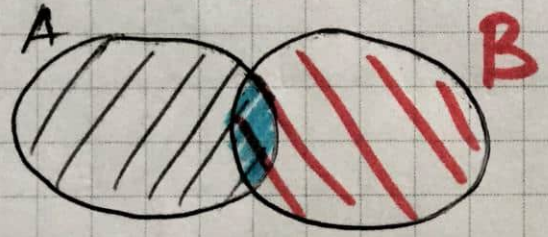
Практичне заняття 2



$A \cup B$

$A \cup B = \{x : (x \in A) \vee (x \in B)\}$

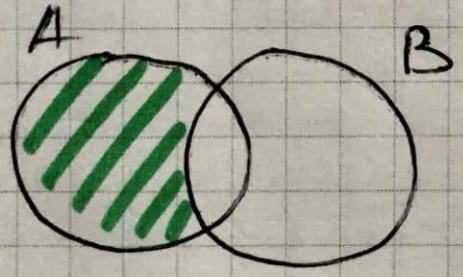
об'єднання



$A \cap B$

$A \cap B = \{x : (x \in A) \wedge (x \in B)\}$

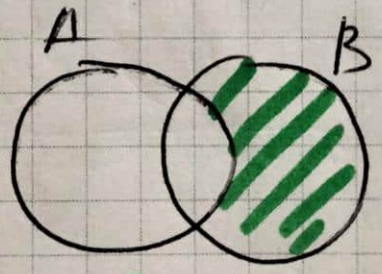
перетин



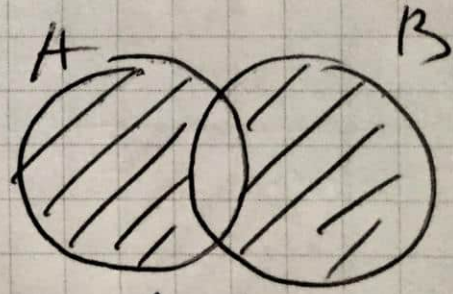
$A \setminus B$

$A \setminus B = \{x : (x \in A) \wedge (x \notin B)\}$

"без"



$B \setminus A$



$A \triangle B$

симетр. різниця

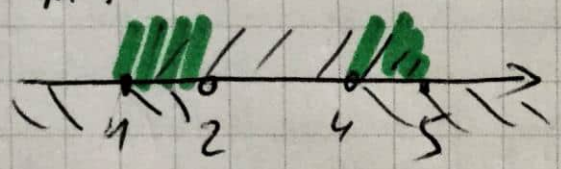
$A \triangle B = (A \setminus B) \cup (B \setminus A)$

1.60 $A = \{x \in \mathbb{R}: 1 < |x-3| \leq 2\}$ $B = \{x \in \mathbb{R}: 2|x| < 3\}$

$A: \begin{cases} |x-3| \leq 2 \\ |x-3| > 1 \end{cases} \quad \begin{cases} -2 \leq x-3 \leq 2 \\ \left[\begin{array}{l} x-3 > 1 \\ x-3 < -1 \end{array}\right. \end{cases} \quad \begin{cases} 1 \leq x \leq 5 \\ \left[\begin{array}{l} x > 4 \\ x < 2 \end{array}\right. \end{cases}$

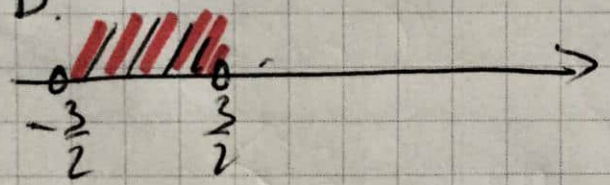
$x \in [1; 2) \cup (4; 5]$

A:



B: $|x| < \frac{3}{2} \quad -\frac{3}{2} < x < \frac{3}{2}$

B:

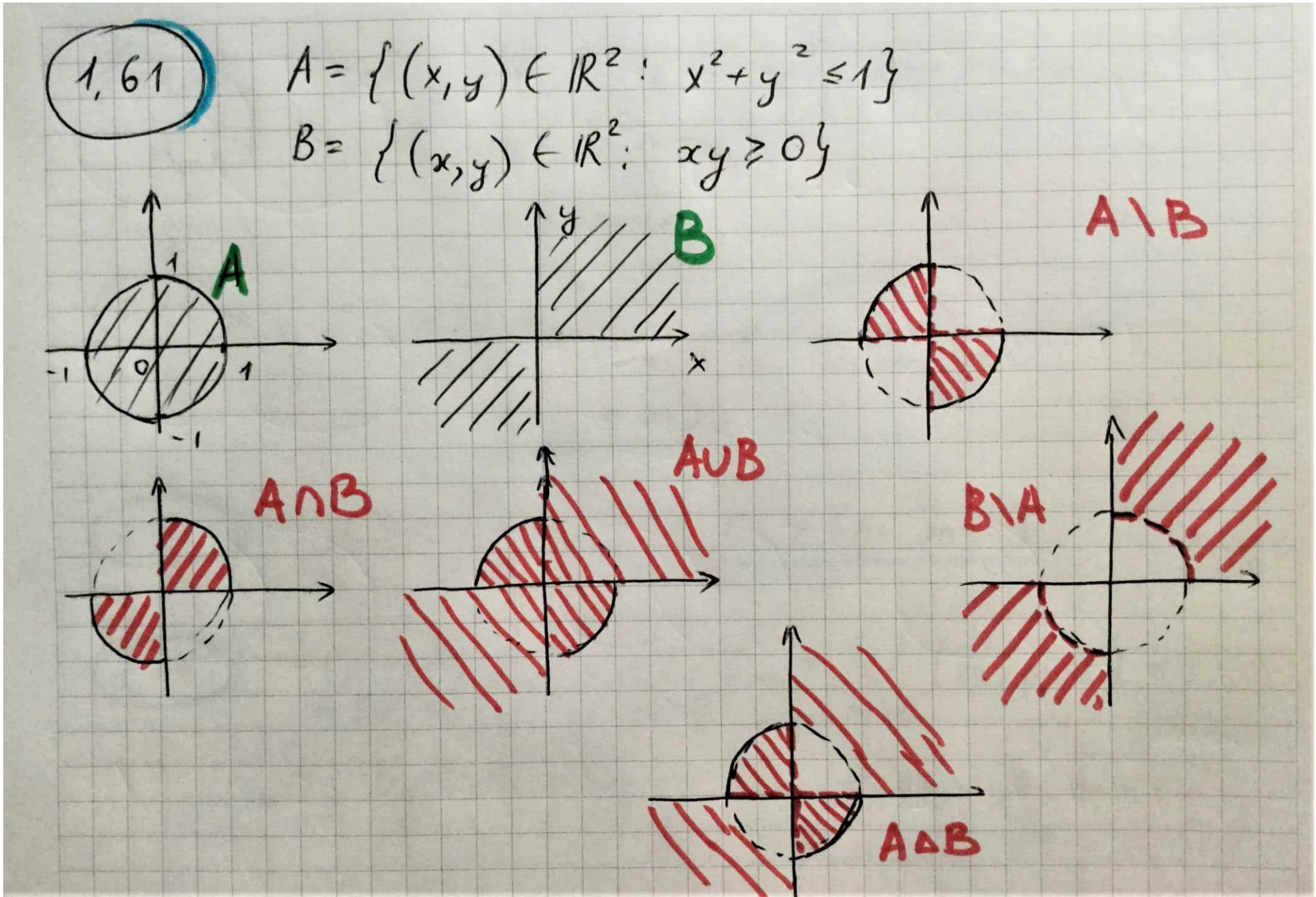


$A \cup B = \left(-\frac{3}{2}; 2\right) \cup (4; 5]$

$A \cap B = [1; 1{,}5)$

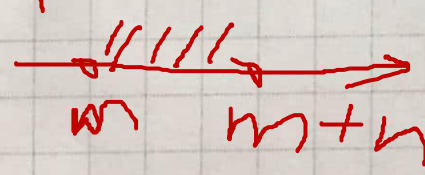
$A \setminus B = [1{,}5; 2) \cup (4; 5]$

$B \setminus A = (-1{,}5; 1)$ $\quad A \triangle B = (-1{,}5; 1) \cup [1{,}5; 2) \cup (4; 5]$


1,61
$A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$
$B = \{(x,y) \in \mathbb{R}^2 : xy \geq 0\}$
A
B
A \ B
A ∩ B
A ∪ B
B \ A
A △ B

Нехай для $m \in \mathbb{Z}$ і $n \in \mathbb{N}$ множина

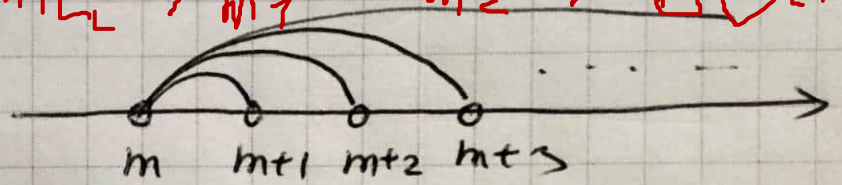
$$A_{mn} = \{x \in \mathbb{R} : m < x < m+n\}.$$



Визначити множини:

1.74 $B_m = \bigcup_{n=1}^{\infty} A_{mn}$

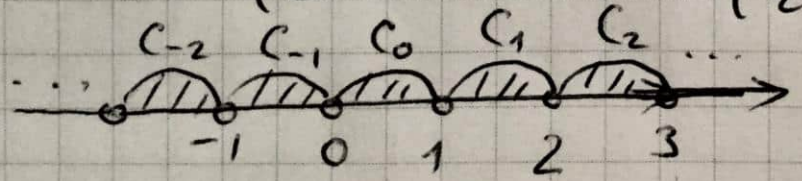
$\bigcup_{n=1}^{\infty} A_{mn} = A_{m1} \cup A_{m2} \cup A_{m3} \cup \dots$



$B_m = (m; +\infty)$

1.75 $C_m = \bigcap_{n=1}^{\infty} A_{mn}$ $\quad C_m = (m, m+1)$

1.79 $\bigcup_{m \in \mathbb{Z}} \bigcap_{n=1}^{\infty} A_{mn} = \bigcup_{m \in \mathbb{Z}} C_m = \bigcup_{m \in \mathbb{Z}} (m, m+1) =$



$= \mathbb{R} \setminus \mathbb{Z}$

1.64 Визначити множину A, якщо

$$A = \{x \in \mathbb{R} : \exists y \in \mathbb{R},\ |x| + |y| = 1\}$$

$|y| = 1 - |x|$ р-ння має роз-зон відносно змінної y ($\exists y \in \mathbb{R}$), якщо

$$1 - |x| \geq 0$$

тобто $|x| \leq 1$

$$-1 \leq x \leq 1$$

$$A = [-1; 1]$$

1.80 Функцію $f: \mathbb{Z} \to \mathbb{N}$ задано співвідношенням $f(n) = 1 + n^2$. Визначити:

1) $f(0) = 1$

$f(\{0\}) = \{1\}$

$f(\{-2; 2\}) = \{5\}$

2) $f^{-1}(\{1\})$ $\quad 1 + n^2 = 1 \Rightarrow n = 0$

$\underline{f^{-1}(\{1\}) = \{0\}}$

$f^{-1}(\{10, 17\})$ $\quad 1 + n^2 = 10 \Rightarrow n^2 = 9 \Rightarrow n = \pm 3,$ бо $n \in \mathbb{Z}$

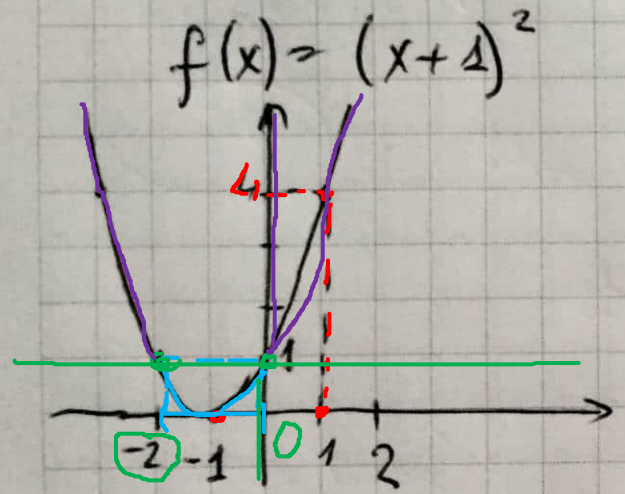
$1 + n^2 = 17 \Rightarrow n^2 = 16 \Rightarrow n = \pm 4,$

$f^{-1}(\{10, 17\}) = \{-4; -3; 3; 4\}.$

1.81 Для функції $f(x) = x^2 + 2x + 1$ визначити:

а) $f(\{-1; 1\}) = \{0, 4\}$

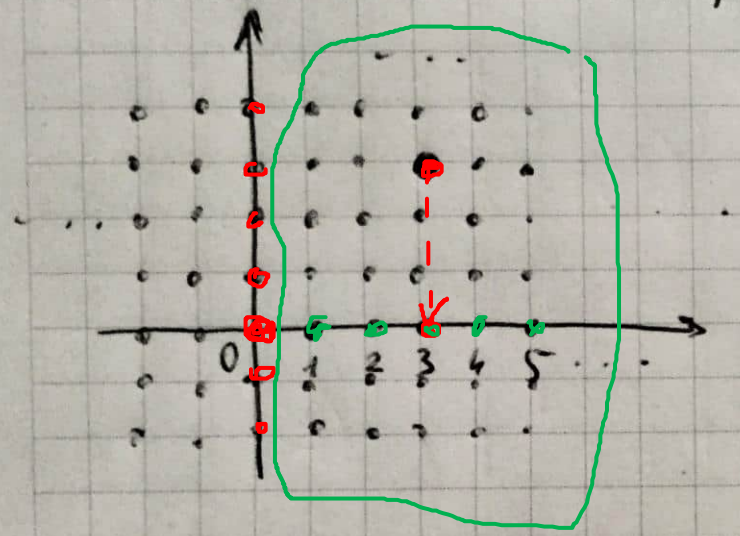
$f((-2; 0]) = [0; 1]$



б) $f^{-1}(\{1\}) = \{-2, 0\}$

$f^{-1}([1; +\infty)) = (-\infty; -2] \cup [0; +\infty)$

1.82 Функцію $f: \mathbb{Z}^2 \to \mathbb{Z}^2$ задано співвідношенням $f((m;n)) = (m;0)$.



а) $f(A) = \{(0;0)\}$
де $A = \{(0;n) : n \in \mathbb{Z}\}$

б) $f^{-1}(C) = \{(m;n) : m \in \mathbb{N}, n \in \mathbb{Z}\}$
де $C = \{(m;0) : m \in \mathbb{N}\}$

Домашнє завдання:
1.59-1.66, 1.76-1.78,
1.80-1.82, 1.89